ON-LINE SUPERVISED ADJUSTMENT OF THE CORRECTING GAINS OF FRACTIONAL ORDER HOLDS

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Abstract: A discrete control using different possible discretization models of a continuous plant is presented. The different models of the scheme are obtained from a set of different discretizations of a continuous transfer function under a fractional-order-hold of correcting gain $\beta \in [-1,1]$ (β-FROH). The objective is to design a supervisory scheme which is able to find the most appropriate value for the gain β in an intelligent design framework. A tracking performance index evaluates each possible discretization and the scheme chooses the one with the lowest value. Two different methods of adjusting this value are presented and discussed. The first one selects it among a fixed set of possible values, while in the second one the value of β can be updated by adding or subtracting a small quantity. Simulations are presented to show the usefulness of the scheme.

1 INTRODUCTION

This paper deals with the problem of controlling a known continuous plant using an appropriate discrete controller selected from a group of possible ones. Each possible controller is associated to a different discretization of the plant (Alonso-Quesada and De la Sen, 2004; Broeser, 1995; Ibeas et al. 2002; Middleton et al. 1988; Narendra and Balakrishnan, 1995 and 1998). A fractional order hold is used in order to generate the continuous input from the discrete signal. The choice of the β gain of the FROH device should be taken into account, since the discretization zeros depend on its election (Åström and Wittenmark, 1984; Bárcena et al., 2000; Ishitobi, 1996). In order to find the most appropriate discretization of the continuous plant, two different methods are proposed. In the first one, a set of fixed values of β are used to generate a group of discretization models. Obviously, only one value of β can be used at each time, but since the plant is supposed to be known we can simulate the behavior of all the possible discretization. A tracking index evaluates the performance of all of them and the system uses the one with the lowest value to implement the FROH device and the control law. In the second method, we only allow the system to update the value of β to a close one. The behavior of the current discretization is compared with the behavior of other two possible discretizations. One model using a gain β being slightly larger and the other one using a β being a bit smaller. The second method has a slower convergence to an optimum value of β, but it avoids the transient in the output which may occurred when β is changed a large value. Some computer simulations about two practical cases will be given in order to highlight the usefulness of the proposed multi-model scheme. In the first case, a DC motor is simulated. In the second one, we deal with a LCR parallel circuit (Tank circuit).

2 PROBLEM DESCRIPTION

In this paper, we are dealing with the problem of controlling a continuous plant by using a set of discrete controllers. Each of those controllers is associated to a different discretization of the continuous plant under a fractional order hold (FROH) with a correcting gain β. The plant continuous input signal obtained from a β-FROH follows this equation,

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$$u(t) = u_k + \beta\left[\frac{u_k - u_{k-1}}{T_s}\right](t - kT_s) \qquad (1)$$

for $kT_s \le t < (k+1)T_s$, where T_s is the sampling time, $\beta \in [-1,1]$ and $u_k = u(kT_s)$ is the input signal to the hold at $t = kT_s$ for each integer $k \ge 0$. For $\beta = 0$ and $\beta = 1$ the zero-order-hold (ZOH) and the first-order-hold (FOH) are obtained respectively. The discrete transfer function of a continuous one, $G(s) = \frac{N(s)}{D(s)}$, results to be as follows,

$$H(z) = Z\left[h_\beta(s)\cdot G(s)\right] = Z\left[h_\beta(s)\frac{N(s)}{D(s)}\right] = \frac{B(z)}{A(z)} =$$

$$\frac{b_m z^m + b_{m-1}z^{m-1} + \ldots + b_0}{z^n + a_{n-1}z^{n-1} + \ldots + a_0} \qquad (2)$$

where, $h_\beta(s) = \left(1 - \beta e^{-sT_s} + \frac{\beta(1-e^{-sT_s})}{T_s s}\right)\frac{1-e^{-sT_s}}{s}$ is the transfer function of a β-FROH, Z the Z-transform and the polynomial degrees,

$$n = \deg(A) = \begin{cases}\deg(D) & if \quad \beta = 0 \\ \deg(D)+1 & if \quad \beta \ne 0\end{cases} \quad \text{and}$$

$m = \deg(B)$. Moreover, $n = m+1$ if $\deg(N) < \deg(D)$ or $n = m$ if $\deg(N) = \deg(D)$. Since the use of a ZOH is more common in practice than the use of a FROH, $H(z)$ may be calculated just using ZOH devices in the following way,

$$H(z) = \frac{z-\beta}{z}Z\left[h_0(s)\cdot G(s)\right] + \frac{\beta(z-1)}{T_s z}Z\left[h_0(s)\cdot\frac{G(s)}{s}\right] \qquad (3)$$

where, $h_0(s) = \frac{1-e^{-sT}}{s}$ is the transfer function of a ZOH device. The following standard assumptions are made:

1-It is assumed that both polynomials $N(s)$ and $D(s)$ are known.

2-The reference model, $H_m = B_m / A_m$, is exponentially stable, i.e. all the roots of A_m satisfy $|z| \le 1-\delta$ for some $\delta \in (0,1]$.

3-There exists a known convex and compact subset $D \subseteq \Re^{2n}$ of the parameter space where the real parameter vectors belong to so that for all plant parameterization in D the polynomials A and B are coprime.

2.1 Basic adaptive controller

The transfer function of the reference model is,

$$H_m(z) = \frac{B^-(z)B_m^{'}(z)A_0(z)}{A_m(z)A_0(z)} = \frac{B_m A_0}{A_m A_0} \qquad (4)$$

where $B_m^{'}(z)$ contains the free-design reference model zeros, $B^-(z)$ is formed by the unstable(assumed known) plant zeros and $A_0(z)$ is a polynomial including the eventual closed-loop stable pole-zero cancellations which are introduced when necessary to guarantee that the relative degree of the reference model is non less than that of the closed-loop system so that the synthesized controller is causal. A basic control scheme is displayed in figure 2. Then, we will consider the polynomials R_k, S_k and T (T depends only on the reference model zeros polynomial which is of constant coefficients) where $T = B_m^{'}A_0$ and R_k (monic), S_k are the unique solutions with degrees fulfilling $\deg(R_k) = 2n - i$, $\deg(S_k) = i-1$, $\deg(A_m A_0) = 2n$ of the polynomial diophantine equation

$$A_k R_k + B_k S_k = B_k^+ A_m A_0 \Leftrightarrow A_k R_{1,k} + B^- S_k = A_m A_0 \qquad (5)$$

with $R_k = B_k R_{1,k}$ at every sampling instant. Figure 1 shows the control scheme.

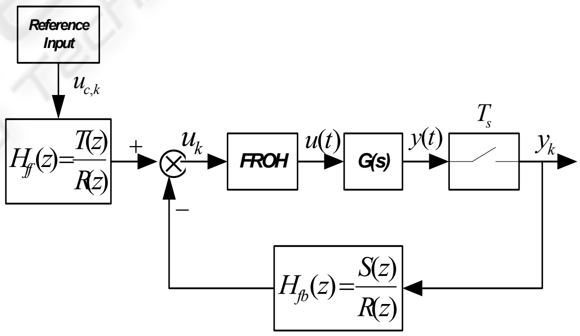

Figure 1: Basic control scheme

From (4)-(5), perfect matching is achieved through the control signal:

$$u_k = \frac{T}{R}u_{c,k} - \frac{S}{R}y_k \qquad (6)$$

2.2 Multimodel scheme A

In order to find the most appropriate value of the gain β, we consider a set of possible design values of it and the corresponding discrete transfer function is obtained,

$$H_\ell(z) = Z\left[h_{\beta^{(\ell)}}(s)\cdot G(s)\right] \text{ for } 1 \le \ell \le n_m \tag{7}$$

where n_m is the number of possible values of β and $\beta^{(\ell)}$ is used to indicate its value for the ℓ^{th} model. Once all the possible discrete transfer function are obtained for the whole set of intended values, we can simulate the performance of all of them when the previous control law is applied and compared their responses with the desired reference output.

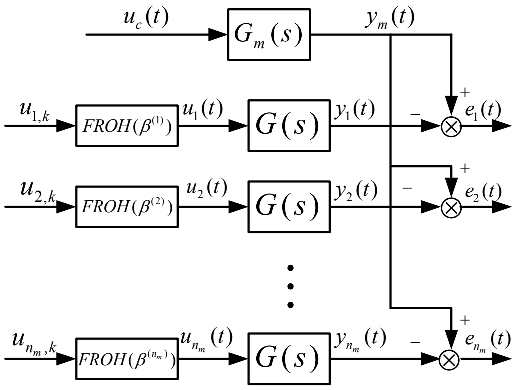

Figure 2: Multimodel scheme

Each plant input is generated by using the control law (6), where the reference transfer function (4), which should be matched, is the corresponding FROH discretization of $G_m(s)$. In other words, we have as many reference transfer functions as the number of possible values of β we are considering.

2.3 Switching rule and identification performance index

The objective of the supervisor is to evaluate the tracking performance of the possible controllers operating on the plant for the given reference model with the aim of choosing the current controller from the set of parallel controllers. The proposed performance index is:

$$J_k^{(\ell)} = \sum_{j=k-M}^{k} \lambda^{k-j} \int_{(j-1)T_s}^{jT_s} \left(y_m(\tau) - y_\ell(\tau)\right) d\tau \tag{8}$$

for $1 \le \ell < n_m$, where, $\lambda \in (0,1]$ and $M > 0$ are design real parameters. λ is a forgetting factor which allows us to give more importance to the last time interval.

2.4 Multimodel scheme B

In this case, instead of letting the system to choose among any of the possible values of β, we only allow the system to change to a close value. The system starts with an arbitrary value and the tracking performance is compared with the tracking performance of two possible close values of β. One being a bit larger and the other one a bit lower. With this method, when we compare their responses, the system can only choose among three possible cases. In other words, n_m is always three. However, these three possible values are not going to be constant and they are updated. If the system chooses one of the two close values of β, then it becomes the active one and the other two are chosen by adding and subtracting a quantity to it. If the system chooses to maintain the same value of β, then the other two possible values are updated as well by considering other two closer values of β. In order to explain this method, the following algorithm describes how it works,

a) At k^{th} sample the active value of β is β_k. Other two values, $\beta_k^{\sup} = \beta_k + \Delta\beta$ and $\beta_k^{\inf} = \beta_k - \Delta\beta$ are used for simulation. Suppose that the last β switching took place at $\bar{k}^{th}$ sample.

b) If $(k+1)T - \bar{k}T \ge MT$, then the tracking performance of the three possible discretizations are compared and the one with the lowest value of (8) is used in the FROH device.

c) If the system chooses to maintain the same value of β, first $\Delta\beta$ is decreased and then $\beta^{\sup}$ and $\beta^{\inf}$ are updated.

- if $\beta_{k+1} = \beta_k$ then $\Delta\beta = \Delta\beta / mf$ (with $mf > 1$) and

$$\beta_{k+1}^{\sup} = \beta_k + \Delta\beta, \quad \beta_{k+1}^{\inf} = \beta_k - \Delta\beta$$

d) If the system chooses another value, $\Delta\beta$ maintains its value and $\beta^{\sup}$ and $\beta^{\inf}$ are calculated by adding and subtracting this value

- if $\beta_{k+1} = \beta_k^{\sup}$ then $\beta_{k+1}^{\sup} = \beta_k + 2\Delta\beta$, $\beta_{k+1}^{\inf} = \beta_k$
- if $\beta_{k+1} = \beta_k^{\inf}$ then $\beta_{k+1}^{\sup} = \beta_k$, $\beta_{k+1}^{\inf} = \beta_k - 2\Delta\beta$

Note that there are two supervisory hierarchized levels of action of this intelligent system, namely:

1) ***The basic control:*** It consists of generating via u_k from (5) for each of the discrete models integrated in the multi-model scheme.
2) ***The choice of*** β ***:*** The model and gain β of the FROH is on-line selected by a switching rule via minimization of the supervisory performance index (8).

3 SIMULATION RESULTS

In this section two different cases are presented in order to show the usefulness of the proposed scheme. The first one simulates a DC motor, while the second one deals with a resonant circuit. In both cases, the two different multi model methods are used.

3.1 DC motor

A simple model of a DC motor driving an inertial loads shows the angular rate of the load, $w(t)$, as the output and applied voltage, $v_{app}(t)$, as the input. The objective is to control the angular rate by varying the applied voltage (Krishnan, 2001). Fig 3 shows a standard model of the DC motor.

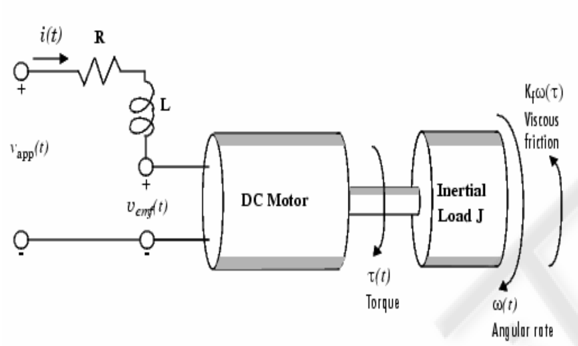

Figure 3: DC motor model

The transfer function of a DC motor can be described as:

$$G(s)=\frac{w(s)}{v_{app}(s)}=\frac{k_m}{LJs^2+(LK_f+JR)s+RK_f+k_bk_m}$$

The simulation is done by using the following parameters,

$R=0.5\Omega$, $L=1.5mH$, $k_m=0.05Nm/A$, $J=0.00025Nm/rad/s^2$, $k_f=0.0001Nm/rd/s$, $k_b=0.025V/rad/s$, which give the continuous transfer function,

$$G(s)=\frac{0.05}{3.75\cdot10^{-7}s^2+0.0001252s+0.0013}$$

The first simulation uses the first multimodel case. The set of possible gains β are:

$$\beta^{(i)}=1-(i-1)/10 \quad \text{for } 1\le i\le 21$$

The sampling time is chosen 0.1s and the residence time is 5 samples. The reference output is obtained from the following continuous transfer function,

$$G_m(s)=\frac{500000}{s^2+200s+12500}$$

Figures 4 shows the plant output when the β value is maintained fixed and when it is updated. The dotted line indicates the desired reference output. Figure 5 shows the active value of β during the whole simulation. It is obvious that the tracking performance is improved by selecting an appropriate value of the gain β.

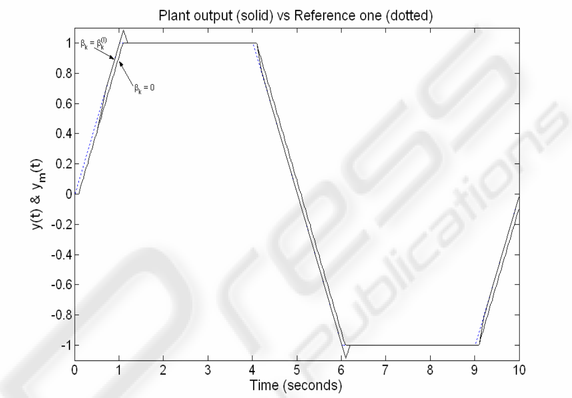

Figure 4 a: Plant output with method A

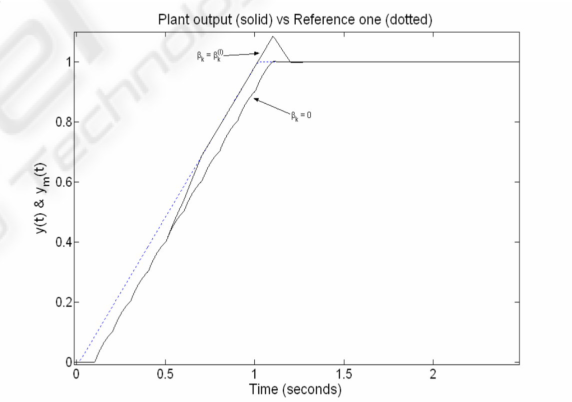

Figure 4 b: Plant output with method A (zoom over first seconds)

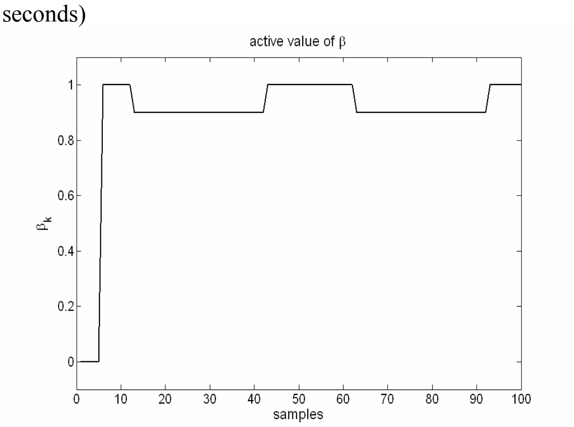

Figure 5: active value of β with method A

The simulation is repeated with the second multimodel method. Figure 6 shows the plant output and figure 7 the on-line active value of β selecting via switchings using (8). The initial value for $\Delta\beta$ is 0.2 and m_f is 1.2.

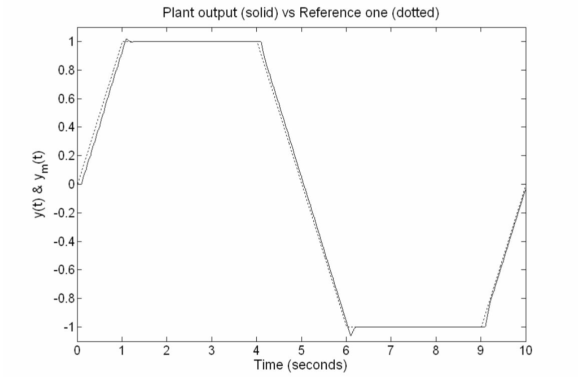

Figure 6: Plant output with method B

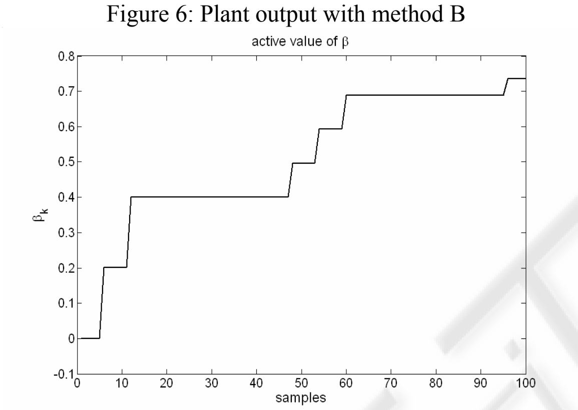

Figure 7: Active value of β with method B

In this case, it takes more time to achieve a good value of β as we do not let it to take the best one in the first switching. One could think that this is a bad option. However, next simulation will show that sometimes this method have a better performance.

3.2 Resonant Circuit (Tank Circuit)

A resonant circuit is simply an LCR circuit with a zero-pole cancellation at $s=0$ at the resonance frequency (Floyd, 2003).

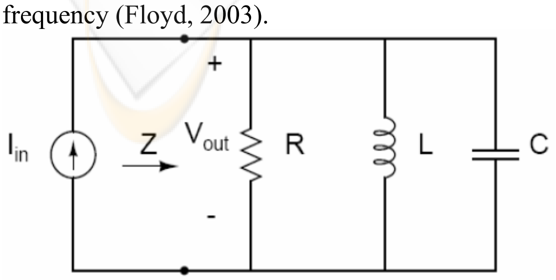

Figure 8: Parallel RLC circuit

Usually the effect of the resistance is small compared to the size of the inductance and the capacitance. This leads to highly resonant behavior. In this work, we will consider a parallel RLC circuit with a transfer function,

$$G(s) = \frac{V_{out}(s)}{I_{in}(s)} = \frac{\frac{s}{C}}{s^2 + \frac{s}{RC} + \frac{1}{LC}}$$

The resonant frequency w_m of a resonant circuit is the frequency corresponding to the peak value of the transfer function and it occurs when $w_m = 1/LC$.

Simulations are performed for a circuit with parameters $R = 100\Omega$, $L = 2mH$ and $C = 300pF$. Replacing these values, the transfer function results to be:

$$G(s) = \frac{3.333 \cdot 10^7 s}{s^2 + 3.3333 \cdot 10^7 s + 1.667 \cdot 10^{12}}$$

The reference output is obtained from the following continuous transfer function:

$$G_m(s) = \frac{1.667 \cdot 10^6 s}{s^2 + 1.667 \cdot 10^6 s + 1.667 \cdot 10^{12}}$$

The resonant frequency is located in $f_m = \frac{w_m}{2\pi} = 205kHz$. Both multi-model schemes with the same parameters as in previous section are simulated. The reference input is generated as the sum of four of four sinusoidal signals of different frequencies $\{0.1w_m, w_m, 10w_m, 100w_m\}$. It is suited for the circuit to select the one at the resonant frequency. Figure 9 shows the plant output in both cases together with the reference output. In this case, although both outputs tend to the desired one, the second multi-model has a better transient behavior in the first time interval. This occurs, because when we change the value of β in a big quantity, the plant behavior suffers a little transient as it can be shown. However, with small changes this transient is found to be smaller.

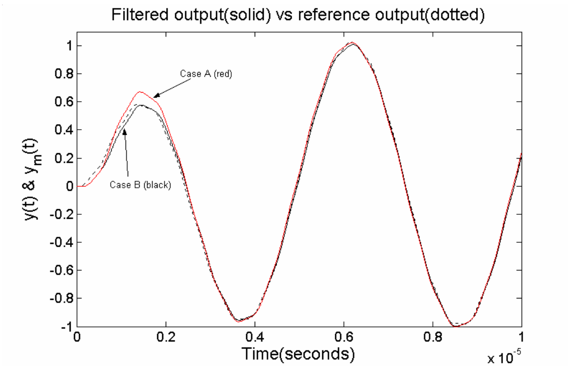

Figure 9: Plant outputs using method 1 and 2

Finally, figures 10 and 11 show the active value of β in both simulations.

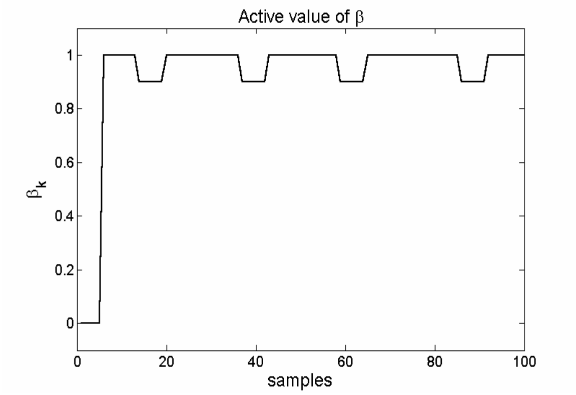

Figure 10: Active value of β with method 1

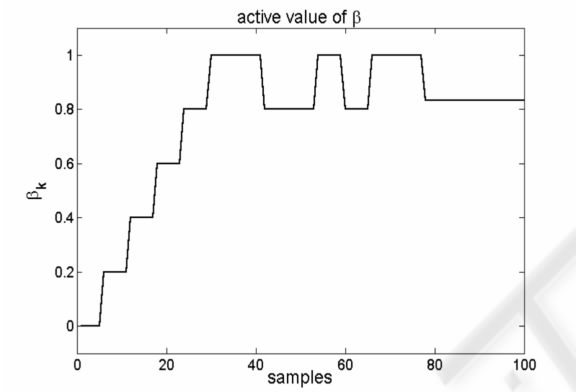

Figure 11: Active value of β with method 2

4 CONCLUSIONS

In this paper, a multi-model based discrete control scheme for a continuous plant has been presented. The different discrete models are obtained by discretizing the continuous plant under a FROH device. The scheme is designed to find the value of the gain β which leads to the best tracking performance. Two different methods have been presented for this purpose. The first one selects the current value of the gain among a fixed set of possible values. The second one updates β only to a close value, avoiding bad transients which may occur when the changing is big. Finally, the proposed schemes have been used in two practical cases. Simulations showed that an appropriate choice of the value of β leads to a good tracking performance, even if a continuous plant is under control by a discrete controller. Moreover, the advantages and disadvantages of both methods have been figured out through the simulation results.

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