SELECTIVE IMAGE DIFFUSION FOR ORIENTED PATTERN EXTRACTION

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Abstract: Anisotropic regularization PDE's (Partial Differential Equation) raised a strong interest in the field of image processing. The benefit of PDE-based regularization methods lies in the ability to smooth data in a nonlinear way, allowing the preservation of important image features (contours, corners or other discontinuities). In this article, a selective diffusion approach based on the framework of Extreme Physical Information theory is presented. It is shown that this particular framework leads to a particular regularization PDE which makes it possible integration of prior knowledge within diffusion scheme. As a proof a feasibility, results of oriented pattern extractions are presented on ad hoc images. This approach may find applicability in vision in robotics.

1 INTRODUCTION

Since the pioneering work of Perona-Malik (Perona and Malik, 1990), anisotropic regularization PDE's raised a strong interest in the field of image processing. The benefit of PDE-based regularization methods lies in the ability to smooth data in a nonlinear way, allowing the preservation of important image features (contours, corners or other discontinuities). Thus, many regularization schemes have been presented so far in the literature, particularly for the problem of scalar image restoration (Perona and Malik, 1990; Alvarez et al., 1992; Catté et al., 1992; Geman and Reynolds, 1992; Nitzberg and Shiota, 1992; Whitaker and Pizer, 1993; Weickert, 1995; Deriche and Faugeras, 1996; Weickert, 1998; Terebes et al., 2002; Tschumperle and Deriche, 2002; Tschumperle and Deriche, 2005). In (Deriche and Faugeras, 1996) Deriche and *al.* propose a unique PDE to express the whole principle.

If we denote $\psi(\mathbf{r},0) : \mathbb{R}^2 \times \mathbb{R}^+ \to \mathbb{R}$ the intensity function of an image, to regularize the considered image is equivalent to a minimization problem of a particular PDE which can be seen as the superposition of two monodimensionnal heat equations, respectively oriented in the orthogonal direction of the gradient and in the tangential direction (Eq. (1 and Fig. 1) :

$$\frac{\partial\psi}{\partial t} = \frac{\phi'(\|\nabla\psi\|)}{\|\nabla\psi\|}\psi_{\xi\xi} + \phi''(\|\nabla\psi\|)\psi_{\eta\eta} \quad , \quad (1)$$

where $\eta = \nabla\psi/\|\nabla\psi\|$ and $\xi \perp \eta$ and $\phi : \mathbb{R} \to \mathbb{R}$ is a decreasing function.

This PDE is characterized by an anisotropic diffusive effect in the privileged directions ξ and η allowing a denoising of scalar image.

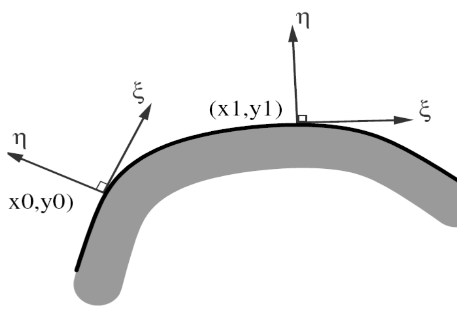

Figure 1: An image contour and its moving vector basis (ξ,η). Taken from (Tschumperle and Deriche, 2002).

The major limitations of this diffusion process is its high dependance to the intrinsic quality of the original image and the impossibility to integrate prior information on the pattern to be restored if it can be

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characterized by particular data (orientation for example). Moreover, no characterization of the uncertainty/inaccuracy compromise can be made on the studied pixel, since the scale parameter is not directly integrated in the minimisation problem in which relies the common diffusion equations (Nordstrom, 1990).

In this article we propose an original PDE directly integrating the scale parameter and allowing the taking into account of *a priori* knowledge on pattern to restore. We propose more particularly, to derive this PDE, to use a recent theory known as Extreme Physical Information (EPI) recently developed by Frieden (Frieden, 1998) and applied to image processing by Courboulay and *al.* (Courboulay et al., 2002).

The second section of this article is dealing with the presentation of EPI and with the obtaining of the particular PDE. The third one presents a direct application to the presented diffusion process which may find applicability in robotics and automation. Last part is dedicated to discussion.

2 EPI AND IMAGE DIFFUSION

2.1 EPI

Developed by Frieden, the principle of Extreme Physical Information (EPI) is aimed at defining a new theory of measurement. The key element of this new theory is that it takes into account the effect of an observer on a measurement scenario. As stated by Frieden (Frieden, 1996; Frieden, 1998), ”EPI is an observer-based theory of physics”. By observing, the observer is both a collector of data and an interference that affects the physical phenomenon which produces the data. Although the EPI principle brings new concepts, it still has to rest on the definition of information. Fisher information was chosen for its ability to effectively represent the quality of a measurement. Fisher information measure was introduced by Fisher in 1922 (Fisher, 1922) in the context of statistical estimation. In the last ten years, a growing interest for this information measure has arisen in theoretical physics. In his recent book (Frieden, 1998), Frieden has characterized Fisher information measure as a versatile tool to describe the evolution laws of physical systems; one of his major results is that the classical evolution equations as the Shrodinger wave equation, the Klein-Gordon equation, the Helmotz wave equation, or the diffusion equation, can be derived from the minimization of Fisher information measure under proper constraint.

Practically speaking, EPI principle can be seen as an optimization of the information transfer from the system under measurement to the observer, each one being characterized by a Fisher Information measure denoted respectively I and J. The first one is representative of the quality of the estimation of the data, and the second one allows to take into account the effect of the subjectivity of the observer on the measure. The existence of this transfer leads to create fluctuations on the acquired data compared to the real ones. In fact, this information channel leads to the loss of accuracy on the measure whereas the certainty is increased.

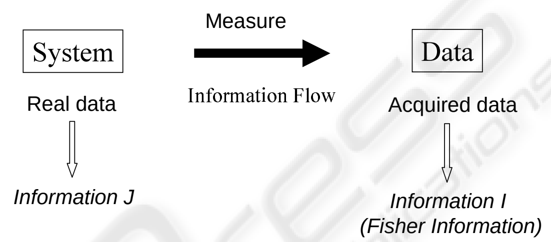


Figure 2: Fisher Information.

The goal of EPI is then to extremize the difference $I-J$ (*i.e.* the uncertainty/inaccurracy compromise) denoted K, called Physical Information of the system, in order to optimized the information flow.

2.2 Application to Image Diffusion

Application to image diffusion can be illustrated by Fig. (3).

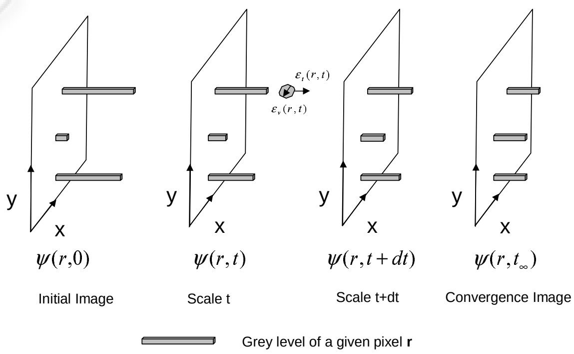


Figure 3: Uncertainty/inaccuracy compromise and isotropic image diffusion. When parameter $t \longrightarrow \infty$, luminance of all pixels of the corresponding image is the same and equal to the spatial average of the initial image.

As far as isotropic image diffusion is concerned, the uncertainty deals with the fluctuations of the grey level of a given pixel compared with its real value, whereas the inaccuracy deals with the fluctuations of the spatial localisation of a given pixel compared with the real one. The two different errors ($\varepsilon_r(t)$ and $\varepsilon_v(t)$) of Fig. (3) which are introduced all along the diffusion process are characterized by a measure of Fisher information. Intrinsic Fisher information J will be an integration of the diffusion constrained we impose on the processing.

Then, we can apply EPI to image diffusion process by considering an image as a measure of characteristics (as luminance, brightness, contrast) of a particular scene, and diffusion as the observer of this measure at a given scale. Extreme Physical Information K is then defined as follows (Frieden, 1998):

$$K(\psi) = \int\int d\Omega dt \times \left[(\nabla - \mathbf{A})(\nabla - \mathbf{A})\psi^2 + (\frac{\partial \psi}{\partial t})^2 - \psi^2 \right], \tag{2}$$

where $\psi(\mathbf{r}, 0) : \mathbb{R}^2 \times \mathbb{R}^+ \rightarrow \mathbb{R}$ is the luminance function of the original image and $\mathbf{A}$ a potential vector representing the parameterizable constrain integrated within diffusion process.

Extremizing K by Lagrangian approach leads to a particular diffusion equation given by :

$$\frac{\partial \psi}{\partial t} = \frac{1}{2}(\nabla - \mathbf{A}).(\nabla - \mathbf{A})\psi \quad . \tag{3}$$

As a consequence, thanks to the possible parameterization of $\mathbf{A}$, it is possible to take into account particular characterized pattern to preserve from the diffusion process.

2.3 About A

The $\mathbf{A}$ potential allows to control the diffusion process and introduce some prior constrains during image evolution. For instance, if no constrain are to be taken into account, we set $\mathbf{A}$ as vector null and (Eq. 3) becomes :

$$\frac{\partial \psi}{\partial t} = \nabla.\nabla\psi = \triangle\psi \quad . \tag{4}$$

which is the well known heat equation characterized by an isotropic smoothing of the data processed.

In order to enlarge the possibility given by Eq. (3), the choice we make for $\mathbf{A}$ is based on the fact that Eq. (3) allows a weighting of the diffusion process with the difference of orientation between the local calculated gradient and $\mathbf{A}$. More precisely, to explain the way $\mathbf{A}$ is practically implemented, let consider Fig. 4.

The expression of the local gradient $\nabla\psi$ in terms of θ" is, considering Fig. 4 :

$$\nabla\psi = \begin{pmatrix} \|\nabla\psi\| \cos\theta'' \\ \|\nabla\psi\| \sin\theta'' \end{pmatrix}, \tag{5}$$

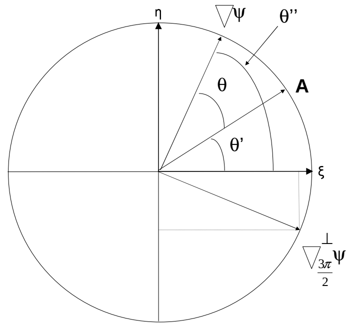


Figure 4: Local geometrical implementation of $\mathbf{A}$ in terms of the local gradient $\nabla\psi$.

and an expression of $\mathbf{A}$ in terms of θ' is :

$$\mathbf{A} = \begin{pmatrix} \|\nabla\psi\| \cos\theta' \\ \|\nabla\psi\| \sin\theta' \end{pmatrix}. \tag{6}$$

Norm of $\mathbf{A}$ is imposed in order to make it possible the comparison with the gradient. To this point, the most interesting expression of $\mathbf{A}$ would be the one in terms of θ, which represents the difference angle between $\mathbf{A}$ and the local gradient. If we made so, using trigonometrical properties and noticing that $\theta = |\theta'' - \theta'|$, we obtain a new expression for $\mathbf{A}$:

$$\mathbf{A} = \begin{pmatrix} \|\nabla\psi\|(\cos\theta'' \cos\theta + \sin\theta'' \sin\theta) \\ \|\nabla\psi\|(\sin\theta'' \cos\theta - \cos\theta'' \sin\theta) \end{pmatrix}. \tag{7}$$

Eq. (7) could be simplified by integrating the vectorial expression of the local gradient (Eq. (5)) :

$$\mathbf{A} = \nabla\psi.\cos\theta + \nabla^{\perp}_{\frac{3\pi}{2}}\psi.\sin\theta\,. \tag{8}$$

From Eq. (8), we could then derive a general expression for $\mathbf{A}$ considering it as a vectorial operator :

$$\mathbf{A} = \nabla.\cos\theta + \nabla^{\perp}_{\frac{3\pi}{2}}.\sin\theta\,, \tag{9}$$

with θ the relative angle between $\mathbf{A}$ et $\nabla\psi$ for a given pixel and $\nabla^{\perp}$ the local vector orthogonal to ∇ (Fig. 4). This expression only represents the way it is possible to reexpress $\mathbf{A}$ by an orthogonal projection in the local base. Considering it, Eq. (3) becomes :

$$\frac{\partial \psi}{\partial t} = \frac{\partial^2 \psi}{\partial \eta^2}.(1 - \cos\theta) + \frac{\partial^2 \psi}{\partial \xi^2}.(1 - \cos\theta) \quad . \tag{10}$$

One can notice on Eq. (10) that when angle $\theta = 0$ (*i.e.* $\mathbf{A}$ and $\nabla\psi$ are colinear), the studied pixel will not be diffused for $\frac{\partial\psi}{\partial t} = 0$. On the contrary, a non-zero value of θ will lead to a weighted diffusion of the considered neighborhood of the pixel (Eq. (10)).

As a consequence, by imposing local θ values, it is possible to preserve particular patterns from the diffusive effect within the processed image.

3 APPLICATION TO ORIENTED PATTERN EXTRACTION

In this section, we present results obtained on simple images in order to show the restoration and denoising potential of the method.

For practical numerical implementation, the process of Eq. (10) is discretized with a time step τ. The images $\psi(t_n)$ are calculated, with Eq. (10), at discrete instant $t_n = n\tau$ with n the number of iterations in the process.

Let first consider an image showing vertical, horizontal, and 45°-oriented dark stripes on a uniform background (Fig. 5).

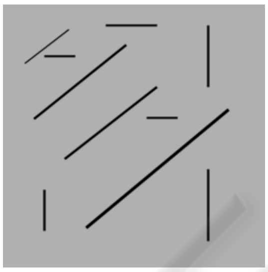


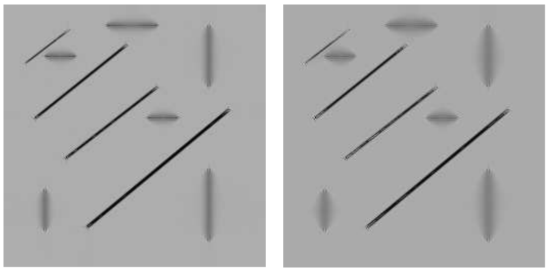
Figure 5: Image 1: Dark stripes with various orientations on a uniform background.

Considering Eq. (10), by imposing two possible orientations for **A** (135°, 325°) which corresponds to the gradient orientations of the diagonal stripes, one could expect to preserve them from isotropic diffusion. Diffusion results are presented Fig. 6.

As one was expected it, the vertical and horizontal dark stripes in diffused images tend to disappear whereas the diagonal stripes are preserved all along the diffusion process.

Let now consider a noisy simple grid diagonally oriented corrupted by a Gaussian noise of standard deviation set to 0.3.

If we apply the same diffusion process of Eq. (10) to this noisy simple grid imposing this time four possible orientations for **A** corresponding to the four possible gradient orientations of the grid, it is then possible to show the denoising effect of the diffusion process (Fig. 8).



(a) (b)

Figure 6: Diffusion of ”Image 1” (Fig. 5) for (a) n=100 and (b) n=200. **A.** is chosen in order to preserve only diagonal stripes from isotropic diffusion process. Time step τ is fixed to 0.2.

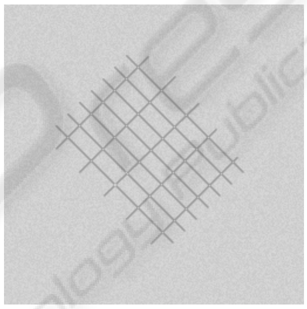
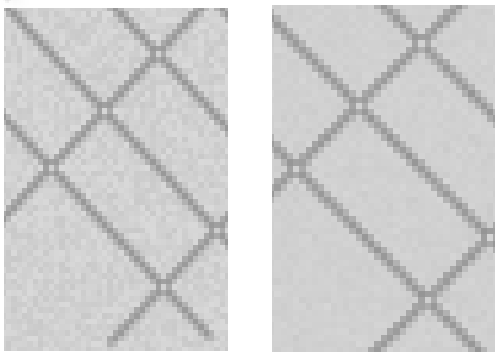


Figure 7: Image 2: Noisy diagonally oriented grid (Gaussian noise). PSNR (calculated with the non corrupted version of the grid as reference) is equal to 68 dB.



(a) (b)

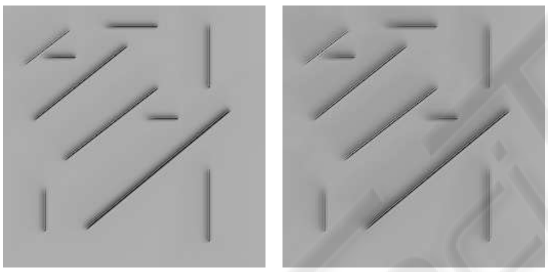
Figure 8: Diffusion of ”Image 2” (a) (Fig. 7) for (b) n=50. As one can notice, the grid itself is preserved from the diffusive effect of Eq. (3) whereas noise is iteration after iteration removed. Time step τ is fixed to 0.2.

As intended, the grid itself is not diffused at all and the increase of the Peak Signal to Noise Ratio (PSNR) from 68 dB to 84 dB, shows that the added Gaussian noise is removed iteration after iteration.

4 DISCUSSION

In this article an original diffusion method, based on the use of a particular PDE (Eq. (3)) derived from EPI theory, has been presented. It has been shown that the integration of the potential vector **A** within the formulation of this PDE makes it possible the integration within the diffusion scheme of particular constrains. This has been assimilated to integration of selectiveness within classical isotropic diffusion process. Examples on ad hoc images have been presented to show the potential of the presented method in the areas of denoising and extraction of oriented patterns.

Applications presented can be discussed, for frequential filterings or Gabor-filters convolution can lead to similar results. Considering that, it is necessary to keep in mind that processed image have been chosen in an ad hoc way to show the potential of the method. Nevertheless, one major difference must be noticed. Let consider again Fig. 5. If **A** is chosen in order to preserve only one direction of the diagonal stripes, implementation of Eq. (3) leads to result presented Fig. 9.



(a) (b)

Figure 9: Diffusion of "Image 1" (Fig. 5) for (a) n=20 and (b) n=50. As one can notice, Eq. (9) makes it possible to only preserve one gradient direction of the diagonal stripes. Time step τ is fixed to 0.2.

That kind of results can not be obtained by classical methods and enlarge the possible applications of Eq. (3).

As a conlusion, an alternative method for oriented pattern extraction has been presented in this article. It has been demonstrated, as a proof a feasibility, on ad hoc images that the developed approach may find applicability in robotics and visions as far extraction of oriented pattern is still an open problem. Industrial control quality check can also be an other area of applications.

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