SPHERICAL IMAGE DENOISING AND ITS APPLICATION TO OMNIDIRECTIONAL IMAGING

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Keywords: Omnidirectional imaging, Spherical filters, Spherical harmonics, Wiener filter, Tikhonov regularization.

Abstract: This paper addresses the problem of spherical image processing. Thanks to projective geometry, the omnidirectional image can be presented as a function on sphere S^2. The target application includes omnidirectional image smoothing. We describe a new method of smoothing for spherical images. For that purpose, we introduce a suitable Wiener filter and we use the Tikhonov method to these images. In order to compare their performances, we present the most used classical spherical kernels.
We present several examples for filtering real and synthetical spherical images. [a].

[a] This project was financed by the D.G.A.

1 INTRODUCTION

Images defined on a spherical surface rather than in the image plane arise in various domains like environment mapping, and medical imaging (Chung, 2006).... Omnidirectional images appear with distorsion (because of the used mirror) in the image plane and are naturally defined on the sphere as shown in the model of Geyer (Geyer and Daniilidis, 2001). Most approaches analysing omnidirectional images applied classical operators. However the analysis or the filtering of these images by traditional methods produces erroneous results. In (Daniilidis et al., 2002), Daniilidis proposed of the very first approaches of adaptation of the operations of filtering to the omnidirectional images. The proposal of the authors is to project the image plane to a non-deformed space (unit sphere) (Geyer and Daniilidis, 2000).

The spherical images are obtained by stereographic projection of omnidirectional image, as shown in Fig.1.Daniilidis then define kernels in the space of the spherical coordinates and propose to carry out the convolution on the surface of the sphere. In (Strauss and Comby, 2005), the authors present new morphological operators that use the projective property of omnidirectional sensor. These operators make use of a structural element with a varying shape. Another work (Ieng et al., 2003), advocates the principle of kernel with varying shape to match indices from two omnidirectional images. Except a few attempts to adapt the method of processing omnidirectional images, no normalization method has been established for these images.

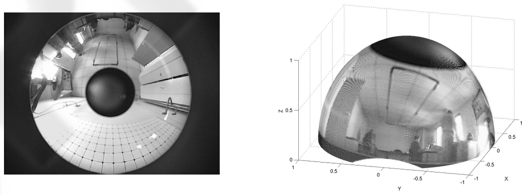


Figure 1: Omnidirectional image of Office and the corresponding spherical image.

A simple technique for smoothing planar images is the convolution of the image with a Gaussian kernel as

$$g(x;t) = \frac{1}{4\pi kt}\exp\left(-\frac{x^2}{4kt}\right)$$

Rather then just using this kernel for noise-removal, a linear scale-space theory can be built upon the application of this convolution kernel. This idea has been

Bigot S., Kachi D., Durand S. and Mustapha Mouaddib E. (2007).
SPHERICAL IMAGE DENOISING AND ITS APPLICATION TO OMNIDIRECTIONAL IMAGING.
In *Proceedings of the Second International Conference on Computer Vision Theory and Applications - IFP/IA*, pages 101-108

introduced by Witkin (Witkin, 1984) and Lindeberg (Lindeberg, 1994). The linear scale space of an image is defined as the set of smoothed images created by convolving the image with Gaussian functions of different scales.
The Gaussian function is the Green function of the linear diffusion equation, i.e. the solution of equation (1) with the δ-impulse as initial condition.

$$\Delta u(x;t) = \frac{1}{k}\frac{\partial}{\partial t}u(x;t) \tag{1}$$

In the case of omnidirectional images, the major difficulties of these approaches have to deal with explicitly include shrinkage, dependency of the smoothing results on the mesh connectivity. In order to be able to deal with well defined globally parameterized domain we restrict the class of omnidirectional images to those which can be described as functions on the sphere, as defined in the model of Geyer.
Diffusion smoothing of surfaces then corresponds to convolution of the surface with the spherical Gaussian.
We propose in this paper to adapt the classical Wiener filter and the Tikhonov regularization to the spherical images. We compare the denoising performances of these methods with the classical spherical kernels. The remainder of this paper is structured as follows. In the next section, we recall mathematical framework related to spherical harmonics, which will be used for the solution of the spherical diffusion equation, and convolution on the sphere. In Section 3, we define the most important classical spherical kernels, and we describe the spherical form of Wiener and Tikhonov filtering. Section 4 shows results of the spherical filtering process applied to synthetical and real indoor omnidirectional images. Finally we give some conclusions and future work.

2 SPHERICAL THEORY

Filtering is based on convolution operators. We present in this section the various definitions of this operation and the mathematical tools used for its implementation.

2.1 Spherical Fourier Transform

We parametrize the unit sphere, embedded in $\mathbb{R}^3$, by using the spherical coordinates $\eta \in S^2$: $\eta(\theta,\varphi) = (\cos(\varphi)\sin(\theta), \sin(\varphi)\sin(\theta), \cos(\theta))$ with $\varphi \in [0,2\pi[$, angle of longitude and $\theta \in [0,\pi]$, angle of colatitude (latitude + $\pi/2$).
The effect of planar diffusion smoothing can be well understood in the frequency domain as a low-pass filter. Since we are going to carry out the corresponding analysis on the sphere we need a spherical analog of the Fourier transform. Such a tool exists in the expansion of a function into a series of spherical harmonic functions.
Notice by Y_{lm} the spherical harmonic of $l (\in \mathbb{N})$ degree and order m as follows

$$Y_{lm}(\theta,\varphi) =$$

$$\sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}}P_l^m(\cos(\theta))e^{im\varphi} \quad \text{for } m \geq 0$$

where the $P_l^m(x)$ are the polynomials of Legendre associated with l degree and order m. We can notice that the spherical harmonics of l degree form a subspace of $L^2(S^2)$ of dimension $2l+1$ which is invariant under rotations of the sphere. Since the spherical harmonics form an orthonormal basis for $L^2(S^2)$, we have

$$\widehat{f}_{lm} = \widehat{f}(l,m) = \langle f, Y_{lm}\rangle$$

where the scalar product on the sphere is defined as

$$\langle f,h\rangle = \int_0^{2\pi}\int_0^{\pi} f(\theta,\varphi)\overline{h(\theta,\varphi)}\sin(\theta)d\theta d\varphi$$

The set of the coefficients $\widehat{f}_{lm}$ is called spherical Fourier transform or spectrum of f. For the implementation, we will use the sampling theorem (Healy et al., 1998).
Theorem: Let f $\in L^2(S^2)$ be a bandlimited function of bandwith B, then:

$$\widehat{f}_{lm} = \frac{\sqrt{2\pi}}{2B}\sum_{j=0}^{2B-1}\sum_{k=0}^{2B-1} a_j^{(B)} f(\theta_j,\varphi_k)\overline{Y_{lm}(\theta_j,\varphi_k)}$$

for $|m| \leq l < B$. The sampling grid is the equiangular or lat-lon grid with $\theta_j = \frac{\pi(2j+1)}{4B}$ et $\varphi_k = \frac{\pi k}{B}$.

2.2 The Convolution on the Sphere

Two definitions were proposed to carry out a product of Convolution on the sphere : that introduced by Driscoll-Healy (Driscoll and Healy, 1994) and that used, by Daniilidis (Daniilidis et al., 2002) and Wandelt (Wandelt and Górski, 2001).
We begin by introducing some notations.
We represent the sphere S^2 as the quotient $SO(3)/SO(2)$ where $SO(3)$ is the group of rotations which acts on the sphere (Vilenkin, 1969). The rotation of a function $f \in L^2(S^2)$ by an element $g \in SO(3)$ is then defined with the operator Λ_g such as

$$\Lambda_g f(\eta) = f(g^{-1}\eta)$$

Let $f \in L^2(S^2)$ and $g \in L^1(S^2)$
The convolution product between f et g is the function on $SO(3)$ defined by

$$(f\tilde{*}g)(R) = \int_{S^2} f(R^{-1}\eta)g(\eta)d\eta$$

We have

$$f\tilde{*}g \in L^2(SO(3))$$

with $L^2(SO(3)) \equiv L^2(SO(3), dR)$ where dR is the Haar measure on $SO(3)$.
An important problem arises; whereas the functions f and g are defined on the sphere, the product of convolution is defined on the group of rotations. Consequently, it is clear that the product of convolution is not associative. Moreover

$$f\tilde{*}\delta_0(R) = f(R^{-1}n_0)$$

that implies a loss of symmetry in this definition of convolution.
Now let us see another definition of the convolution, introduced by Driscoll and Healy. Let $f, h \in L^2(S^2)$, we have

$$(f * h)(\eta) = \int_{SO(3)} f(Rn_0)h(R^{-1}\eta)dR$$

where $\eta \in S^2$, $n_0 = (0,0,1)$ represent the North Pole of the unit sphere.
The effectiveness of this second definition becomes obvious thanks to the theorem of convolution, shown by Driscoll and Healy. For $f, h \in L^2(S^2)$, we have that

$$\widehat{(f*h)}_{lm} = 2\pi\sqrt{\frac{4\pi}{2l+1}}\widehat{f}_{lm}\widehat{h}_{l0} \qquad (2)$$

We can observe that only the coefficients of spherical transform of the filter $\widehat{h}_{lm}$ with $m = 0$ are used in expression (2). These coefficients correspond to the zonal harmonics Y_{l0} and thus represent the invariant part by rotation of the filter h. So the filtering operator is invariant under the actions of rotations.

3 SPHERICAL FILTERS

3.1 Spherical Kernels

The question which we put is to know to define Gaussian on the sphere. Bülow (Bulow, 2004) proposed to determine the Green function, as a solution of the spherical diffusion equation. The spherical equation of diffusion is :

$$\Delta_{\mathbb{S}^2} u(\theta,\phi,t) = \frac{1}{k}\frac{\partial}{\partial t}u(\theta,\phi,t)$$

where

$$\Delta_{S^2} = \frac{1}{\sin(\theta)}\frac{\partial}{\partial\theta}\Big(\sin(\theta)\frac{\partial}{\partial\theta}\Big) + \frac{1}{\sin^2(\theta)}\frac{\partial^2}{\partial^2\varphi}$$

Let us recall that the spherical harmonics are the eigenfunctions of the spherical Laplace operator :

$$\Delta_{\mathbb{S}^2} Y_{l,m} = -l(l+1)Y_{l,m}$$

Consequently, it is checked easily that the functions

$$u_{l,m}(\theta,\phi,t) = Y_{l,m}(\theta,\phi)exp(-l(l+1)kt)$$

are solutions of the spherical equation of diffusion.
To obtain the function of Green G, we force like initial condition :

$$u(\theta,\phi,0) = G(\theta,\phi,0) := \delta_{\mathbb{S}^2}(\theta,\phi)$$

where δ is defined by $f(n_0) = \int_{S^2} f(\eta)\delta_{\mathbb{S}^2}(\eta)d\eta$.
By using the decomposition of spherical Dirac in the basis of the spherical harmonics, we obtain

$$G(.,0) = \delta_{\mathbb{S}^2} = \sum_{l\in\mathbb{N}}\sqrt{\frac{2l+1}{4\pi}}Y_{l,0} = \sum_{l\in\mathbb{N}}\sqrt{\frac{2l+1}{4\pi}}u_{l,0}(.,0)$$

From where finally for the Green function

$$G(\theta,\phi,t) = \sum_{l\in\mathbb{N}}\frac{2l+1}{4\pi}P_l^0(\cos(\theta))e^{-l(l+1)kt}$$

we carry out then the spherical Fourier transform to find

$$\widehat{G(.,t)}(l,m) = \begin{cases} \sqrt{\frac{2l+1}{4\pi}}e^{-l(l+1)kt} & \text{if } m = 0 \\ 0 & \text{if not} \end{cases}$$

Let us define another functions candidates to build Gaussian on the sphere.

3.2 Spherical Gaussian

If we start with the definition of the Gaussian on a plane, we can obtain the spherical Gaussian using the inverse stereographic projection

$$G_s(\theta,\phi,t) = e^{-(\tan(\theta/2)/t)^2}$$

3.3 Poisson Kernel

This Kernel is given by the function

$$P(\theta,\phi,t) = \sum_{n\in\mathbb{N}}(2n+1)t^n P_n^0(\cos(\theta))$$

$$= \frac{1-t^2}{(1-2t\cos(\theta)+t^2)^{3/2}} \quad \text{for} \quad t \in [0,1]$$

This equation is the solution of the equation

$$\Delta_{\mathbb{S}^2} P(\theta,\varphi,t) = -t\frac{\partial^2}{\partial t^2}(hP(\theta,\varphi,t))$$

Let us calculate the Fourier transform of this function.

$$\widehat{P(.,t)}(l,m) = \begin{cases} \sqrt{(2l+1)4\pi} \quad t^l & \text{if } m=0 \\ 0 & \text{else} \end{cases}$$

For more details, we refer to (Bulow, 2004).

3.4 Spherical Form of the Wiener Filter

We present in this section a Wiener filter adapted to spherical functions.
We consider our original image $f \in L^2(S^2)$, corrupted by an additive white noise n. We denotes $x = f + n$, the data, i.e. the corrupted image.
We seek to obtain the best possible estimate g of f starting from our data x.
For that purpose we proceed to obtain the maximum of the SNR defined as

$$SNR = 10\log_{10}\frac{\mathbb{E}(\|f\|^2)}{\mathbb{E}(\|f-g\|^2)}$$

by minimizing the average quadratic error

$$e = \mathbb{E}(\|f-g\|^2)$$

The image f and the estimator g are related to $L^2(S^2)$, whose spherical harmonics form a basis. As a consequence, we can use the theorem of Riesz-Fischer to obtain

$$e = \mathbb{E}(\sum_{l\in\mathbb{N}}\sum_{|m|\le l}\left|\widehat{f}(l,m)-\widehat{g}(l,m)\right|^2)$$

We propose to search g as the result of a filter (of impulse response h) applied to the data x, which is equivalent to write g as

$$g = \frac{1}{2\pi}x * h$$

We apply the convolution theorem of Driscoll-Healy, we obtain :

$$e = \mathbb{E}\Big(\sum_{l\in\mathbb{N}}\sum_{|m|\le l}\left|\widehat{f}(l,m)-\sqrt{\frac{4\pi}{2l+1}}\widehat{x}(l,m)\widehat{h}(l,0)\right|^2\Big) \tag{3}$$

We must thus find the filter h which minimizes e, defined in (3). We wish that this filter to be invariant by rotation, i.e. of form

$$h(\eta) = \sum_{k\in\mathbb{N}}\widehat{h}(k,0)Y_{k,0}(\eta)$$

That amounts determining the $\widehat{h}(k,0)$ which minimizes e, and so solve the equation

$$\frac{\partial e}{\partial\widehat{h}(k,0)} = 0$$

By considering that noise is a white noise with standard deviation σ, we have $\mathbb{E}|\widehat{n}(k,m)|^2 = \sigma^2$.

We assume

$$\mathbb{E}|\widehat{f}(k,m)|^2 = \frac{c}{k^2}$$

where c is a constant to be chosen, we are led with

$$\widehat{h}(k,0) = \sqrt{\frac{2k+1}{4\pi}}\frac{1}{1+\frac{\sigma^2k^2}{c}}$$

3.5 Regularization Process of Tikhonov

In 1977, Arsenin and Tikhonov (Tikhonov and Arsenin, 1977) have proposed a method for determining the best possible estimator.
The idea is to find

$$\min_{f\in H^1(S^2)}\int_{S^2}|\nabla f|^2+\lambda\|f-x\|^2$$

where as above, $\|.\|$ represents the standard $L^2(S^2)$ norm. We recall that for a smooth function f with compact support, we have

$$\int|\nabla f|^2 = -\int f\Delta f$$

Therefore :

$$\int_{S^2}|\nabla f|^2+\lambda\|f-x\|^2 = -\int_{S^2}f\Delta f+\lambda\|f-x\|^2$$

However the spherical harmonics, which form an orthonormal basis of $L^2(S^2)$, are the eigenfunctions of Laplace-Beltrami operator. Consequently, we obtain

$$\int_{S^2}|\nabla f|^2+\lambda\|f-x\|^2 = \sum_{l\in\mathbb{N}}\sum_{|m|\le l}\widehat{f}^2(l,m)l(l+1)+\lambda\|f-x\|^2$$

Then by using the Riesz-Fischer theorem, we have

$$J = \int_{S^2}|\nabla f|^2+\lambda\|f-x\|^2 =$$

$$\sum_{l\in\mathbb{N}}\sum_{|m|\le l}\widehat{f}^2(l,m)l(l+1)+\lambda\sum_{l\in\mathbb{N}}\sum_{|m|\le l}\left|\widehat{f}(l,m)-\widehat{x}(l,m)\right|^2$$

Tikhonov methods aims to find the function f where the minimum is achieved. However by using the expansion of f in the basis of the spherical harmonics,

$$f(\eta) = \sum_{l\in\mathbb{N}}\sum_{|m|\le l}\widehat{f}(l,m)Y_{l,m}(\eta) \quad \forall\eta\in S^2$$

We have to estimate the function J according to $\widehat{f}(k,p)$.

$$\frac{\partial J}{\partial\widehat{f}(k,p)} = 2k(k+1)\widehat{f}(k,p)+2\lambda(\widehat{f}(k,p)-\widehat{x}(k,p))$$

Then

$$\frac{\partial J}{\partial \widehat{f}(k,p)} = 0$$

$$\Leftrightarrow \widehat{f}(k,p) = \frac{\lambda}{\lambda + k(k+1)} \widehat{x}(k,p)$$

Let g be the estimator which minimizes the function J. We infer from the computations above that

$$\widehat{g}(k,p) = \frac{\lambda}{\lambda + k(k+1)} \widehat{x}(k,p)$$

As for the Wiener filter, we suppose that the estimator is the result of a filtering of the data, i.e.,

$$g = \frac{1}{2\pi} x * h$$

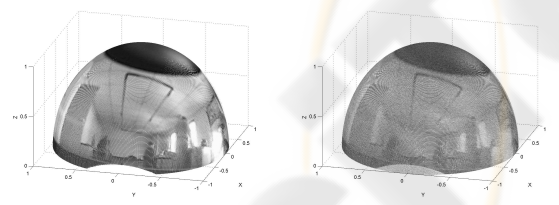
with a h that is the impulse response of the filter of restoration. We thus find

$$\widehat{h}(k,0) = \sqrt{\frac{2k+1}{4\pi}} \frac{1}{1 + \frac{k(k+1)}{\lambda}}$$

The filter h of restoration has a form similar to the Wiener filter, that is a classical result in the framework of the planar images denoising.

4 EXPERIMENTAL RESULTS

We have compared our denoising process on synthetical, real indoor and outdoor omnidirectional images. The used mirror is parabolic and verifies the property of single effective viewpoint. So we can apply the Geyer model and process the filtering on spherical images. For the implementation, omnidirectional images are projected on S^2 and redefined on the lat-lon grid. The spherical transform of images and kernels are computed by using the yawtb toolbox (http://rhea.tele.ucl.ac.be/yawtb/). The noise is an additive white noise with varying standard deviation σ and is added directly to the spherical images as seen in Fig.2. The scale is an important parameter in the process, and the result of filtering depends on this value. Fig. 3 shows an example of filtering results (Green filter) with different scale value. For each filter, we search (numerically) the optimal scale, which provides the highest SNR. This SNR is calculated on the whole sphere, while the projected omnidirectional image is defined on the hemisphere. The optimal SNR is approximately the same whatever the filter and the type of images (indoor or outdoor) as proved in Fig.4, Fig.5, and Table 1, even if the Green function and the gaussian kernel provide often ligthly higher SNR. We have obtained similar results for other types of noise (like exponential noise...).



(a) Office image (b) Office image noised

Figure 2: Noised Office image with additive noise $N(\sigma,\mu) = N(40,0)$.

Table 1: Optimal SNR for Office image $N(\sigma,\mu) = N(40,0)$.

Kernel	Scale t	SNR
Green	$2.6650 * 10^{-4}$	19.7852
Gaussian spherical	0.0164	19.7842
Wiener	2903000	19.1174
Tikhonov	1900	19.1013
Poisson	0.9850	18.6118

However if we carry out a zoom in some images, see Fig.6 and Fig.7 we can notice (visually) that the Wiener filter and the method of Tikhonov give further precision on edges. This precision is independent of position on the sphere, (close or far from the pole of the sphere). The SNR calculated at different regions on the sphere are similar, see Fig. 6. We can also show this conclusion for various images. These results have a considerable benefit if we want to detect edges, or extract important primitive from images.

5 CONCLUSION AND FUTURE WORK

We presented a panorama on the tools and the methods to treat the spherical images. We adapted the Wiener filter and Tikhonov method in that case and compared their performances to classical spherical gaussian filters. The SNR obtained for various types of images prove the effectiveness of these filters for the denoising application. Moreover the treatments can be carried out in real time. The filters are invariant by rotation, the results are thus independent of the position on the sphere. We also noticed that the Wiener filter and Tikhonov regularization are relatively better, if we want to analyse precisely some parts of images for edge detection for example.

For the continuation of our work, we propose to