ON THE BALANCING CONTROL OF HUMANOID ROBOT

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Abstract: This paper proposes the kinematic resolution method of CoM(center of mass) Jacobian with embedded motions and the design method of posture/walking controller for humanoid robots. The kinematic resolution of CoM Jacobian with embedded motions makes a humanoid robot balanced automatically during movement of all other limbs. Actually, it offers an ability of WBC(whole body coordination) to humanoid robot. Also, we prove that the proposed posture/walking controller brings the ISS(disturbance input-to-state stability) for the simplified bipedal walking robot model.

1 INTRODUCTION

Recently, there have been many researches about humanoid motion control, for example, walking control(Choi et al., 2006; Kajita et al., 2001), and whole body coordination(Sugihara and Nakamura, 2002). Especially, the WBC(whole body coordination) algorithm with good performance becomes the essential part in the development of humanoid robot because it offers the enhanced stability and flexibility to the humanoid motion planning. In this paper, we suggest the kinematic resolution method of CoM Jacobian with embedded motions, actually, which offers the ability of WBC to humanoid robot. For example, if humanoid robot stretches two arms forward, then the position of CoM(center of mass) of humanoid robot moves forward and its ZMP(zero moment point) swings back and forth. In this case, the proposed kinematic resolution method of CoM Jacobian with embedded (stretching arms) motion offers the joint configurations of supporting limb(s) calculated automatically to maintain the position of CoM fixed at one point.

Also, we will simplify the dynamics of bipedal robot as the equation of motion of a point mass concentrated on the position of CoM. First, let us assume that the motion of CoM is constrained on the surface $z = c_z$, then the rolling sphere model with the concentrated point mass m can be obtained as the simplified model for bipedal robot as shown in Fig. 1. The motion of the rolling sphere on a massless plate is described by the position of CoM, $c = [c_x, c_y, c_z]^T$, and the ZMP is described by the position on the ground, $p = [p_x, p_y, 0]^T$. Second, let us take the moments about origin on the ground of the linear equations of motion for the rolling sphere (with a point mass = m) confined to motion on a plane $z = c_z$ as shown in Fig. 1, then the following equations are obtained:

$$\tau_x = mgc_y - m\ddot{c}_y c_z \tag{1}$$
$$\tau_y = -mgc_x + m\ddot{c}_x c_z \tag{2}$$
$$\tau_z = -m\ddot{c}_x c_y + m\ddot{c}_y c_x \tag{3}$$

where g is the acceleration of gravity, c_z is a height constant of constraint plane and τ_i is the moment about i-coordinate axis, for $i = x, y, z$. Now, if we introduce the conventional definition of ZMP as following forms:

$$p_x \triangleq -\frac{\tau_y}{mg} \quad \text{and} \quad p_y \triangleq \frac{\tau_x}{mg}$$

to two equations (1) and (2), then ZMP equations can be obtained as two differential equations:

$$p_i = c_i - \frac{1}{\omega_n^2}\ddot{c}_i \qquad \text{for } i = x, y \tag{4}$$

where $\omega_n \triangleq \sqrt{g/c_z}$ is the natural radian frequency of the simplified biped walking robot system. Above

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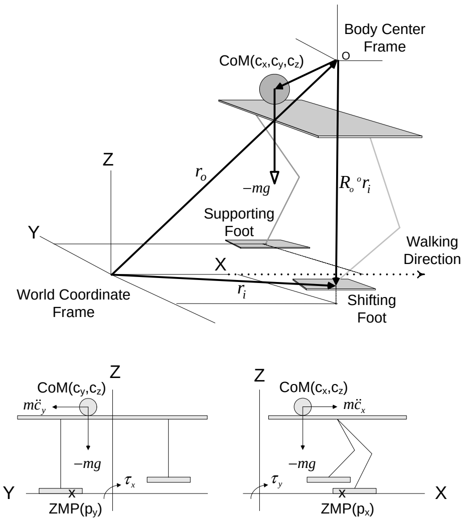

Figure 1: Rolling Sphere Model for Dynamic Walking.

equations will be used to prove the stability of the posture/walking controller in the following sections.

2 KINEMATIC RESOLUTION

Let a robot has n limbs and the first limb be the base limb. The base limb can be any limb but it should be on the ground to support the body. Each limb of a robot is hereafter considered as an independent limb. In general, the i-th limb has the following relation:

$$ {}^o\dot{x}_i = {}^oJ_i\dot{q}_i \tag{5}$$

for $i = 1,2,\cdots,n$, where ${}^o\dot{x}_i \in \Re^6$ is the velocity of the end point of i-th limb, $\dot{q}_i \in \Re^{n_i}$ is the joint velocity of i-th limb, ${}^oJ_i \in \Re^{6\times n_i}$ is the usual Jacobian matrix of i-th limb, and n_i means the number of active links of i-th limb. The leading superscript o implies that the elements are represented on the body center coordinate system shown in Fig. 1, which is fixed on a humanoid robot.

In the humanoid robot, the body center is floating, and thus the end point motion of i-th limb about the world coordinate system is written as follows:

$$\dot{x}_i = X_i^{-1}\dot{x}_o + X_o{}^oJ_i\dot{q}_i \tag{6}$$

where $\dot{x}_o = [\dot{r}_o^T; \omega_o{}^T]^T \in \Re^6$ is the velocity of the body center represented on the world coordinate system, and

$$X_i = \begin{bmatrix} I_3 & [R_o{}^o r_i\times] \\ 0_3 & I_3 \end{bmatrix} \in \Re^{6\times 6} \tag{7}$$

is a (6×6) matrix which relates the body center velocity and the i-th limb velocity. I_3 and 0_3 are an (3×3) identity and zero matrix, respectively. $R_o{}^o r_i$ is the position vector from the body center to the end point of the i-th limb represented on the world coordinate frame. $[(\cdot)\times]$ is a skew-symmetric matrix for the cross product. The transformation matrix X_o is

$$X_o = \begin{bmatrix} R_o & 0_3 \\ 0_3 & R_o \end{bmatrix} \in \Re^{6\times 6} \tag{8}$$

where $R_o \in \Re^{3\times 3}$ is the orientation of the body center represented on the world coordinate frame, and hereafter, we will use the relation $J_i \stackrel{\triangle}{=} X_o{}^oJ_i$.

All the limbs in a robot should have the same body center velocity, in other words, from Eq. (6), we can see that all the limbs should satisfy the compatibility condition that the body center velocity is the same, and thus, i-th limb and j-th limb should satisfy the following relation:

$$X_i(\dot{x}_i - J_i\dot{q}_i) = X_j(\dot{x}_j - J_j\dot{q}_j). \tag{9}$$

From Eq. (9), the joint velocity of any limb can be represented by the joint velocity of the base limb and cartesian motions of limbs. Actually, the base limb should be chosen to be the support leg in single support phase or one of both legs in double support phase. Let us express the base limb with the subscript 1, then the joint velocity of i-th limb is expressed as:

$$\dot{q}_i = J_i^+\dot{x}_i - J_i^+ X_{i1}(\dot{x}_1 - J_1\dot{q}_1), \tag{10}$$

for $i = 2,\cdots,n$, where J_i^+ means the Moore-Penrose pseudoinverse of J_i and

$$X_{i1} \stackrel{\triangle}{=} X_i^{-1}X_1 = \begin{bmatrix} I_3 & [R_o({}^o r_1 - {}^o r_i)\times] \\ 0_3 & I_3 \end{bmatrix}. \tag{11}$$

The position of CoM represented on the world coordinate frame, in Fig. 1, is given by

$$c = r_o + \sum_{i=1}^{n} R_o{}^o c_i \tag{12}$$

where n is the number of limbs, c is the position vector of CoM represented on the world coordinate system, and ${}^o c_i$ means the CoM position vector of i-th limb represented on the body center coordinate frame which is composed of n_i active links. Now, let us differentiate Eq. (12), then the it is obtained as follows:

$$\dot{c} = \dot{r}_o + \omega_o \times (c - r_o) + \sum_{i=1}^{n} R_o{}^oJ_{c_i}\dot{q}_i. \tag{13}$$

where ${}^oJ_{c_i} \in \Re^{3\times n_i}$ means CoM Jacobian matrix of i-th limb represented on the body center coordinate frame, and hereafter, we will use the relation $J_{c_i} \stackrel{\triangle}{=} R_o{}^oJ_{c_i}$.

Remark **1** *The CoM Jacobian matrix of i-th limb represented on the body center frame is expressed by*

$$ {}^{o}J_{c_i} \triangleq \sum_{k=1}^{n_i} \mu_{i,k} \frac{\partial^{o} c_{i,k}}{\partial q_i}, \tag{14} $$

where ${}^{o}c_{i,k} \in \Re^3$ means the position vector of center of mass of k-th link in i-th limb represented on the body center frame and the mass influence coefficient of k-th link in i-th limb is defined as follow:

$$ \mu_{i,k} \triangleq \frac{\text{mass of k-th link in i-th limb}}{\text{total mass}}. \tag{15} $$

The motion of body center frame can be obtained by using Eq. (6) for the base limb as follows:

$$ \begin{aligned} \dot{x}_o &= X_1 \{\dot{x}_1 - J_1 \dot{q}_1\} \\ \begin{bmatrix} \dot{r}_o \\ \omega_o \end{bmatrix} &= \begin{bmatrix} I_3 & [R_o {}^{o}r_1 \times] \\ 0_3 & I_3 \end{bmatrix} \left\{ \begin{bmatrix} \dot{r}_1 \\ \omega_1 \end{bmatrix} - \begin{bmatrix} J_{v_1} \\ J_{\omega_1} \end{bmatrix} \dot{q}_1 \right\}, \end{aligned} \tag{16} $$

where J_{v_1} and J_{ω_1} are the linear and angular velocity part of the base limb Jacobian J_1 expressed on the world coordinate frame, respectively. Now, if Eq. (10) is applied to Eq. (13) for all limbs except the base limb with subscript 1, the CoM motion is rearranged as follows:

$$ \begin{aligned} \dot{c} = \; & \dot{r}_o + \omega_o \times (c - r_o) + J_{c_1} \dot{q}_1 \\ & + \sum_{i=2}^{n} J_{c_i} J_i^{+} (\dot{x}_i - X_{i1} \dot{x}_1) + \sum_{i=2}^{n} J_{c_i} J_i^{+} X_{i1} J_1 \dot{q}_1. \end{aligned} \tag{17} $$

Here, if Eq. (16) is applied to Eq. (17), then the CoM motion is only related with the motion of base limb. Also, if the base limb has the face contact with the ground (the end-point of base limb represented on world coordinate frame is fixed, $\dot{x}_1 = 0$, namely, $\dot{r}_1 = 0$, $\omega_1 = 0$), then Eq. (17) is simplified as follows:

$$ \begin{aligned} \dot{c} - \sum_{i=2}^{n} J_{c_i} J_i^{+} \dot{x}_i = \; & -J_{v_1} \dot{q}_1 + r_{c1} \times J_{\omega_1} \dot{q}_1 + J_{c_1} \dot{q}_1 \\ & + \sum_{i=2}^{n} J_{c_i} J_i^{+} X_{i1} J_1 \dot{q}_1. \end{aligned} \tag{18} $$

where $r_{c1} = c - r_1$.

Finally, $3 \times n_1$ CoM Jacobian matrix with embedded motions can be rewritten like usual kinematic Jacobian of base limb:

$$ \dot{c}_{\text{fsem}} = J_{\text{fsem}} \, \dot{q}_1, \tag{19} $$

where

$$ \dot{c}_{\text{fsem}} \triangleq \dot{c} - \sum_{i=2}^{n} J_{c_i} J_i^{+} \dot{x}_i, \tag{20} $$

$$ J_{\text{fsem}} \triangleq -J_{v_1} + r_{c1} \times J_{\omega_1} + J_{c_1} + \sum_{i=2}^{n} J_{c_i} J_i^{+} X_{i1} J_1. \tag{21} $$

Here, if the CoM Jacobian is augmented with the orientation Jacobian of body center ($\omega_o = -J_{\omega_1} q_1$) and all desired cartesian motions are embedded in Eq. (20), then the desired joint configurations of base limb (support limb) are resolved as follows:

$$ \dot{q}_{1,d} = \begin{bmatrix} J_{\text{fsem}} \\ -J_{\omega_1} \end{bmatrix}^{+} \begin{bmatrix} \dot{c}_{\text{fsem},d} \\ \omega_{o,d} \end{bmatrix}, \tag{22} $$

where the subscript d means the desired motion and

$$ \dot{c}_{\text{fsem},d} \quad = \dot{c}_d - \textstyle\sum_{i=2}^{n} J_{c_i} J_i^{+} \dot{x}_{i,d}. \tag{23} $$

All the given desired limb motions, $\dot{x}_{i,d}$ are embedded in the relation of CoM Jacobian, thus the effect of the CoM movement generated by the given limb motion is compensated by the base limb. The CoM motion with fully specified embedded motions,

After solving Eq. (22), the desired joint motion of the base limb is obtained. The resulting base limb motion makes a humanoid robot balanced automatically during the movement of the all other limbs. With the desired joint motion of base limb, the desired joint motions of all other limbs can be obtained by Eq. (10) as follow:

$$ \dot{q}_{i,d} = J_i^{+} (\dot{x}_{i,d} + X_{i1} J_1 \dot{q}_{1,d}), \quad \text{for } i = 2, \cdots, n. \tag{24} $$

The resulting motion follows the given desired motions, regardless of balancing motion by base limb. In other words, the suggested kinematic resolution method of CoM Jacobian with embedded motion offers the WBC(whole body coordination) function to the humanoid robot automatically.

3 STABILITY

The control system is said to be disturbance input-to-state stable (ISS), if there exists a smooth positive definite radially unbounded function $V(e,t)$, a class $\mathcal{K}_\infty$ function γ_1 and a class $\mathcal{K}$ function γ_2 such that the following dissipativity inequality is satisfied:

$$ \dot{V} \le -\gamma_1(|e|) + \gamma_2(|\varepsilon|), \tag{25} $$

where $\dot{V}$ represents the total derivative for Lyapunov function, e the error state vector and ε disturbance input vector.

In this section, we propose the posture/walking controller for bipedal robot systems as shown in Fig. 2. In this figure, first, the ZMP Planer and CoM Planer generate the desired trajectories satisfying the following differential equation:

$$ p_{i,d} = c_{i,d} - 1/\omega_n^2 \ddot{c}_{i,d} \quad \text{for} \quad i = x, y. \tag{26} $$

Second, the simplified model for the real bipedal walking robot has the following dynamics:

$$ \begin{aligned} \dot{c}_i &= u_i + \varepsilon_i \\ p_i &= c_i - 1/\omega_n^2 \ddot{c}_i \quad \text{for} \quad i = x, y, \end{aligned} \tag{27} $$

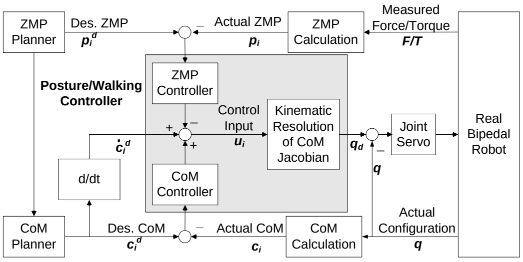

Figure 2: Posture/Walking Controller for Humanoid Robot.

where ε_i is the disturbance input produced by actual control error, u_i is the control input, c_i and p_i are the actual positions of CoM and ZMP measured from the real bipedal robot, respectively. Here, we assume that the the disturbance produced by control error is bounded and its differentiation is also bounded, namely, $|\varepsilon_i| < a$ and $|\dot{\varepsilon}_i| < b$ with any positive constants a and b. Also, we should notice that the control error always exists in real robot systems and its magnitude depends on the performance of embedded local joint servos. The following theorem proves the stability of the posture/walking controller to be suggested for the simplified robot model.

Theorem **1** *Let us define the ZMP and CoM error for the simplified bipedal robot control system (27) as follows:*

$$e_{p,i} \triangleq p_{i,d} - p_i \tag{28}$$

$$e_{c,i} \triangleq c_{i,d} - c_i \quad \text{for} \quad i = x, y. \tag{29}$$

If the posture/walking control input u_i in Fig. 2 has the following form:

$$u_i = \dot{c}_i^d - k_{p,i}e_{p,i} + k_{c,i}e_{c,i} \tag{30}$$

under the gain conditions:

$$k_{c,i} > \omega_n \quad \text{and} \quad 0 < k_{p,i} < \left(\frac{\omega_n^2 - \beta^2}{\omega_n} - \gamma^2\right) \tag{31}$$

satisfying the following conditions:

$$\beta < \omega_n \quad \text{and} \quad \gamma < \sqrt{\frac{\omega_n^2 - \beta^2}{\omega_n}}, \tag{32}$$

then the posture/walking controller gives the disturbance input($\varepsilon_i, \dot{\varepsilon}_i$)-to-state($e_{p,i}, e_{c,i}$) stability (ISS) to a simplified bipedal robot, where, the $k_{p,i}$ is the proportional gain of ZMP controller and $k_{c,i}$ is that of CoM controller in Fig. 2.

Proof. First, we get the error dynamics from Eq. (26) and (27) as follows:

$$\ddot{e}_{c,i} = \omega_n^2(e_{c,i} - e_{p,i}). \tag{33}$$

Second, another error dynamics is obtained by using Eq. (27) and (30) as follows:

$$k_{p,i}e_{p,i} = \dot{e}_{c,i} + k_{c,i}e_{c,i} + \varepsilon_i, \tag{34}$$

also, this equation can be rearranged for $\dot{e}_c$:

$$\dot{e}_{c,i} = k_{p,i}e_{p,i} - k_{c,i}e_{c,i} - \varepsilon_i. \tag{35}$$

Third, by differentiating the equation (34) and by using equations (33) and (35), we get the following:

$$\begin{aligned}
\dot{e}_{p,i} &= 1/k_{p,i}\,(\ddot{e}_{c,i} + k_{c,i}\dot{e}_{c,i} + \dot{\varepsilon}_i) \\
&= \omega_n^2/k_{p,i}(e_{c,i} - e_{p,i}) \\
&\quad + k_{c,i}/k_{p,i}(k_{p,i}e_{p,i} - k_{c,i}e_{c,i} - \varepsilon_i) + (1/k_{p,i})\dot{\varepsilon}_i \\
&= \left(\frac{\omega_n^2 - k_{c,i}^2}{k_{p,i}}\right)e_{c,i} - \left(\frac{\omega_n^2 - k_{p,i}k_{c,i}}{k_{p,i}}\right)e_{p,i} \\
&\quad + \frac{1}{k_{p,i}}(\dot{\varepsilon}_i - k_{c,i}\varepsilon_i).
\end{aligned} \tag{36}$$

Fourth, let us consider the following Lyapunov function:

$$V(e_{c,i}, e_{p,i}) \triangleq \frac{1}{2}\left[(k_{c,i}^2 - \omega_n^2)e_{c,i}^2 + k_{p,i}^2 e_{p,i}^2\right], \tag{37}$$

where $V(e_c, e_p)$ is the positive definite function for $k_{p,i} > 0$ and $k_{c,i} > \omega_n$, except $e_{c,i} = 0$ and $e_{p,i} = 0$. Now, let us differentiate the above Lyapunov function, then we can get the following:

$$\begin{aligned}
\dot{V} \le\; & -(k_{c,i} - \alpha^2)(k_{c,i}^2 - \omega_n^2)e_{c,i}^2 \\
& -k_{p,i}[\omega_n^2 - (k_{p,i} + \gamma^2)k_{c,i} - \beta^2]e_{p,i}^2 \\
& + \left[\frac{(k_{c,i}^2 - \omega_n^2)}{4\alpha^2} + \frac{k_{p,i}k_{c,i}}{4\gamma^2}\right]\varepsilon_i^2 + \frac{k_{p,i}}{4\beta^2}\dot{\varepsilon}_i^2
\end{aligned} \tag{38}$$

where $e_{c,i}^2$ term is negative definite with any positive constant satisfying $\alpha < \sqrt{\omega_n}$ and $e_{p,i}^2$ term is negative definite under the given conditions (31). Here, since the inequality (38) follows the ISS property (25), we concludes that the proposed posture/walking controller gives the disturbance input($\varepsilon_i, \dot{\varepsilon}_i$)-to-state($e_{p,i}, e_{c,i}$) stability (ISS) to the simplified control system model of bipedal robot. □

To make active use of the suggested control scheme, the control input u of Eq. (30) suggested in Theorem 1 is applied to the place of the term $\dot{c}_d$ in Eq. (23). In other words, equation (23)is modified to include the ZMP and CoM controllers as following forms:

$$\dot{c}_{\text{fsem},d} = u - \sum_{i=2}^{n} J_{c_i} J_i^{+} \dot{x}_{i,d} \tag{39}$$

where $u \triangleq \dot{c}_d - k_p e_p + k_c e_c$. And then, the suggested kinematic resolution method of Eq. (22) and (24) are utilized to obtain the desired base limb and other limb motions in the joint space as shown in Fig. 2.

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