MULTIPLE MODEL ADAPTIVE EXTENDED KALMAN FILTER FOR THE ROBUST LOCALIZATION OF A MOBILE ROBOT

Y. Touati, Y. Amirat, Z. Djama and A. Ali Chérif*
LISSI Laboratory, Paris 12 University, 120-122 rue Paul Armangot
94400 Vitry-sur-Seine, France

**Department of Computing, University of Paris 8, 2 rue de la liberté - 93526 Saint-Denis cedex*

Keywords: Mobile robot, Robust Localization, Multiple Model, Hybrid Systems, Kalman Filtering, Data Fusion.

Abstract: This paper focuses on robust pose estimation for mobile robot localization. The main idea of the approach proposed here is to consider the localization process as a hybrid process which evolves according to a model among a set of models with jumps between these models according to a Markov chain. In order to improve the robustness of the localization process, an on line adaptive estimation approach of noise statistics (state and observation), is applied for each mode. To demonstrate the validity of the proposed approach and to show its effectiveness, we've compared it to the standard approaches. For this purpose, simulations were carried out to analyze the performances of each approach in various scenarios.

1 INTRODUCTION

Localization constitutes a key problem in mobile robotics (Borenstein, 1996). It consists of estimating the robot's pose (position, orientation) with respect to its environment from sensor data. Therefore, a better sensory data exploitation is required to increase robot's autonomy. The simplest way to estimate the pose parameters is the integration of odometric data which, however, is associated with unbounded errors, resulting from uneven floors, wheel slippage, limited resolution of encoders, etc. However, such a technique is not reliable due to cumulative errors occurring over the long run. Therefore, a mobile robot must also be able to localize or estimate its parameters with respect to the internal world model by using the information obtained with its external sensors.

The use of sensory data from a range of disparate multiple sensors, is to automatically extract the maximum amount of possible information about the sensed environment under all operating conditions. The main idea of data fusion methods is to provide a reliable estimation of robot's pose, taking into account the advantages of the different sensors (Harris, 1998). The Kalman filter is the best known and most widely applied parameter and state estimation algorithm in data fusion methods (Gao, 2002). Such a technique can be implemented from the kinematic model of the robot and the observation (or measurement) model, associated to external sensors (gyroscope, camera, telemeter, etc.). Basically, the Kalman filter gives a linear, unbiased, and minimum error variance recursive algorithm to optimally estimate the unknown state of a linear dynamic system from Gaussian distributed noisy observations. The Kalman filtering process can be considered as a prediction-update formulation. The algorithm uses a predefined linear model of the system to predict the state at the next time step. The prediction and updates are combined using the Kalman gain which is computed to minimize the Mean Square Error (MSE) of the state estimate. The Extended Kalman Filter (EKF) is a version of the Kalman filter that can handle non-linear dynamics or non-linear measurement equations. Various approaches based on EKF have been developed. These approaches work well as long as the used information can be described by simple statistics well enough. The lack of relevant information is compensated by using models of various processes. However, such model-based approaches require assumptions about parameters which might be very difficult to determine (white Gaussian noise and initial uncertainty over Gaussian distribution). Assumptions that guarantee optimum convergence are often violated and, therefore, the process is not

Touati Y., Amirat Y., Djama Z. and Ali Chérif A. (2007).
MULTIPLE MODEL ADAPTIVE EXTENDED KALMAN FILTER FOR THE ROBUST LOCALIZATION OF A MOBILE ROBOT.
In *Proceedings of the Fourth International Conference on Informatics in Control, Automation and Robotics*, pages 446-454
DOI: 10.5220/0001633704460454

optimal or it can not even converge. In fact, many approaches are based on fixed values of the measurement and state noise covariance matrices. However, such information is not a priori available, especially if the trajectory of the robot is not elementary and if changes occur in the environment. Moreover, it has been demonstrated in the literature that how poor knowledge of noise statistics (noise covariance on state and measurement vectors) may seriously degrade the Kalman filter performance (Jetto, 1999). In the same manner, the filter initialization, the signal-to-noise ratio, the state and observation processes constitute critical parameters, which may affect the filtering quality. The stochastic Kalman filtering techniques were widely used in localization (Gao, 2002) (Chui, 1987) (Arras, 2001)(Borthwick, 1993) (Jensfelt, 2001) (Neira, 1999) (Perez, 1999) (Borges, 2003). Such approaches rely on approximative filtering, which requires ad hoc tuning of stochastic modelling parameters, such as covariance matrices, in order to deal with the model approximation errors and bias on the predicted pose. In order to compensate such error sources, local iterations (Kleeman, 1992), adaptive models (Jetto 1999) and covariance intersection filtering (Julier, 1997)(Xu, 2001) have been proposed. An interesting approach solution was proposed in (Jetto, 1999), where observation of the pose corrections is used for updating of the covariance matrices. However, this approach seems to be vulnerable to significant geometric inconsistencies of the world models, since inconsistent information can influence the estimated covariance matrices.

In the literature, the localization problem is often formulated by using a single model, from both state and observation processes point of view. Such an approach, introduces inevitably modelling errors which degrade filtering performances, particularly, when signal-to-noise ratio is low and noise variances have been estimated poorly. Moreover, to optimize the observation process, it is important to characterize each external sensor not only from statistic parameters estimation perspective but also from robustness of observation process perspective. It is then interesting to introduce an adequate model for each observation area in order to reject unreliable readings. In the same manner, a wrong observation leads to a wrong estimation of the state vector and consequently degrades the performance of localization algorithm. Multiple-Model estimation has received a great deal of attention in recent years due to its distinctive power and great recent success in handling problems with both structural and parametric uncertainties and/or changes, and in decomposing a complex problem into simpler sub-problems, ranging from target tracking to process control (Blom, 1988)(Li, 2000) (Li, 1993)(Mazor, 1996).

This paper focuses on robust pose estimation for mobile robot localization. The main idea of the approach proposed here is to consider the localization process as a hybrid process which evolves according to a model among a set of models with jumps between these models according to a Markov chain (Djama, 1999)(Djama, 2001). A close approach for multiple model filtering is proposed in (Oussalah 2001). In our approach, models refer here to both state and observation processes. The data fusion algorithm which is proposed is inspired by the approach proposed in (Dufour 1994). We generalized the latter for multi mode processes by introducing multi mode observations. We also introduced iterative and adaptive EKFs for estimating noise statistics. Compared to a single model-based approach, such an approach allows the reduction of modelling errors and variables, an optimal management of sensors and a better control of observations in adequacy with the probabilistic hypotheses associated to these observations. For this purpose and in order to improve the robustness of the localization process, an on line adaptive estimation approach of noise statistics (state and observation) proposed in (Jetto, 1999), is applied to each mode. The data fusion is performed by using Adaptive Linear Kalman Filters for linear processes and Adaptive Extended Kalman Filters for nonlinear processes.

The reminder of this article is organized as follows. Section 2 discusses the problem statement of multi-sensor data fusion for the localization of a mobile robot. We develop the proposed robust pose estimation algorithm in section 3 and its application is demonstrated in section 4. Experimental results and a comparative analysis with standard existing approaches are also presented in this section.

2 PROBLEM STATEMENT

This paper deals with the problem of multi sensor filtering and data fusion for the robust localization of a mobile robot. In our present study, we consider a robot equipped with two telemeters placed perpendicularly, for absolute position measurements of the robot with respect to its environment, a gyroscope for measuring robot’s orientation, two drive wheels and two separate encoder wheels attached with optical shaft encoders for odometry measurements (Figure 1). The environment where the mobile robot moves is a rectangular room without obstacles (Figure 2). The aim is not to develop a new method for environment reconstruction or modelling from data sensors; rather, the goal is to propose a new approach to improve existing data fusion and filtering techniques for robust localization of a mobile robot. For an environment with a more complex shape, the observation model, which has to be employed at a given time, will depend on the robot's situation (robot's trajectory, robot's pose with respect to its environment) and on the geometric or symbolic model of environment.

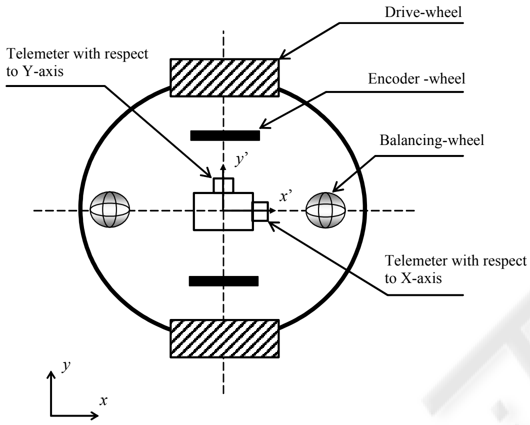

Figure 1: Mobile robot description.

Odometric model: Let $X_e(k)=[x(k)\ \ y(k)\ \ \theta(k)]^T$ be the state vector at time k, describing the robot's pose with respect to the fixed coordinate system. The kinematic model of the robot is described by the following equations:

$$x_{k+1} = x_k + l_k \cdot \cos(\theta_k + \Delta\theta_k/2) \quad (1)$$

$$y_{k+1} = y_k + l_k \sin(\theta_k + \Delta\theta_k/2) \quad (2)$$

$$\theta_{k+1} = \theta_k + \Delta\theta_k \quad (3)$$

with: $l_k = (l_k^r + l_k^l)/2$ and $\Delta\theta_k = (l_k^r - l_k^l)/d$. l_k^r and l_k^l are the elementary displacements of the right and the left wheels; d the distance between the two encoder wheels.

Observation model of telemeters: As the environment is a rectangular room, the telemeters measurements correspond to the distances from the robot location to walls (Fig. 2.).

Then, the observation model of telemeters is described as follows:

for $0 \le \theta(k) < \theta^l$:

$$d(k) = \left(d^x - x(k)\right)/\cos(\theta(k)) \text{ with respect to } X \text{ axis} \quad (4)$$

for $\theta^l \le \theta(k) \le \theta^m$:

$$d(k) = \left(d^y - y(k)\right)/\sin(\theta(k)) \text{ with respect to } Y \text{ axis.} \quad (5)$$

with:

- d^x and d^y, respectively the length and the width of the experimental site;
- θ^l and θ^m, respectively the angular bounds of observation domain with respect to X and Y axes;
- $d(k)$ is the distance between the robot and the observed wall with respect to X or Y axes at time k.

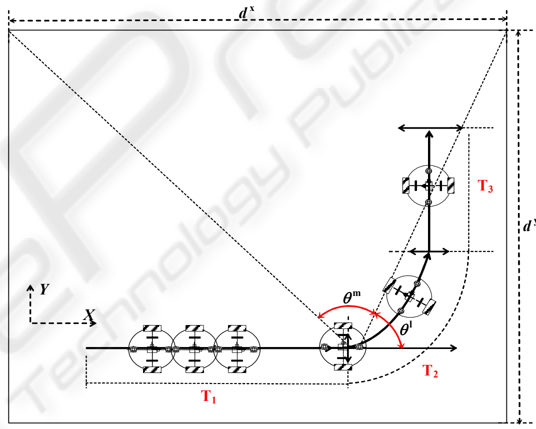

Figure 2: Telemeters measurements –Nominal trajectory composed of sub trajectories T1-T2 and T3.

Observation model of gyroscope: By integrating the rotational velocity, the gyroscope model can be expressed by the following equation:

$$\theta_l(k) = \theta(k) \quad (6)$$

Each sensor described above is subject to random noise. For instance, the encoders introduce incremental errors (slippage), which particularly affect the estimation of the orientation. For a telemeter, let's note various sources of errors: geometric shape and surface roughness of the target, beam width. For a gyroscope, the sources of errors are: the bias drift, the nonlinearity in the scale factor and the gyro's susceptibility to changes in ambient temperature. So, both the odometric and observation models must integrate additional terms representing these noises. Models inaccuracies induce also noises which must be taken into account. It is well known that the odometric model is subject to inaccuracies caused by factors such as: measured wheel diameters, unequal wheel-diameters, trajectory approximation of robot between two consecutive samples. These noises are usually assumed to be Zero-mean white Gaussian with known covariance. This hypothesis is discussed and reconsidered in the proposed approach. Besides, an estimation error of orientation introduces an ambiguity in the telemeters measurements (one telemeter is assumed to measure along X axis while it is measuring along Y axis and vice-versa). This situation is particularly true when the orientation is near angular bounds θ^l and θ^m. This justifies the use of multiple model to reduce measuring errors and efficiently manage robot's sensors. For this purpose, we have introduced the concept of observation domain (boundary angles) as defined in equations (4) and (5).

3 ROBUST MULTIPLE MODEL FILTERING APPROACH

In this section, we present the data fusion and filtering approach for the localization of a mobile robot. In order to increase the robustness of the localization and as discussed in section 2, the localization process is decomposed into multiple models. Each model is associated with a mode and an interval of validity corresponding to the observation domain; the aim is to consider only reliable information by filtering erroneous information. The localization is then considered as a hybrid process. A Markov chain is employed for the prediction of each model according to the robot mode. The multiple model approach is best understandable in terms of stochastic hybrid systems. The state of a hybrid system consists of two parts: a continuously varying base-state component and a modal state component, also known as system mode, that may only jump among points, rather than vary continuously, in a (usually discrete) set. The base state components are the usual state variables in a conventional system. The system mode is a mathematical description of a certain behavior pattern or structure of the system. In our study, the mode corresponds to the robot's orientation. In fact, the latter parameter governs the observation model of telemeters along with observation domain. Other parameters, like velocity or acceleration, could also be taken into account for mode's definition. Updating of mode's probability is carried out either from a given criterion or from given laws (probability or process). In this study, we assume that each Markovian jump (mode) is observable (Djama, 2001)(Dufour, 1994). The mode is observable and measurable from the gyroscope.

3.1 Multiple Model Formulation

Let us consider a stochastic hybrid system. For a linear process, the state and observation processes are given by:

$$\begin{aligned} X_e(k/k-1,\alpha_k) &= A(\alpha_k)\cdot X_e(k-1/k-1,\alpha_k) \\ &+ B(k,\alpha_k)\cdot U(k-1,\alpha_k) + W(k,\alpha_k) \end{aligned} \tag{7}$$

$$Y_e(k,\alpha_k) = C(\alpha_k)\cdot X_e(k/k-1,\alpha_k) + V(k,\alpha_k) \tag{8}$$

For a nonlinear process, the state and observation processes are described by:

$$\begin{aligned} X_e(k/k-1,\alpha_k) &= F\big(X_e(k-1/k-1),\alpha_k,U(k-1)\big) \\ &+ W(k,\alpha_k) \end{aligned} \tag{9}$$

$$Y_e(k,\alpha_k) = G_e\big(X_e(k/k-1),\alpha_k\big) + V(k,\alpha_k) \tag{10}$$

where: X_e is the base state vector;
Y_e is the noisy observation vector;
U is the input vector;
α_k is the modal state or system mode at time k, which denotes the mode during the k[th] sampling period;
W and V are the mode-dependent state and measurement noise sequences, respectively.

The system mode sequence $\langle \alpha_k \rangle$ is assumed for simplicity to be a first-order homogeneous Markov chain with the transition probabilities:

$$P\{\alpha_{k+1}^j \mid \alpha_k^i\} = \pi_{ij} \qquad \forall \alpha_i, \alpha_j \in S$$

where α_k^j denotes that mode α_j is in effect at time k and S is the set of all possible system modes, called mode space.

The state and measurement noises are of Gaussian white type. In our approach, the state and measurement processes are assumed to be governed by the same Markov chain. However, it's possible to define differently a Markov chain for each process. The Markov chain transition matrix is stationary and well defined.

3.2 Variance Estimation Algorithm

It is well known that how poor estimates of noise statistics may lead to the divergence of Kalman filter and degrade its performance. To prevent this divergence, we apply an adaptive algorithm for the adjustment of the state and measurement noise covariance matrices.

a. Estimation of measurement noise variance

Let $R = \left(\sigma_{v,i}^2(k)\right)\ (i = 1:n_0)$, be the measurement noise variance at time k for each component of the observation vector. n_0 denotes the number of observers (sensors number).

Let $\hat{\beta}(k)$ the squared mean error for stable measurement noise variance:

$$\hat{\beta}(k) = \frac{1}{n}\sum_{j=0}^{n}\gamma_i^2(k-1) \tag{11}$$

where $\gamma(k)$ represents the innovation.

For $n+1$ samples, the variance of $\hat{\beta}(k)$ can be written as:

$$E\left(\hat{\beta}(k)\right) = \frac{1}{n+1}\sum_{j=0}^{n}\begin{pmatrix} C_i(k-j)\cdot P(k-j,k-j-1)\cdot \\ C_i(k-j)^T + \sigma_{v,i}^2 \end{pmatrix} \tag{12}$$

Then, we obtain the estimation of the measurement noise variance:

$$\hat{\sigma}_{v,i}^2 = \max\left\{\frac{1}{n}\sum_{j=0}^{n}\begin{pmatrix}\gamma_i^2(k-j) - \frac{n}{n+1}\cdot C_i(k-j)\cdot \\ P(k-j,k-j-1)\cdot C_i(k-j)^T\end{pmatrix}, 0\right\} \tag{13}$$

The restriction with respect to zero is related to the notion of variance.

A recursive formulation of the previous estimation can be written:

$$\hat{\sigma}_{v,i}^2(k) = \max\left\{\hat{\sigma}_{v,i}^2(k-1) + \frac{1}{n}\cdot\begin{pmatrix}\gamma_i^2(k) \\ -\gamma_i^2(k-(n+1)) \\ -\frac{n}{n+1}\cdot\Psi\end{pmatrix}, 0\right\} \tag{14}$$

where:

$$\begin{aligned}\Psi = {} & C_i(k)\cdot P(k,k-1)\cdot C_i(k)^T - C_i(k-(n+1))\cdot \\ & P(k-(n+1),k-(n+1)-1)\cdot C_i(k-(n+1))^T\end{aligned} \tag{15}$$

b. Estimation of state noise variance

To estimate the state noise variance, we use the same principle as in subsection a. One can write:

$$\hat{Q}_e(k) = \hat{\sigma}_{n,i}^2(k)\cdot Q_d \tag{16}$$

By assuming that noises on the two encoder wheels measurements obey to the same law and have the same variance, the estimation of state noise variance can be written:

$$\hat{\sigma}_{n,i}^2(k) = \max\left\{\begin{matrix}\dfrac{\gamma_i^2(k-1) - C_i(k+1)\cdot P(k+1,k)\cdot C_i(k+1)^T - \hat{\sigma}_{v,i}^2(k+1)}{C_i(k+1)\cdot Q_d\cdot C_i(k+1)^T}, \\ 0\end{matrix}\right\} \tag{17}$$

with:

$$\hat{Q}_d(k) = B(k)\cdot B(k)^T \tag{18}$$

By replacing the measurement noise variance by its estimate, we obtain a mean value given by the following equation:

$$\hat{\sigma}_n^2(k) = \max\left\{\frac{1}{(m+1)\cdot n_0}\sum_{j=1}^{m}\sum_{i=1}^{n_0}\hat{\sigma}_{n,i}^2(k-j), 0\right\} \tag{19}$$

Where m represents the sample number.

The algorithm described above carries out an on line estimation of state and measurement noise variances. Parameters n and m are chosen according to the number of samples used at time k. The noises variances are initialized from an "*a priori"* information and then updated on line. In this approach, variances are updated according the robot's mode and the measurement models.

For an efficient estimation of noise variances, an ad hoc technique consisting in a measure selection is employed. This technique consists of filtering unreliable readings by excluding readings with weak probability like the appearance of fast fluctuations. For instance, in the case of Gaussian distribution, we know that about 95% of the data are concentrated in the interval of confidence $[m-2\sigma, m+2\sigma]$ where m represents the mean value and σ the variance.

The sequence in which the filtering of the state vector components is carried out is important. Once the step of filtering completed, the probabilities of each mode are updated from the observers (sensors). One can note that the approach used here is close, on one hand, to the Bayesian filter by the extrapolation of the state probabilities, and on the other to the filter with specific observation of the mode.

4 IMPLEMENTATION AND SIMULATION RESULTS

The approach described above for robust localization was applied for the mobile robot described in section 2. The nominal trajectory of the mobile robot includes three sub trajectories T1, T2 and T3, defining respectively a displacement along X axis, a curve and a displacement along Y axis (Fig. 2.). Note that the proposed approach remains valid for any type of trajectory (any trajectory can be approximated by a set of linear and circular sub trajectories). In our study, we have considered three models. This number can be modified according to the environment's structure, the type of trajectory (robot rotating around itself, forward or backward displacement, etc.) and to the number of observers (sensors). Notice that the number of models (observation and state) has no impact on the validity of the proposed approach.

To demonstrate the validity of the proposed approach (noticed AMM for Adaptive Multiple-Model) and to show its effectiveness, we've compared it to the following standard approaches: Single-Model based EKF without estimation variance (noticed SM), single-model based IEKF (noticed SMI). For this purpose, simulations were carried out to analyze the performances of each approach in various scenarios.

For sub trajectories T1 and T3, filtering and data fusion are carried out by iterative linear Kalman filters due to linearity of the models, and for sub trajectory T2, by iterative and extended Kalman filters. The observation selection technique is applied for each observer before the filtering step in order to control, on one side, the estimation errors of variances, and on the other, after each iteration, to update the state noise variance. If an unreliable reading is rejected at a given filtering iteration, this has for origin either a bad estimation of the next component of the state vector and of the prediction of the corresponding observation, or a bad updating of the corresponding state noise variance. The iterative filtering is optimal when it is carried out for each observer and no reading is rejected. In the implementation of the proposed approach, the state noise variance is updated, for a given mode i, is carried out according to the following filtering sequence: x, y and then θ.

Notation:
- εx, εy and $\varepsilon\theta$: the estimation errors corresponding to x, y and θ respectively;
- Ndx, Ndy and $Nd\theta$: the percentage of selected data for filtering, corresponding to components x, y and θ respectively;
- $Ndxe$, $Ndye$ and $Nd\theta e$: the percentage of selected data for estimation of the variances of state and measurement noises, corresponding to components x, y and θ respectively.

-+:SMI; °: SMI, --:AMM

Scenario 1

-Noise-to-signal Ratio of odometric sensors: right encoder: 8%, left encoder: 8%

-Noise-to-signal Ratio of Gyroscope: 3%

-Noise-to-signal Ratio of telemeter 1: 10% of the odometric elementary step

-Noise-to-signal Ratio of telemeter 2: 10% the odometric elementary step

-"A priori" knowledge on the variance in initial state: Good

-"A priori" knowledge on noise statistics (measurement and state variances): Good

In this scenario, the telemeters measurement noise is higher than state noise. We notice that performances of AMM filter are better that those of SM and SMI filters concerning x and y-components (Table 1; Fig. 3-5). In sub trajectory T3, the orientation's estimation error relating to AMM filter (Table 1) has no influence on filtering quality of the remaining components of state vector. Besides, one can note that this error decreases in this sub trajectory (Figure 6). In this case, only gyroscope is used for the prediction and updating the Markov chain probabilities. In sub trajectory T2, we notice that the estimation error along x-Axis for AMM filter is lightly higher than those relating to other filters. This error is concentrated on first half of T2 sub trajectory (Figure 7) and decreases then on second half of the trajectory. This can be explained by the fact that on one hand, the estimation variances algorithm rejected 0.7% of data, and on the other, the filtering step has rejected the same percentage of data. This justifies that neither the variances updating, nor the x-coordinate correction, were carried out.

Note that unlike filters SM and SMI, filter AMM has a robust behavior concerning pose estimation even when the signal-to-noise ratio is weak. By introducing the concept of observation domain for observation models, we obtain a better modeling of observation and a better management of robot's sensors. The last remark is related to the bad performances of filters SM and SMI when the signal-to-noise ratio is weak. This ratio degrades the estimation of the orientation angle, observation matrices, Kalman filter gain along with the prediction of the observations.

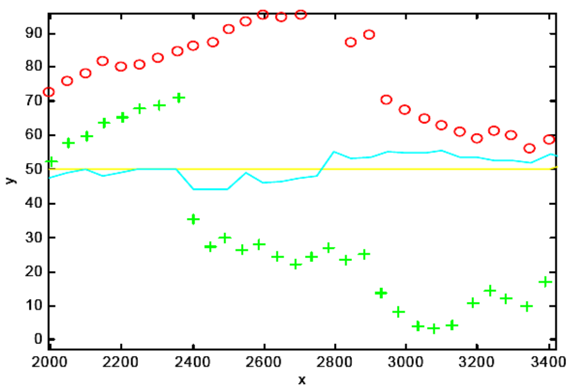

Figure 3: Estimated trajectories (sub trajectory T1).

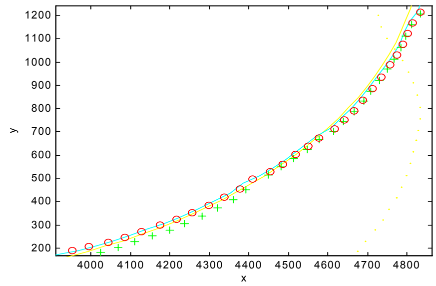

Figure 4: Estimated trajectories (sub trajectory T2).

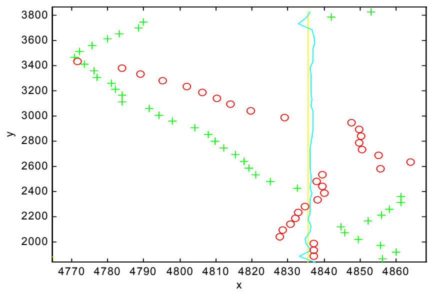

Figure 5: Estimated trajectories (sub trajectory T3).

Table 1: Average estimation errors (Scenario 1).

	T1			T2			T3		
	SM	SMI	AMM	SM	SMI	AMM	SM	SMI	AMM
εx (cm)	6.25	3.23	2.5	13.2	10.8	15.3	31.9	31.2	1.2
εy (cm)	13.6	16.7	2.3	23.9	11.9	8.25	19.2	5.75	3.23
$\varepsilon\theta$ (10^{-3} rad)	81.1	66.9	3.8	32.2	39.9	35.6	136	125	267.9

Ndx =99.37%, Ndy = 84.37%, $Nd\theta$ =99.37%,
$Ndxe$ =99.37%, $Ndye$ =97.5%, $Nd\theta e$ =99.37%.

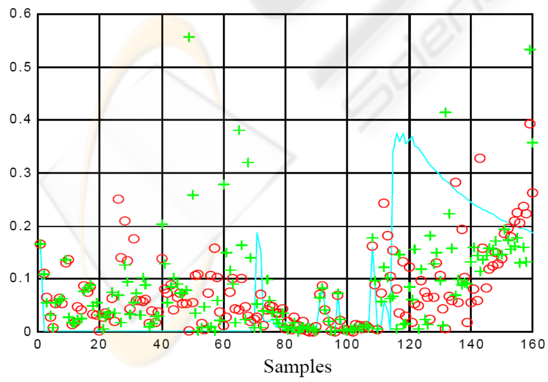

Figure 6: Orientation error.

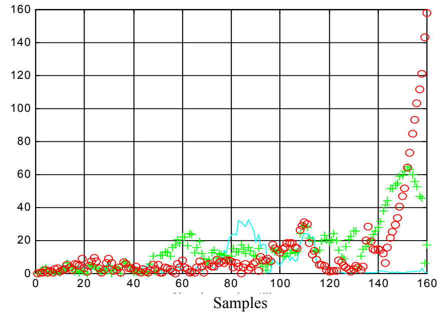

Figure 7: Position error with respect to X axis.

Scenario 2

-Noise-to-signal Ratio of odometric sensors: right encoder: 10%, left encoder: 10%

-Noise-to-signal Ratio of Gyroscope: 3%

-Noise-to-signal Ratio of telemeters: 4% of the odometric elementary step (40% of the state noise)

-"A priori" knowledge on the variance in initial state: Good

-"A priori" knowledge on noise variances (i) telemeters and state: Good; (ii) gyroscope: Bad

The results presented here (Table 2 and Fig. 8-10) show the influence of signal-to-noise ratio and the estimation of noise variances on performances of SM and SMI filters. In this scenario, the initial variance of measurement noise of the gyroscope is incorrectly estimated. Contrary to AMM approach, filters SM and SMI do not carry out any adaptation of this variance, leading to unsatisfactory performance.

Figure 11 illustrates the evolution of state noise variance estimate compared to the average variance. Note that the ratio between variances reaches 1.7 on sub trajectory T1, 3.0 on sub trajectory T2, and 3.3 on sub trajectory T3. It is important to mention that the algorithm proposed for estimation of variances estimates the actual value of state noise variance and not its average value. These results are related to the fact that the signal-to-noise ratio is weak both for the odometer and the telemeters.

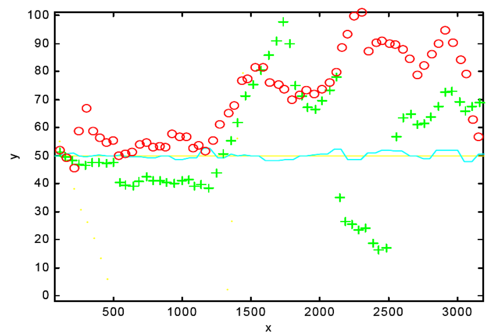

Figure 8: Estimated trajectories (sub trajectory T1).

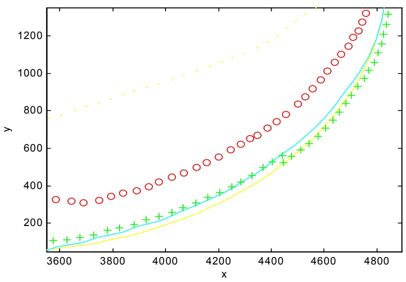

Figure 9: Estimated trajectories (sub trajectory T2).

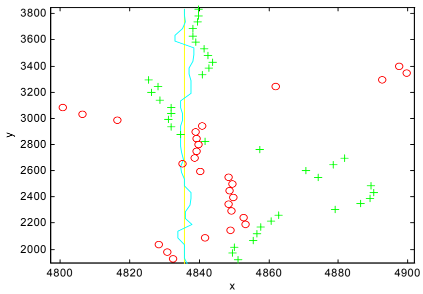

Figure 10: Estimated trajectories (sub trajectory T3).

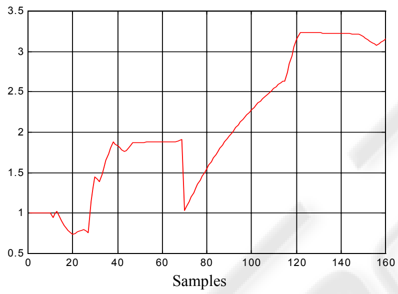

Figure 11: Ratio between the estimate of state noise variance and the average variance.

Table 2: Average estimation errors (Scenario 2).

	T1			T2			T3		
	SM	SMI	AMM	SM	SMI	AMM	SM	SMI	AMM
εx (cm)	11.7	11	1.8	19	75	13.6	17.3	40	1.3
εy (cm)	16.7	21	1	39	179	17.4	15.7	117	1.93
$\varepsilon\theta$ (10^{-3} rad)	99.3	129	1.5	42.9	175	35.4	97.5	167	37.8

Ndx =87.5%, Ndy =66%, $Nd\theta$ =99.37%, $Ndxe$ =87.5%, $Ndye$ =82.5%, $Nd\theta e$ =99.37%.

5 CONCLUSIONS

We presented in this paper a multiple model approach for the robust localization of a mobile robot. In this approach, the localization is considered as a hybrid process, which is decomposed into multiple models. Each model is associated with a mode and an interval of validity corresponding to the observation domain. A Markov chain is employed for the prediction of each model according to the robot mode. To prevent divergence of standard Kalman Filtering, we proposed the application of an adaptive algorithm for the adjustment of the state and measurement noise covariance matrices. For an efficient estimation of noise variances, we used an ad hoc technique consisting of a measure selection for filtering unreliable readings. The simulation results which we obtain in different scenarios show better performances of the proposed approach compared to standard existing filters. These investigations into utilizing multiple model technique for robust localization show promise and demand continuing research.

REFERENCES

J. Borenstein, B. Everett, L. Feng, Navigating Mobile Robots: Systems and Techniques. A.K. Peters, Ltd., Wellesley, MA, 1996.

C. Harris, A. Bailley, T. Dodd, Multi-sensor data fusion in defense and aerospace, *Journal of Royal Aerospace Society*, 162 (1015) (1998) 229-244.

J.B. Gao, C.J. Harris, Some remarks on Kalman filters for the multi-sensor fusion, *Journal of Information Fusion*, 3 (2002) 191-201.

C. Chui, G. Chen, Kalman filtering with real time applications, *Springer Series in Information Sciences, Springer-Verlag*, New-York 17 (1987) 23-24.

K.O. Arras, N. Tomatis, B.T. Jensen, R. Siegwart, Multisensor on-the-fly localization: precision and reliability for applications, *Robotics and Autonomous Systems*, 34 (2001) 131–143.

S. Borthwick, M. Stevens, H. Durrant-Whyte, Position estimation and tracking using optical range data, *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, 1993, pp. 2172–2177.

P. Jensfelt, H.I. Christensen, Pose tracking using laser scanning and minimalistic environment models, *IEEE Transactions on Robotics and Automation*, 17 (2) (2001) 138–147.

J. Neira, J.D. Tardós, J. Horn, G. Schmidt, Fusing range and intensity images for mobile robot localization, *IEEE Transactions on Robotics and Automation*, 15 (1) (1999) 76–84.

J.A. Pérez, J.A. Castellanos, J.M.M. Montiel, J. Neira, J.D. Tardós, Continuous mobile robot localization: vision vs. laser, *Proceedings of the IEEE International Conference on Robotics and Automation,* 1999, pp. 2917–2923.

G.A. Borges, M.J. Aldon, Robustified estimation algorithms for mobile robot localization based geometrical environment maps, *Robotics and Autonomous Systems,* 45 (2003) 131-159.

L. Kleeman, Optimal estimation of position and heading for mobile robots using ultrasonic beacons and dead-reckoning, *Proceedings of the IEEE International Conference on Robotics and Automation*, 1992, pp. 2582–2587.

L. Jetto, S. Longhi, G. Venturini, Development and experimental validation of an adaptive Kalman filter for the localization of mobile robots, *IEEE Transactions on Robotics and Automation*, 15 (2) (1999) 219–229.

S.J. Julier, J.K. Uhlmann, A non-divergent estimation algorithm in the presence of unknown correlations, *Proceedings of the American Control Conference*, 1997.

X. Xu, S. Negahdaripour, Application of extended covariance intersection principle for mosaic-based optical positioning and navigation of underwater vehicles, *Proceedings of the IEEE International Conference on Robotics and Automation*, 2001, pp. 2759–2766.

Z. Djama, Y. Amirat, Multi-model approach for the localisation of mobile robots by multisensor fusion, *Proceedings of the 32th International Symposium On Automotive Technology and Automation*, 14th-18th june 1999, Vienna, Autriche, pp. 247-260.

Z. Djama, Approche multi modèle à sauts Markoviens et fusion multi capteurs pour la localisation d'un robot mobile. *PhD Thesis, University of Paris XII*, May 2001.

H. A. P. Blom and Y. Bar-Shalom, The interacting multiple model algorithm for systems with Markovian switching coefficients, *IEEE Trans. Automat. Contr.*, vol. 33, pp. 780–783, Aug. 1988.

X. R. Li, Engineer's guide to variable-structure multiple-model estimation for tracking, in Multitarget-Multisensor Tracking: Applications and Advances, *Y. Bar-Shalom and D.W. Blair,* Eds. Boston, MA: Artech House, 2000, vol. III, ch. 10, pp. 499–567.

X. R. Li, Hybrid estimation techniques, Control and Dynamic Systems: *Advances in Theory and Applications, C. T. Leondes*, Ed. New York: Academic, 1996, vol. 76, pp. 213–287.

E. Mazor, A. Averbuch, Y. Bar-Shalom, and J. Dayan, Interacting multiple model methods in target tracking: A survey, *IEEE Trans. Aerosp. Electron. Syst.*, vol. 34, no. 1, pp. 103–123, 1996.

F. Dufour, Contribution à l'étude des systèmes linéaire à saut markoviens, *PhD Thesis*, University of Paris Sud University, France, 1994.

M. Oussalah, Suboptimal Multiple Model Filter for Mobile Robot Localization, International Journal of Robotics Research, vol. 20, no. 12, December 2001, pp. 977-989.