# FORECASTING TOTAL SALES OF HIGH-TECH PRODUCTS
## *Daily Diffusion Models and a Genetic Algorithm*

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Abstract: In recent years, the release interval of high-tech consumer products such as mobile phones and portable media players is getting shorter. New models of mobile phones are released three times a year in Japan. The manufactures have to avoid dead stock because the value of the previous model drops sharply after the launch of the new model. In this paper, we propose a method to forecast the total sales of the products. The method utilizes diffusion models for forecasting. Only short-term sales record is available since the sales are forecasted one month after the release. In order to make effective use of the available data, we use a day as the time unit of forecasting. To apply the diffusion models to daily demand forecasting, we derive the difference equation representation of the models and propose discrete-time diffusion models. Day-of-week-dependent parameters are introduced to the models. The proposed method is tested on the data provided by a high-tech consumer products manufacturer. The result shows that the proposed method has an excellent forecasting ability.

## 1 INTRODUCTION

In recent years, the release interval of high-tech consumer products such as mobile phones, portable media players, and PDAs is getting shorter. After the launch of a new model of the products, the commercial value and the sales of its previous model drop sharply. Thus, the manufacturers have to sell out the previous model before the launch of the new model. At the same time, they have to avoid opportunity losses in order to maximize profitability. Consequently, accurate forecasting of the total sales of the products just after the launch is desired.

New models of mobile phones are released three times a year in Japan. That means their effective sales period is only four months. In this paper, we propose a method that the total sales of high-tech products in four months are forecasted one month after the release of the products.

Only short-term sales record is available since the total sales are forecasted just one month after the release. In our case, only one-month sales record is available. If the unit of time is a week, only four sales records are available. However, if it is a day, so are 28 or more sales records. Thus, we use a day as the time unit of forecasting.

Diffusion models(Mahajan et al., 2000) are used to forecast the demand of new products. In order to apply diffusion models to daily demand forecast, we derive the difference equation representation of diffusion models and discretize the models with respect to time. A real-valued genetic algorithm is employed for estimation of the parameters of the models.

## 2 DIFFUSION MODELS

In this section, four diffusion models are briefly reviewed. $x(t)$ denotes the cumulative sales amount at time $t$ and $dx(t)/dt$ represents the sales rate at time $t$. On the assumption that a certain proportion $p$ of the consumers who have not yet bought the product buy the product at time $t$, sales rate can be stated as follows:

$$\frac{dx(t)}{dt} = p\,(m - x(t)) \quad (1)$$

where $m$ is the market size. Solving (1), we obtain negative exponential diffusion model as follows:

$$x(t) = f_{\mathrm{NE}}(t; m, p, \tau) = m\left(1 - e^{-p(t-\tau)}\right) \quad (2)$$

Logistic model assumes that the purchase is promoted by word-of-mouth and the influence of word-of-mouth is proportional to penetration rate $x(t)/m$.

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The sales rate can be states as follows:

$$\frac{dx(t)}{dt} = q\left(\frac{x(t)}{m}\right)(m - x(t)) \tag{3}$$

Solving (3), we obtain logistic model as follows:

$$x(t) = f_{\text{Log}}(t; m, q, \tau) = \frac{m}{\left(1 + e^{q(t-\tau)}\right)} \tag{4}$$

Bass model involves both external and internal influences. Its sales rate is stated as follows:

$$\frac{dx(t)}{dt} = \left(p + q\left(\frac{x(t)}{m}\right)\right)(m - x(t)) \tag{5}$$

Equation (5) corresponds to (1) when $q = 0$ and (3) when $p = 0$. Solving (5), Bass model is:

$$x(t) = f_{\text{Bass}}(t; m, p, q, \tau) = m\frac{p - pe^{-(p+q)(t-\tau)}}{p + qe^{-(p+q)(t-\tau)}} \tag{6}$$

PNE(Power of Negative Exponential) model (Munakata and Tezuka, 2008) is an extension of negative exponential model and is written as:

$$x(t) = f_{\text{PNE}}(t; m, p, r, \tau) = m\left(1 - e^{p(t-\tau)}\right)^r \tag{7}$$

# 3 TOTAL SALES FORECASTING ON DAILY BASIS

## 3.1 Problems of Daily Forecasting

Previous studies applied diffusion models to monthly or weekly forecasting, i. e., the time unit of the model is month or week. However, in this paper, the total sales of a product, which are the cumulative sales in four months or in 120 days, are forecasted on the 28th day from the launch of the product.

When time unit of $t$ is a day, sales rate $dx(t)/dt$ depends on the day of the week. Usually, more consumers go to buy products on holidays than on weekdays. Thus, sales rate is higher on holidays than on weekdays. That means the parameters of the diffusion models have to be time-variant.

## 3.2 Derivation of Discrete-time Diffusion Models

In order to apply the diffusion models to sales forecasting on daily basis, we derive the difference equation representation of diffusion models and discretize the models with respect to time. Then, we introduce parameters depending on the day of the week to the models.

We modify the models and develop a discretized negative exponential, logistic, Bass, and PNE models as:

$$x_{t+1} = \hat{f}_{\text{NE}}(x_t, m, p) = m\left(\left(1 - e^{-p}\right) + \frac{x_t}{m}e^{-p}\right), \tag{8}$$

$$x_{t+1} = \hat{f}_{\text{Log}}(x_t, m, q) = \frac{m}{1 + \left(\frac{m}{x_{t-1}} - 1\right)e^q}, \tag{9}$$

$$x_{t+1} = \hat{f}_{\text{Bass}}(x_t, m, p, q) = m\frac{\left(q + pe^{-(p+q)}\right)x_{t-1} + \left(p - pe^{-(p+q)}\right)m}{\left(q - qe^{-(p+q)}\right)x_{t-1} + \left(p + qe^{-(p+q)}\right)m}, \tag{10}$$

$$x_{t+1} = \hat{f}_{\text{PNE}}(x_t, m, p, r) = m\left(\left(1 - e^{-p}\right) + \left(\frac{x_t}{m}\right)^{\frac{1}{r}}e^{-p}\right)^r. \tag{11}$$

## 3.3 Day-of-Week-Dependent Parameters

Parameter $m$, that is the market size, can vary over time in the long-term depending on the economic condition in the market. However, we assume that $m$ is time-invariant in the case of high-tech products whose sales period is very short.

On the other hand, we assume that $p$, $q$, and $r$ are time-variant. They depend on the day of the week because so do the behavioral pattern of consumers. As mentioned before, they differ between on holidays and on weekdays. Thus, we introduce parameters $p_h$, $q_h$, and $r_h$ for holidays and $p_w$, $q_w$, and $r_w$ for weekdays.

Then, we have discretized diffusion models with time-variant (day-of-week-dependent) parameters:

$$x_{t+1} = g_{\cdot}(x_t; m, \theta_h, \theta_w) = \begin{cases} \hat{f}_{\cdot}(x_t; m, \theta_h) & \text{if } t+1 \text{ is holiday} \\ \hat{f}_{\cdot}(x_t; m, \theta_w) & \text{if } t+1 \text{ is weekday} \end{cases} \tag{12}$$

where $\theta_h$ and $\theta_w$ are $\{p_h\}$ and $\{p_w\}$ for the negative exponential model, $\{q_h\}$ and $\{q_w\}$ for the logistic model, $\{p_h, q_h\}$ and $\{p_w, q_h\}$ for Bass model, and $\{p_h, r_h\}$ and $\{p_w, r_h\}$ for PNE model.

## 3.4 Parameter Estimation With a Genetic Algorithm

Sales record for $T$ periods, $s_1, s_2, \ldots, s_T$ are available. The parameter estimation problem is formulated as follows:

$$\text{Min. } \frac{1}{T-1}\sum_{t=1}^{T-1}\left(g_{\cdot}(s_t; m, \theta_h, \theta_w) - s_{t+1}\right)^2 \tag{13}$$

$$\begin{aligned} \text{subject to } & m - s_T \geq 0 \\ & m, p_h, p_w, q_h, q_w, r_h, r_w \geq 0 \\ & p_h - p_w \geq 0 \\ & q_h - q_w \geq 0 \end{aligned} \quad (14)$$

Market size $m$ can not be negative value and is naturally larger than the latest sales amount $s_T$. The domains of the other parameters also have to be positive real number because the range of the function $g$, that is forecasted demand, have to be positive.

The third and fourth constraints are based on the empirical observation that the sales rate is larger on holidays than on weekdays in our case. This observation certainly depends on the products.

Since $g.(s_t; m, \theta_h, \theta_w)$ is either of (8), (9), (10), or (11) according to (12), the objective function of the parameter estimation problem (13) is nonlinear and complex. It is unable to estimate the parameters by solving normal equations or a linear least-square method. From some preliminary experiments, it is found that the solution obtained by quasi-Newton method such as BFGS method highly depends on the selection of initial search point and has large variance. Thus, we employed real-coded genetic algorithms known as efficient optimization methods for such problems(Eshelman and Schaffer, 1993; Fogel, 1997).

# 4 NUMERICAL EXPERIMENTS

The proposed total sales forecasting method is evaluated on the data provided by a high-tech consumer products manufacturer. The data consist of the sales record of seven models of their products for 120 days from the date of release.

The sales record of the first 28 days, $s_1$, $s_2$, ..., $s_{28}$ are used for the parameter estimation. Then, $x_{29}$, $x_{30}$, ..., $x_{120}$, are forecasted as:

$$x_{t+1} = \begin{cases} g(s_{28}; m, \theta_h, \theta_w), & (t = 28) \\ g(x_t; m, \theta_h, \theta_w), & (otherwise) \end{cases} \quad (15)$$

The objective is to forecast total sales of a high-tech product in four months. Thus, the absolute error on 120th day,

$$\left| \frac{x_{120} - s_{120}}{s_{120}} \right| \quad (16)$$

is evaluated.

Since GAs are stochastic search algorithms and their performance varies from time to time, ten runs are performed with each model. Thus, 70 runs (10 runs multiplied by 7 models) are performed with each diffusion model. Then mean and standard deviation of the absolute error over 70 samples are evaluated.

The computation time required for one run consisting of parameter estimation and demand forecasting is as short as about 1 second on Microsoft Windows XP PC with Intel Core Solo T1300 1.66GHz and 1Gbytes RAM.

Table 1 shows the mean and standard deviation (stdev) of the absolute error over 70 samples. For comparison, the result of the conventional method, which uses the diffusion models with time-invariant parameters, is also shown. The proposed method achieved better performance than the conventional method. The mean forecasting error of the negative exponential model with the proposed method is about 11% while with the conventional method is about 44%. This is a considerable improvement. PNE model with the proposed method also achieved a big improvement.

T-test is conducted and the significance probability between proposed method and conventional time-invariant parameter method is shown. There are significant differences between proposed and conventional method.

Although the forecasting accuracies of the logistic and Bass model are also improved with proposed method, the forecasting error with the models is much higher (worse) than the other models. We consider that the logistic and Bass model themselves do not fit the product we tested.

Figure 1 shows an example of the sales forecasted with proposed and conventional method and an actual sales record. In the figure, actual sales slows down on around 30th day by some unknown reason. After that, however, the sales rate of the actual record and the forecasts with the proposed method are almost same while the sales rate with the conventional method declines gradually and the forecasts deviate from the actual sales.

Figure 2 shows an another example of the forecasted and the actual sales. The figure is a close-up of the data from 29th day to 42nd day. 33rd, 34th, 41st, and 42nd day are holidays and you can see from the figure that the actual sales rate on the days is higher than the other days. The proposed method follows the change of the sales rate while the conventional method does not. Accumulation of the small difference of the sales rate results in a considerable difference of total sales forecasting.

# 5 CONCLUSIONS

In this paper, we proposed a method to forecast the total sales of products whose effective sale period is very short.

Table 1: Absolute errors of proposed method with time-variant parameters and conventional method with time-invariant parameters.

| | Proposed discrete time model with time-variant params. | | Conventional method with time-invariant params. | | significance probability. |
|---|---|---|---|---|---|
| | mean | variance | mean | variance | |
| Negative Exponential | 0.1121 | 0.0892 | 0.4425 | 0.2012 | 0.0000 |
| Logistic | 0.6353 | 0.0394 | 0.6825 | 0.0727 | 0.0000 |
| Bass | 0.3339 | 0.1835 | 0.5364 | 0.2286 | 0.0000 |
| PNE | 0.1563 | 0.1550 | 0.4448 | 0.2616 | 0.0000 |

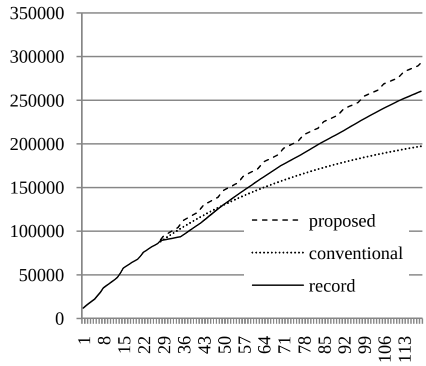


Figure 1: An example of the sales forecasted with proposed and conventional method and an actual sales record.

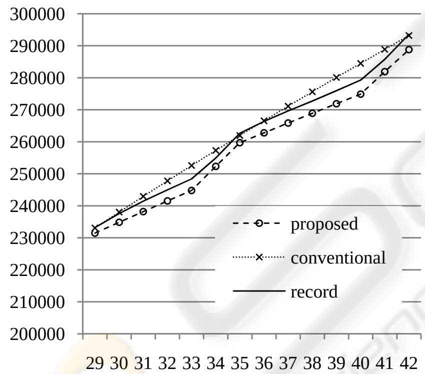


Figure 2: Closeup of another example of the forecasted and the actual sales.

The method uses the diffusion models to forecast demand. It is better to forecast the total sales at the earliest possible time. In this paper, it is forecasted one month after the release of the new model. Since only one-month sales record is available, we use a day as the time unit of forecasting. In order to apply the diffusion model to daily demand forecast, we derive the difference equation representation of diffusion models and discretize the models with respect to time. Then, day-of-week-dependent parameters are introduced to the discrete-time diffusion models.

The parameter estimation is formulated as a linearly constrained non-linear minimization problem. Since the objective function is non-linear, we employed a GA to estimate the parameters. We add several practical constraints in order to reduce the search space and to improve the optimization efficiency.

The proposed method is tested on the data provided by a high-tech consumer products manufacturer. Total sales in 120 days from the release of their products are forecasted and compared to the actual sales record. The result shows that the proposed method has an excellent forecasting ability.

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