Shape Factor's Effect on a Dynamic Cleaners Swarm ⋆

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Abstract. This work discusses an impossibility result for the *Dynamic Cooperative Cleaners* problem, and the relation of a specific geometric feature of the problem, known as the *shape factor*, to the efficiency of the operating swarm. The dynamic cooperative cleaners problem assumes a grid, having "*contamination*" points or tiles that form a connected region of the grid. Several agents move in this contaminated region, each having the ability to "clean" the place it is located in. The "contaminated" tiles expand deterministically, simulating a spreading of contamination, or *fire*. This problem, as well as a cooperative cleaning protocol for it and its analysis, were first introduced in [1]. The equivalence of this problem to another interesting multi agents problem was demonstrated in [2] by utilizing results relevant to the problem in order to design a cooperative hunting protocol for a swarm of UAVs. The results of [1] contain a generic lower bound for the cleaning time of *any* multi agents system which is designed to entirely clean an expanding contaminated area. This work enhances this bound, while discussing the effect of the region's shape factor (i.e. the ratio between the region's boundary and its area) and the swarm's cleaning efficiency. As a result, a tighter lower bound is produced, establishing a new and more generic impossibility result for the problem.

1 Introduction

In recent years significant research efforts have been invested in design and simulation of multi-agent robotics and intelligent swarms systems — see e.g. [3, 4] or [5–7] for biology inspired designs (behavior based control models, flocking and dispersing models and predator-prey approaches, respectively), [8–11] for economics applications and [12] for a physics inspired approach). Unfortunately, the mathematical geometrical theory of such multi-agents systems is far from being satisfactory, as pointed out in [13] and many other papers.

In this work we discuss the dynamic variant of the *Cooperative Cleaners* problem, first presented in [14], in which agents must work in a dynamic environment — where changes may take place, that are independent and certainly not caused by the agents' activity. The problem assumes a grid, part of which is 'dirty', where the 'dirty' part is a connected region of the grid. On this dirty grid region several agents move, each having

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the ability to 'clean' the place ('tile', 'pixel' or 'square') it is located in (similar works appear in [15–17]). The dynamic variant of the cooperative cleaners problem (presented in [1] and described in section 2) involves a deterministic evolution of the environment, simulating a spreading *contamination* (or spreading *fire*). Once again, the goal of the agents is to clean the spreading contamination in as little time as possible. In the spirit of [18] simple robots with only a bounded amount of memory are considered (i.e. a *finite-state-machines*).

A cooperative swarm cleaning protocol for the problem and a basic analysis of, as well as various experimental results are presented in [1], whereas a comparison of this swarm protocol to an A^* based omniscient centralized algorithm is discussed in [19]. A scheme of a cooperative hunting protocol, designed to be used by a swarm of unmanned air vehicles seeking evading targets, which is based on the cleaning protocol mentioned above is described in [2]. This work discusses the effect of a certain geometric feature of the dirty region (known as the *shape factor*) on the cleaning time of the agents (see section 3).

2 The Dynamic Cooperative Cleaners Problem

We shall assume that the time is discrete. Let G be a two dimensional grid, whose vertices have a binary property of '*contamination*'. Let $cont_t(v)$ state the contamination state of the vertex v at time t, taking either the value "*on*" or "*off*". Let F_t be the dirty sub-graph of G at time t, i.e. $F_t = \{v \in G \mid cont_t(v) = on\}$. We assume that F_0 is a single connected component.

Let a group of k agents that can move across the grid G (moving from a vertex to its neighbor in one time step) be placed at time t_0 on F_0 (we focus on the cleaning problem, and not on the discovery problem).

Each agent is equipped with a sensor capable of telling the condition of the tile it is currently located in, as well as the condition of the 8-neighbors of this tile. An agent is also aware of other agents which are located in its current position, and all the agents agree on a common direction. Each tile can contain any number of agents simultaneously.

When an agent moves to a vertex v, it has the possibility of causing $cont(v)$ to become *off*. The agents do not have any prior knowledge of the shape or size of the sub-graph F_0 except that it is a single and simply connected component.

Every d time steps the contamination spreads. That is, if $t = nd$ for some positive integer n, then :

$$\forall v \in F_t \; \forall u \in 4 - neighbors(v) \; , \; cont_{t+1}(u) = on$$

The agents' goal is to clean G by eliminating the contamination entirely, meaning that the agents must ensure that :

$$\exists t_{success} \text{ s.t } F_{t_{success}} = \emptyset$$

In addition, it is desired that this time span $t_{success}$ will be minimal.

3 Results

Since we know no easy way to foretell whether k agents can successfully clean an instance of the *Dynamic Cooperative Cleaners* problem, producing bounds for the proposed cleaning protocol is important for estimating its efficiency.

The completion of the cleaning mission at time t means that $S_t = 0$. By showing that at a specific time t, S_t is always larger than zero, it is shown that the mission could not be completed until that time, regardless of the nature of the cleaning protocol utilized by the agents.

For producing this bound, the contaminated region was assumed to spread in such a way that creates the minimal number of new contaminated tiles. Having no additional information, this can be guaranteed by assuming that whenever the contamination spreads, it is somehow organized as a digital sphere (as was the case in the bound presented in [1]). This, however, is rarely the case, since in the course of the expansions and erosion process of the contamination, the probability for the contaminated region to be accidently maintained in the form of a digital sphere, is very low. As a result, we are interested in examining a variant of this bound, in which the contaminated region is not assumed to be kept in the shape of a digital sphere.

3.1 Definitions

Let S_t denote the size of the contaminated region F at time t, namely the number of grid tiles in F_t. Let d denote the number of time steps between two contamination spreads.

The boundary of the contaminated region F is denoted as ∂F, defined as :

$$\partial F = \{(x, y) \mid (x, y) \in\ F\ \wedge\ (x, y) \textit{ has an 8 neighbor in } (G \setminus F)\}$$

Let $\psi(F_t)$ denote the *shape factor* of F_t, defined as the ratio between the perimeter of F_t and its area, namely :

$$\psi(F_t) = \frac{|\partial F_t|}{S_t}$$

3.2 Detailed Analysis

Note that a lower bound for the cleaning time is in fact an upper bound for the agents' performance. Let us assume that the agents are working in 100% efficiency, meaning, each time step every agent cleans a single tile. After $(d - 1)$ time steps k agents will thus clean $k \cdot (d - 1)$ tiles, and thus we know that $S_{d-1} \geq S_0 - (d - 1) \cdot k$

In the d-th time step, the agents clean another portion of k tiles, but the remaining contaminated tiles spread their contamination to their 4-neighbors and cause new tiles to become contaminated. We are interested in the *minimal* number of tiles which can become contaminated at this stage.

As the assumption that F_t is continuously preserved in the shape of a digital sphere is too rigid, we are interested in constructing a method that will provide us with tighter predictions. For achieving this, we assume that the shape factor of the contaminated region is kept bounded by some value Ψ throughout the entire evolution of F_t, namely :

$$\forall t \quad \psi(F_t) \geq \Psi \tag{1}$$

Since every new contaminated tile is a 4-neighbors of some $v \in F_t$, the total number of new contaminated tiles is at least the number of boundary tiles of F_t, namely $|\partial F_t|$. Since we are interested in the minimal number of new contaminated tiles, we can use the definition of $\psi(F_t)$ and write :

$$S_{t+d} \geq S_t - d \cdot k + \psi(F_t) \cdot S_t \tag{2}$$

Since $\forall t \; \psi(F_t) \geq \Psi$ we can then write :

Lemma 1.

$$S_{t+d} \geq (1 + \Psi) \cdot S_t - d \cdot k$$

As to the explicit value of S_t for some $t = i \cdot d$ we can quickly see that :

Lemma 2.

$$S_t = S_{i \cdot d} \geq (1 + \Psi)^i \cdot S_0 - d \cdot k \cdot \sum_{j=0}^{i-1} (1 + \Psi)^j$$

For finding the time in which the agents may be able to complete the mission successfully (meaning that $S_t \leq 0$) we require that :

$$(1 + \Psi)^i \cdot S_0 - d \cdot k \cdot \sum_{j=0}^{i-1} (1 + \Psi)^j \leq 0 \tag{3}$$

(note that this does not guarantee the completion of the mission, but rather contradicts the impossibility of the completion of the mission, meaning that a successful completion of the mission is *enabled*). This requirement can also be written as follows :

$$\frac{S_0}{d \cdot k} \leq \sum_{j=0}^{i-1} \frac{(1 + \Psi)^j}{(1 + \Psi)^i} = \sum_{j=0}^{i-1} (1 + \Psi)^{j-i} \tag{4}$$

Remembering that $\Psi > 0$, we then use the expression describing the sum of a geometric progression and see that :

$$\sum_{j=0}^{i-1} (1 + \Psi)^{j-i} = \frac{(1 + \Psi)^{-i}((1 + \Psi)^i - 1)}{(1 + \Psi) - 1} = \frac{1 - (1 + \Psi)^{-i}}{\Psi} \tag{5}$$

Combining equations 4 and 5 the following is produced :

Theorem 1. *For a contaminated region F_0 of size S_0 such that F_t spreads every d time steps, and such that $\forall t \quad \psi(F_t) \geq \Psi$, the number of agents required for a successful cleaning of F_0 within at most $(i \cdot d)$ time steps is at least :*

$$k = \frac{S_0 \cdot \Psi}{d \cdot (1 - (1 + \Psi)^{-i})}$$

Note that since $\forall F_t \ |\partial F_t| \leq S_t$ we can see that $\forall F_t \ \psi(F_t) \leq 1$. On the other hand, for any region F_t the minimal value of $\psi(F_t)$ is obtained when F_t is organized in the shape of a digital sphere (let us denote this value by Ψ_{SPHERE}, and note that $\Psi_{SPHERE} > 0$). Hence, we are only interested in $0 < \Psi_{SPHERE} \leq \Psi \leq 1$. Note that for $\Psi = \Psi_{SPHERE}$ a lower bound similar to this of [1] can be derived from Theorem 1.

However, unlike the case of $\Psi = \Psi_{SPHERE}$, using larger values for Ψ yields better estimations for the minimal number of agents which are required for a successful completion of the mission. This means that if it can be shown for some contaminated region F_0 that its shape factor is kept bounded by some Ψ throughout its cleaning process, then a tighter prediction for the minimal k needed for this problem is available.

Let F_0 be a contaminated region of size S_0 such that F_t spreads every d time steps and such that $\forall t \ \psi(F_t) \geq \Psi$ (we know that the number of agents required for a successful cleaning of F_0 within at most $t = (i \cdot d)$ time steps is at least $k_F = \frac{S_0 \cdot \Psi}{d \cdot (1-(1+\Psi)^{-i})}$). Then, for following Corollaries are derived from Theorem 1 :

Corollary 1. *For some contaminated region H_0 of size $\alpha \cdot S_0$ (for some $\alpha \geq 0$) such that H_t spreads every d time steps and such that $\forall t \ \psi(H_t) \geq \Psi$ the number of agents required for a successful cleaning of H_0 within at most $t = (i \cdot d)$ time steps is at least :*

$$k_H = \alpha \frac{S_0 \cdot \Psi}{d \cdot (1-(1+\Psi)^{-i})} = \alpha \cdot k_F$$

Corollary 2. *For some contaminated region H_0 of size S_0 such that H_t spreads every $\alpha \cdot d$ time steps (for some $\alpha > \frac{1}{d}$) and such that $\forall t \ \psi(H_t) \geq \Psi$ the number of agents required for a successful cleaning of H_0 within at most $t = (i \cdot d)$ time steps is at least :*

$$k_H = \alpha^{-1} \frac{S_0 \cdot \Psi}{d \cdot \left(1-(1+\Psi)^{-\frac{i}{\alpha}}\right)} \sim \alpha^{-1} \cdot k_F$$

An example of Corollary 2 appears in Figure 1.

Note that if for some region F_H it holds that $d \to \infty$ (meaning that the contamination does not spread at all, for all practical reasons) then using *De l'Hôpital's rule* on Corollary 2 we see that :

$$\lim_{\alpha \to \infty} \frac{\alpha^{-1} \cdot S_0 \cdot \Psi}{d \cdot \left(1-(1+\Psi)^{-\frac{i}{\alpha}}\right)} = \frac{S_0 \cdot \Psi}{d \cdot i \ln(1+\Psi)}$$

and since for every $0 < \Psi < 1$, $\frac{\Psi}{\ln(1+\Psi)} < 2$ we see that $k_H \geq \frac{S_0}{d \cdot i}$ (which is also intuitively correct).

Corollary 3. *For some contaminated region H_0 of size S_0 such that H_t spreads every d time steps and such that $\forall t \ \psi(H_t) \geq \alpha \cdot \Psi$ (for some $0 < \alpha \leq \frac{1}{\Psi}$) the number of agents required for a successful cleaning of H_0 within at most $t = (i \cdot d)$ time steps is at least :*

$$k_H = \alpha \frac{1-(1+\Psi)^{-i}}{1-(1+\alpha \cdot \Psi)^{-i}} \cdot k_F$$

For large values of i, $\frac{1-(1+\Psi)^{-i}}{1-(1+\alpha \cdot \Psi)^{-i}} = 1$ and so $k_H = \alpha \cdot k_F$.

An example of Corollary 3 appears in Figure 2.

The previous Corollaries as well as Theorem 1 present various ways of predicting a lower value which bounds the number of agents required for successfully solving an instance of the dynamic cooperative cleaners problem. In addition, once such a lower bound was established, the effects of changes in the initial problem's features (e.g. spreading speed, shape factor, etc') on this bound are discussed. Let us assume that a certain cleaning protocol for the problem was constructed, which is able to direct some of its cleaning resources to actively controlling the geometric features of the region to be cleaned. Meaning — instead of cleaning as much tiles as possible, cleaning the shape so its boundary area is kept limited. It is obvious that since the shape factor of the region is artificially controlled, we may expect an acceleration in the operation of the agents using this protocol (due to Corollary 3). However, since some of the agents' resources are diverted from the cleaning mission (since those agents are used for maintaining the required shape factor), this improvement in the agents' performance will be compensated by the resources spent on the maintenance of the region's shape factor. This can be described as follows — let $f(\Psi) \in (0, 1)$ denote the slowdown function of the cleaning protocol caused by maintaining the shape factor bounded by Ψ. Thus, we examine the following variation of Theorem 1 :

$$k = \frac{S_0 \cdot \Psi}{d \cdot \left(1 - (1 + \Psi)^{-i \cdot f(\Psi)}\right)}$$

In order to obtain the minimal number of agents needed for such a cleaning protocol, we first much find the optimal value for the percentage of the cleaning efforts allocated to maintaining the shape factor. Since we assume the cleaning protocol is able to select the level of Ψ in which the region's shape factor is maintain, Theorem 1 can be written as follows :

Theorem 2. *For a contaminated region F_0 of size S_0 such that F_t spreads every d time steps, and assuming that a cleaning protocol which is able to artificially preserve the shape factor of F_t is used (with a slowdown function $f(\Psi)$), the number of agents required for a successful cleaning of F_0 within at most $(i \cdot d)$ time steps is at least :*

$$k = \min\left\{ \frac{S_0 \cdot \Psi}{d \cdot (1 - (1 + \Psi)^{-i \cdot f(\Psi)})} \quad \middle| \quad \Psi_{SPHERE} \leq \Psi \leq 1 \right\}$$

For example, imagine a protocol whose slowdown function is $f(\Psi) = \Psi$. Namely, the protocol suffers no slowdown when it is completely focused on cleaning F_0, while preserving the region to be organized as a digital sphere (i.e. the shape with the minimal shape factor) the time it takes it to complete the cleaning is $\frac{1}{\Psi_{SPHERE}} \cdot t$ the time required without this slowdown. Using Theorem 2 we can see that :

$$k = \min\left\{ \frac{S_0 \cdot \Psi}{d \cdot (1 - (1 + \Psi)^{-i \cdot \Psi})} \quad \middle| \quad \Psi_{SPHERE} \leq \Psi \leq 1 \right\}$$

A short discussing considering this example appears in Figure 3.

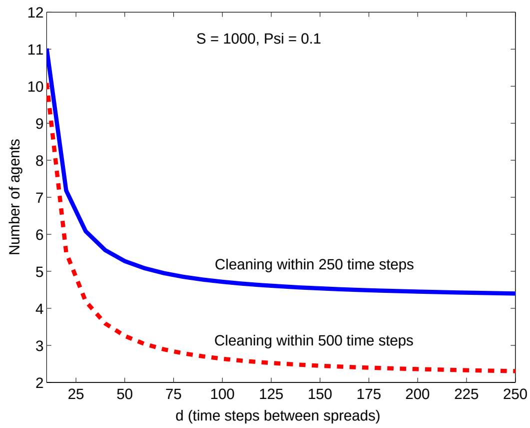

Fig. 1. An example of Corollary 2. The two graphs represent the minimal number of agents as a function of the spreading speed d. In addition, results of the change in the cleaning time permitted, are presented. Notice that for most values of d (number of time steps between spreads) the ratio between the two values of minimal numbers of agents required equals the ratio of the two cleaning times, whereas for faster spreading regions (smaller values of d) the price for demanding faster cleaning is much smaller.

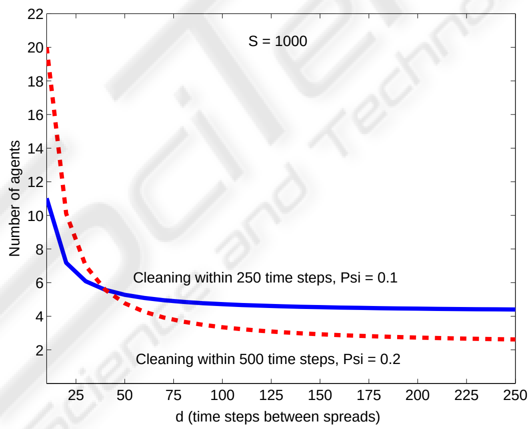

Fig. 2. An example of Corollary 3. The two graphs represent the minimal number of agents as a function of the spreading speed d. In addition, results of the change in the cleaning time allowed and the shape factor of the contaminated region are presented. Notice that for different values of d, sometimes a “simple” shape with less cleaning time produces a smaller requirement of k while in other cases longer cleaning times for higher Ψ values are preferred. This example demonstrates how various features of the problem (in this case — the spreading speed) may significantly influence designers of multi agents systems.

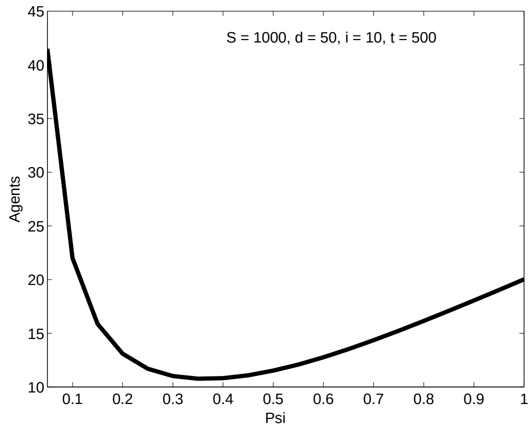

Fig. 3. An example of Theorem 2. For minimizing the number of agents required for a successful completion of the mission, the optimal value of Ψ should be calculated. Once available, it allows the cleaning protocol to optimally partially allocate its resources for maintaining the shape factor of the region. Notice how in this example, using this optimal value results in a minimal requirement for 11 agents, while focusing solely on cleaning the region produces a demand for at least 20 agents (and diverting too much resources towards maintaining the shape factor in the lowest value possible yields a lower bound of 40 agents).

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