BUSINESS ANALYSIS IN THE OLAP CONTEXT

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Keywords: Business Intelligence, Multi-dimensional databases, OLAP, Sensitivity analysis, Explanation, Data mining.

Abstract: Today's multi-dimensional business or OnLine Analytical Processing (OLAP) databases have little support for sensitivity analysis. Sensitivity analysis is the analysis of how the variation in the output of a mathematical model can be apportioned, qualitatively or quantitatively, to different sources of variation in the input of the model. This functionality would give the OLAP analyst the possibility to play with ``What if...?"-questions in an OLAP cube. For example, with questions of the form: ``What happens to an aggregated value in the dimension hierarchy if I change the value of this data cell by so much?" These types of questions are, for example, important for managers that want to analyse the effect of changes in sales, cost, etc., on a product's profitability in an OLAP sales cube. In this paper, we describe an extension to the OnLine Analytical Processing (OLAP) framework for business analysis in the form of sensitivity analysis.

1 INTRODUCTION

In this paper, a new OLAP database operator is described that supports the analyst in answering these managerial sensitivity analysis questions in an OLAP data cube. For example, an analyst while navigating an OLAP cube, might be interested in the question: How is the profit in the year 2008 for a certain product affected when its unit price is changed ceteris paribus (c.p.) with one extra unit or one percent in the sales model? Such question might be `dangerous', when the change is not caused by a variable in the base cube, but by a variable on some intermediate aggregation level in the cube. The latter situation makes the OLAP database mathematically inconsistent.

Consistency in a set of OLAP equations is not trivial because by changing a certain variable (c.p.), a system of equations can become inconsistent. For instance, missing data, dependency relations, and the presence of non-linear relations in the business model can cause an OLAP system to become inconsistent. It is therefore important to investigate the criteria for consistency and solvability in the OLAP context. Our novel OLAP operator corrects for such inconsistencies such that the analysts can still carry out sensitivity analysis in the OLAP context. Moreover, we elaborate on two important mathematical conditions for sensitivity analysis in the OLAP context namely consistency and solvability of the system of OLAP equations. For this purpose, we distinguish between linear systems of OLAP equations, associated with dimension hierarchies, and nonlinear systems of OLAP equations, generally associated with business models.

Sensitivity analysis in the OLAP context is related to the notion of comparative statics in economics or sensitivity analysis in engineering. Where the central issue is to determine how changes in independent variables affect dependent variables in an economic model (Samuelson, 1941). Comparative statics is defined as the comparison of two different equilibrium states solutions, before and after change in one of the independent variables, keeping the other variables at their original values. The basis for comparative statics is an economic model that defines the vector of dependent variables y_1, y_2, …, y_m as functions of the vector of

Caron E. and Daniels H. (2009).
BUSINESS ANALYSIS IN THE OLAP CONTEXT.
In *Proceedings of the 11th International Conference on Enterprise Information Systems - Artificial Intelligence and Decision Support Systems*, pages 325-330
DOI: 10.5220/0001989103250330

independent variables $x_1, x_2, \ldots, x_n$. In this paper we apply comparative statics in the OLAP context where we have a system of linear or nonlinear equations with dependent variables on an aggregated level of the cube, called non-base variables and independent variables on the base level, called base variables.

This research is part of our continued work on extensions for the OLAP framework for business diagnosis. Current OLAP databases have limited capabilities for sensitivity and diagnostic analysis. The goal of our research is to largely automate these manual diagnostic discovery processes (Caron and Daniels, 2007). In (Sarawagi et al., 1998) a similar research approach is taken.

The remainder of this paper is organized as follows. Section 2 introduces our notation for multi-dimensional equations, followed by formal description of consistency and solvability of systems of OLAP equations in Section 3. In addition, we show that systems of OLAP equations are consistent and have a unique solution. In Section 4 the OLAP framework is extended with sensitivity analysis based on the consistency property. Subsequently, we briefly describe a software implementation of our model for OLAP sensitivity analysis. Finally, conclusions are discussed in Section 6.

2 NOTATION AND EQUATIONS

The multi-dimensional OLAP database is a framework used to provide business decision-makers with the ability to perform dynamic data analysis. With OLAP tools, users gain access to the data warehouse. Decision-makers tend to have questions that are often multi-dimensional in nature and demand fast access to large amounts of aggregated data. A typical business question might be: ``What was the profit of product A this year, in region X, per sales office, compared with the previous version of the product, compared to the targeted profit?" For decision-making purposes it might be necessary that the answer to this question is explored further, for example on the quarter, month and week level. This functionality is provided by OLAP.

Two important data schemata for the design of a multi-dimensional database are the star schema and the snowflake schema. OLAP typically uses a star schema, where data is stored in fact tables and dimension tables. In a star schema, one central fact table is linked via foreign keys with several dimension tables. Each dimension has its own single table with a smaller set of data. The other important multi-dimensional design approach, the snowflake schema, is a non-redundant database design that characterises itself by the normalized data approach where data is further split into additional dimension tables (Han and Kamber, 2005).

In both schemata data is organized using the dimensional modelling approach, which classifies data into *measures* (i.e., facts) and *dimensions*. Measures are numeric and dimensions are categorical data types. Measures like are the basic units of interest for analysis. Dimensions correspond to different perspectives for viewing measures. Dimensions are usually organized as *dimension hierarchies*, which offer the possibility to view measures at different dimension levels (e.g. month $\prec$ quarter $\prec$ year is a hierarchy for the Time dimension). Aggregating measures up to a certain dimension level, with functions like sum, count, and average, creates a multidimensional view of the data, also known as the *data cube*. A number of data cube operations exist to explore the multidimensional data cube.

Here we use a generic notation for multi-dimensional data schemata that is particularly suitable for combining the concepts of measures, dimensions, and dimension hierarchies as described in (Caron and Daniels, 2007). Therefore, we define a measure y as a function on multiple domains:

$$y^{i_1 i_2 \ldots i_n} : D_1^{i_1} \times D_2^{i_2} \times \ldots \times D_n^{i_n} \rightarrow \mathbf{R} \qquad (1)$$

Each domain D_i has a number of hierarchies ordered by $D_k^0 \prec D_k^1 \prec \ldots \prec D_k^{i_{\max}}$, where D_k^0 is the lowest level and $D_k^{i_{\max}}$ is the highest level in $D_k^{i_{\max}}$. A dimension’s top level has a single level instance $D_k^{i_{\max}} = \{\text{All}\}$. For example, for the time dimension we could have the following hierarchy $\mathrm{T}^0 \prec \mathrm{T}^1 \prec \mathrm{T}^2$, where $\mathrm{T}^2 = \{\text{All-T}\}$, $\mathrm{T}^1 = \{2000{,}2001\}$, and $T^2 = \{\text{Q1,Q2,Q3,Q4}\}$. A cell in the cube is denoted by $(d_1, d_2, \ldots, d_n)$, where the d_k's are elements of the domain hierarchy at some level, so for example (2000, Amsterdam, Beer) might be a cell in a sales cube. Each cell contains data, which are the values of the measures y like, for example, sales^{211} (2000, Amsterdam, Beer). The measure’s upper indices indicate the level on the associated dimension hierarchies. If no confusion can arise we will leave out the upper indices indicating level hierarchies and write sales (2000, Amsterdam, Beer). Furthermore, the combination of a cell and a measure is called a *data point*. The measure values at the lowest level cells are entries of the *base cube*. If a measure value is on the base cube level, then the hierarchies of the domains can be used to aggregate the measure values using aggregation operators like SUM, COUNT, or, AVG.

By applying suitable equations, we can alter the level of detail and map low level cubes to high level cubes and vice versa. For example, aggregating measure values along the dimension hierarchy (i.e. rollup) creates a multidimensional view on the data, and de-aggregating the measures on the data cube to a lower dimension level, creates a more specific cube.

Here we investigate the common situation where the aggregation operator is the summarization of measures in the dimension hierarchy. So y is an additive measure or OLAP equation (Lenz and Shoshani, 1997) if in each dimension and hierarchy level of the data cube:

$$y^{i_1 \ldots i_{q+1} \ldots i_n}(\ldots, a, \ldots) = \sum_{j=1}^{J} y^{i_1 \ldots i_q \ldots i_n}(\ldots, a_j, \ldots) \tag{2}$$

where $a \in D_k^{q+1}$, $a_j \in D_k^q$, q is some level in the dimension hierarchy, and J represents the number of level instances in D_k^q. An example equation corresponding to two roll-up operations reads:

$$\text{sales}^{212}(2001, \text{All-Locations}, \text{Beer}) =$$
$$\sum_{j=1}^{4}\sum_{k=1}^{20} \text{sales}^{102}(2001.\text{Q}_j, \text{Country}_k, \text{Beer}).$$

3 SOLVABILITY

In comparative statics in economics the central issue is to determine how changes in independent variables affect dependent variables in an economic model. Comparative statics is based on an economic model (i.e., a system of equations) that defines the vector of dependent variables **y** as functions of the vector of independent variables **x**. In the OLAP context we have a system of linear equations with dependent variables on an aggregated level of the cube, called non-base variables and independent variables on the base level, called base variables. A condition for the application of comparative statics is that the underlying system of equations is mathematical *consistent*.

The data structure in an OLAP cube represents a system of additive equations in the form of a *aggregation lattice* (Han and Kamber, 2005). The top of the lattice is the apex cube $y^{i_{\max} i_{\max} \ldots i_{\max}}$ and the bottom of the the lattice is represented by the base variables $\mathbf{x}^{00\ldots0}$. The upset of a base variable in the lattice represents non-base variables on specific levels of aggregation in the OLAP cube. For example, the non-base variable $y^{i_1 i_2 \ldots (i_p+1) \ldots i_n}$ is a parent of the non-base variable $y^{i_1 i_2 \ldots i_p \ldots i_n}$, somewhere in the lattice. Roll-ups can be alternated from one dimension to the next by the data analyst, resulting in multiple paths from a base variable to a non-base variable in the aggregation lattice.

An example aggregation lattice is given for the variable sales ($y^{i_1 i_2 i_3}$) from an example sales datacube in figure 1, where the indices represent the customer, product, and store dimension, respectively. In the lattice the variable y^{101}, which has a number of data instances, has instances of the root in its upset and instances of the variables $\{y^{100}, y^{001}, x^{000}\}$ in its downset. All non-base variables **y** are aggregated from instances of the base variables $x^{000}(\text{customer}, \text{product}, \text{store})$. The length of a path from a non-base variable $y^{i_1 i_2 \ldots i_n}$ in the lattice to a base variable $x^{00\ldots0}$ is $i_1 + i_2 + \ldots + i_n$. Obviously, the sum of the indices of a non-base variable corresponds with the number of aggregations carried out. Non-base variable in the system of OLAP equations are the result of aggregation operators in the lattice structure. Moreover, a non-base variable in the lattice corresponds with a single equation expressed in a unique set of base variables. This property can be easily verified by *substituting* all equations in the downset of a non-base variable from its current level to the base level.

A system of OLAP equations as in equation (2), where the functions are linear, can be written in matrix form as:

$$A\mathbf{z} = \mathbf{c} \tag{3}$$

where A is an $m \times n$ coefficient matrix of constants, **c** is an $m \times 1$ vector of constants, and **z** is an $n \times 1$ vector of variables for which we need solutions. In the next section below, we discuss relevant matrix theory on the conditions under which equation (3) is consistent and solvable. Moreover, we transfer the matrix theory to a system of implicit equations with independent and dependent variables that corresponds to a system of additive OLAP equations. This system of equations lets us study the impact of a change in one or more base variables (c.p.) on a non-base variable. The matrix form of this system of equations is $A\mathbf{z} = 0$; the matrix A is partitioned as $[A_1\ A_2]$, where A_1 is the coefficient sub matrix for non-base variables and A_2 is the coefficient sub matrix for base variables. And the vector of variables **z** is partitioned in non-base variables **y** and base variables **x** as **z'** = [**y x**].

The equations of the OLAP aggregation lattice in (3) are rewritten in the following partioned matrix form:

$$A_1\mathbf{y} + A_2\mathbf{x} = \mathbf{0} \qquad (4)$$

where A_1 is a $m \times n$ matrix of constants, A_2 is a $m \times l$ matrix of constants, $\mathbf{y} \in \mathbf{R}^n$ is vector of all non-base variables, from all levels in the aggregation lattice, for which solutions are needed, and $\mathbf{x}^{00\ldots0} \in \mathbf{R}^l$ is a vector of base variables that are given. The above system of equations in (4) is the collection of *all* possible drilldown equations in the OLAP database by drilling down from the root of the lattice to the base over all possible dimensions and dimension hierarchies. This system of equations is clearly overspecified, because a non-base variable in the lattice might be the right hand side in multiple drilldown equations. In fact, each possible drilldown from one dimension to the next, results in an additional equation for a non-base variable. From the substation argument above it follows that equation (4) has a unique solution **y** for a given set of base variables **x**.

This implies $rank(A_1 | -A_2\mathbf{x}) = rank(A_1)$, see theorem 6.1 from (Schott, 1997), for all **x** so the columns of A_2 are linear combinations of the columns of A_1, so $A_2 = A_1 Z$ where a Z is a $n \times 1$ matrix of constants.

Furthermore, since the solution for **y** is unique we have $rank(A_1) = n$ because the null space of A_1 is $N(A_1) = \{\mathbf{0}\}$. So also Z is unique since $A_1 Z = A_1 Z^*$ would imply $A_1(Z - Z^*) = 0$ and because $N(A_1) = \{\mathbf{0}\}$, we have $Z = Z^*$. It is also easy to show that

$$Z = A_1^- A_2 \qquad (5)$$

where A_1^- is the left generalized inverse (e.g. the Moore-Penrose inverse) of A_1. This exists because $rank(A_1) = n$ and satisfies $A_1^- A_1 = I_n$, see theorem 6.6 from (Schott, 1997).

To show (5) note that $A_1 Z = A_2$ implies:

$$A_1 A_1^- A_2 = A_1 A_1^- A_1 A_2 = A_1 Z = A_2 \qquad (6)$$

So $A_1^- A_2$ is another solution of $A_1 Z = A_2$ and therefore $Z = A_1^- A_2$ by uniqueness. Because of (6) it can be shown that the OLAP aggregation lattice always has a unique solution for the non-base variables for a given a set of base variables.

4 SENSITIVITY ANALYSIS

Because a system of OLAP equations is uniquely solvable, a change in a single base variable (c.p.) in the aggregation lattice will result in a new unique solution for the non-base variables. If a non-variable $y^{i_1\ldots i_q\ldots i_n}(\ldots,a_j,\ldots)$ is changed with some magnitude (c.p.) the aggregation lattice will obviously become inconsistent because its down set variables are not changed accordingly. This is demonstrated with the following 2 example equations that are derived from the aggregation lattice in Figure 1 where an instance of the variable y^{101} is changed with some Δ:

$$1.\ y^{111} + \Delta = \sum_{j=1}^{J} y^{101}(\ldots,a_j,\ldots) + \Delta$$

$$2.\ y^{101}(\ldots,a,\ldots) + \Delta \neq \sum_{j=1}^{J} y^{100}(\ldots,a_j,\ldots).$$

In the first equation we see that variables in the upset of y^{101} incorporate the change resulting in a consistent equation. However, in the second equation we see that the system becomes inconsistent, because in the down set of y^{101}, i.e. the variables on the right hand side of the equations, remain on their initial values. In that case, sensitivity analysis is 'dangerous' because it results in an inconsistent system of OLAP equations.

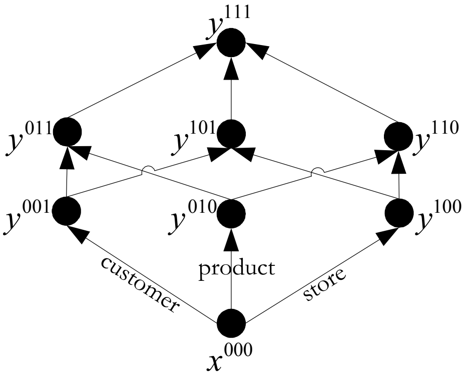

Figure 1: Aggregation lattice for the sales cube.

Now we have to correct the down set of the variable $y^{i_1\ldots i_q\ldots i_n}(\ldots,a_j,\ldots)$ for the change from each associated lower level aggregation level to the base cube level. This correction makes the sensitivity procedure again useful for the complete aggregation lattice. In the correction procedure all variables in the down sets of siblings of $y^{i_1\ldots i_q\ldots i_n}(\ldots,a_j,\ldots)$ have to remain on their norm values and one variable on each level of the down set of $y^{i_1\ldots i_q\ldots i_n}(\ldots,a_j,\ldots)$ has to be corrected with the specified change. The variables on the base cube level are corrected in the final step.

5 SOFTWARE IMPLEMENTAION

In this section, we shortly present the most important concepts of the prototype software implementation of the sensitivity analysis model in MS Excel/ Access in combination with Visual Basic. This application is initially programmed to perform experiments and analyses necessary for a case study. Fig. 2 depicts the UML use case of the program for OLAP sensitivity analysis and Fig. 3 depicts the GUI in an MS Excel environment.

For the user it is important to have an interface that is easy in use. An organized lay-out will help the user in getting maximum results. Another important functionality of the prototype is the dynamic environment. Different databases, measures, dimensions and tables should all be handled in an easy consistent manner. With this dynamic prototype, the most important aspect of the program, the sensitivity analysis, should not be forgotten. The user needs a set of tools, which can be used in order to perform the sensitivity analysis. Features like the undo operation and error handling, must also be taken into account. In order to get a working prototype in Microsoft Excel, some constraints must be made. The first constraint applies to the input of the program. The database must be a Microsoft Access database or some other database that be accessed with ODBC, that is modelled via a star schema. From this database, one single measure can be selected for analysis. In order to keep the data in the database valid, all sensitivity analysis operations are done on a copy of the original database. This makes sure that the original data will not be modified and the user is able to 'play' with the data as much and extreme as he or she wants. The copy has to be made on the background without the notice of the user. After each sensitivity analysis, the selected and changed cell will be highlighted. From this point, a new sensitivity analysis can be made by the business analyst.

6 CONCLUSIONS

In this paper, an extension in the OLAP framework has been developed and implemented in a prototype application. The model for sensitivity analysis describes the theoretical framework of this subject. The prototype software implementation for sensitivity analysis is an additional tool for business analysts that want to analyse their company data interactively. With this tool, they are able to 'play' with the data by doing sensitivity analyses.

ACKNOWLEDGEMENTS

We would like to thank Wim Zuiderwijk and Arno van den Berg for their contributions to this research.

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APPENDIX

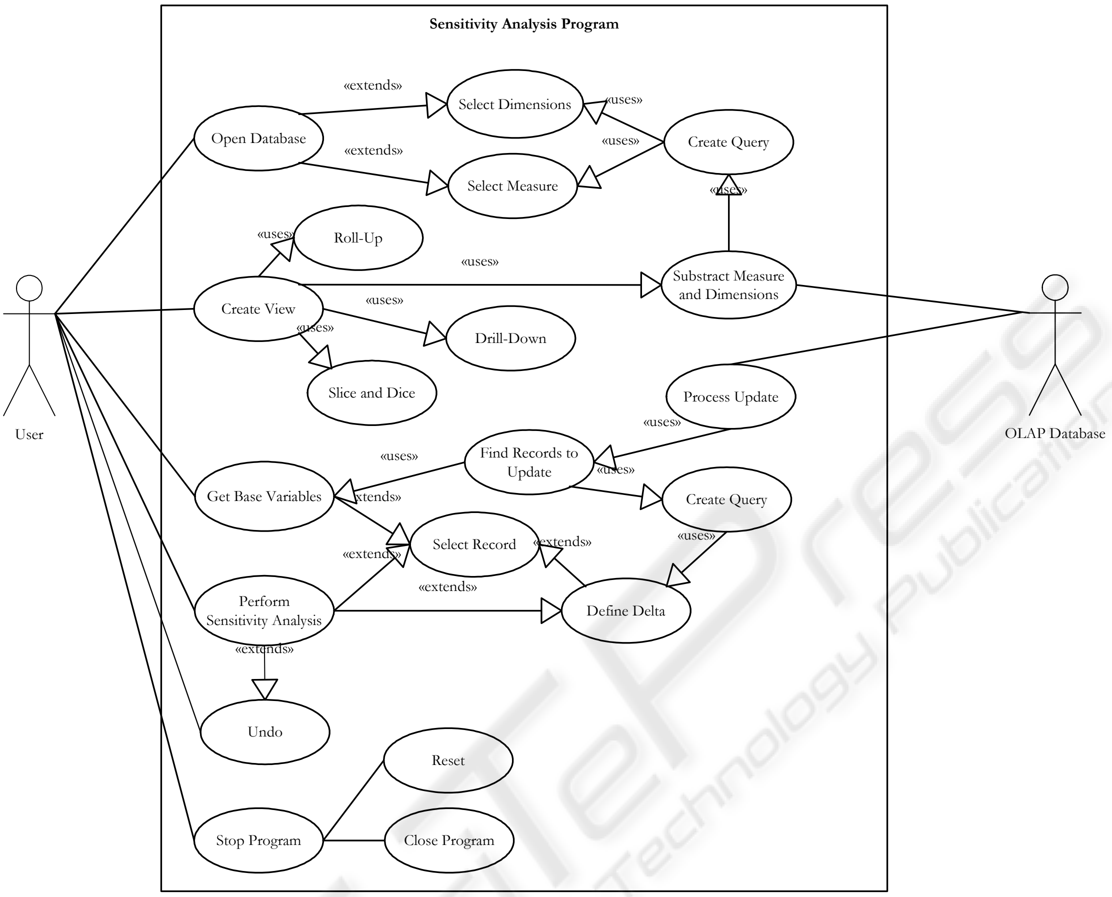
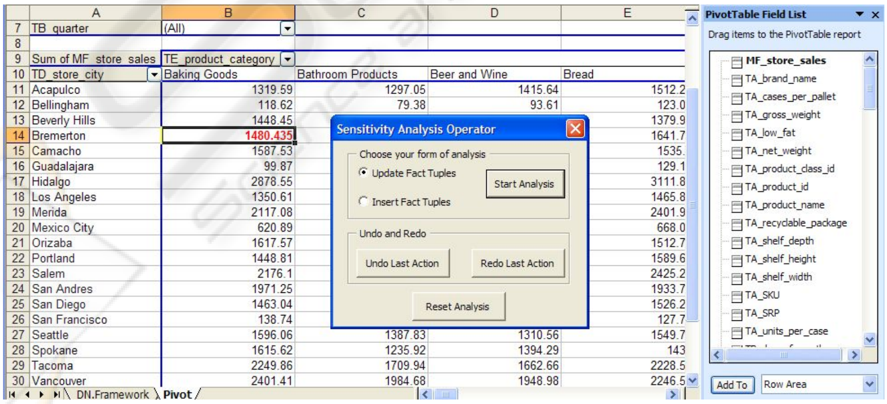


Figure 2: Use cases for the OLAP sensitivity analysis application.



	A	B	C	D	E
7	TB_quarter	(All)			
8					
9	Sum of MF_store_sales	TE_product_category			
10	TD_store_city	Baking Goods	Bathroom Products	Beer and Wine	Bread
11	Acapulco	1319.59	1297.05	1415.64	1512.2
12	Bellingham	118.62	79.38	93.61	123.0
13	Beverly Hills	1448.45			1379.9
14	Bremerton	1480.435			1641.7
15	Camacho	1587.53			1535.
16	Guadalajara	99.87			129.1
17	Hidalgo	2878.55			3111.8
18	Los Angeles	1350.61			1465.8
19	Merida	2117.08			2401.9
20	Mexico City	620.89			668.0
21	Orizaba	1617.57			1512.7
22	Portland	1448.81			1589.6
23	Salem	2176.1			2425.2
24	San Andres	1971.25			1933.7
25	San Diego	1463.04			1526.2
26	San Francisco	138.74			127.7
27	Seattle	1596.06	1387.83	1310.56	1549.7
28	Spokane	1615.62	1235.92	1394.29	143
29	Tacoma	2249.86	1709.94	1662.66	2228.5
30	Vancouver	2401.41	1984.68	1948.98	2246.5

Figure 3: GUI OLAP sensitivity analysis application.