Topologies and Meaning Generating Capacities of Neural Networks

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Abstract. The paper introduces the concept of meaning generating capacity (MC) of neural nets, i.e. a measure of information processing, depending on the size of basins of attraction. It can be shown that there is a significant relation between the variance values of the weight matrix of a network and its MC-values. By the concept of MC network characteristics like robustness and generalizing capability can be explained.

1 Introduction

The analysis of topological characteristics of complex dynamical systems frequently enables important insights into the behavior, i.e. the dynamics of such systems. By "topology" we here mean that set of system's rules that determine, which elements of the respective systems interact with which other elements. In the classical mathematical meaning of topology these rules define the neighborhood relations of the respective elements, which are at the core of, e.g., the fundamental Hausdorff axioms of topology. In the case of neural networks the topology is usually defined by the according weight matrix, which determines the degree of interaction between the different elements, including the limiting case of interaction degree equal to zero.

In [1] we introduced the concept of the *meaning processing capacity* (MC) of a complex dynamical system. This definition was motivated by some informal remarks of Wolfram [3] about the "information processing capacity" of complex dynamical systems. With this term Wolfram described the fact that frequently different initial states of a system generate the same final attractor state; other systems in contrast generate different final states if the initial states are different. In other words, the information processing capacity refers to the different sizes of the "basins of attraction" of a system, i.e. the sets of initial states that generate the same final attractor state.

In [1] we defined the concept of the "meaning" of a message by the final attractor state a system generates when receiving this message; in other words, a system processes a message and generates an according meaning. Therefore, we now use the term of *meaning generating capacity* (MC), i.e. the capacity to generate more or less different meanings when receiving different inputs.

The MC-value of a complex dynamical system is now defined as the proportion between the size m of the set of all final attractor states and the size n of the set of all

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initial states of a system, i.e., MC = m/n. Obviously $0 < MC \le 1$: MC = 0 is impossible because each complex system has at least one final state, even if it is an attractor state with a very large period. The according limiting case hence is MC = 1/n. If MC is very small then many different initial states will generate the same final states – the according attractors are characterized by large basins of attraction. If MC = 1 then each different initial state will generate a different final attractor state. This is the other limiting case, where the basins of attraction all are of size 1. In other words, small values of MC mean large basins of attractions and vice versa. It must be noted that we refer only to discrete systems, i.e. systems with only a finite number of initial states.

There are at least three main reasons why this concept is important: On the one hand it is possible via the usage of MC to analyze complex dynamical systems like neural networks with respect to their informational complexity. In this sense MC allows for new approaches in the theory of computability. On the other hand an important and frequently mentioned characteristic of neural networks can be understood in a new and more differentiated way: In all textbooks on neural networks there are statements like "one of the main advantages of neural networks is their robustness, i.e. their tolerance with respect to faulty inputs" or something equivalent. We shall show that via the definition of MC not only a theoretical explanation of this advantage can be given but also a measurement of this robustness; in particular by the variation of MC specific neural networks can be generated that are either very robust, less robust or not at all robust in the sense of error tolerance.

Last but not least it is possible to give by the usage of MC an explanation for phenomena known from the field of human information processing. It is well known that different humans react in a significant different way to the same messages. This can be illustrated by the examples of fanatics who refer all messages to the same cause, e.g. the enmity of Western Capitalism to religious movements. The psychiatrist Sacks [2] for another example describes a rather intelligent and well-educated man who is unable to distinguish little children from fire hydrants. The definition of MC can be a useful approach to construct mathematical models for the explanation of such phenomena.

In contrast to dynamical systems like, e.g., cellular automata and Boolean networks neural networks are not often analyzed in terms of complex dynamical systems. Therefore, it is necessary to clarify what we understand by "initial states" and "final attractor states" when speaking of neural networks.

In a strict systems theoretical sense all initial states of neural networks are the same, i.e. the activation values of all neurons are equal to zero, and regardless to which layer(s) they belong. Because this fact would make the definition of different initial states quite useless we define the initial state of a neural net as the state where the neurons of the input layer have been externally activated with certain input values and where the activation values of all other neurons still are equal to zero, in particular those of the output layer. An initial state S_i of a neural net, hence, is formally defined by $S_i = ((A_i), (0))$, if (A_i) is the input vector and (0) is the output vector, i.e. it denotes the fact that the values of the output neurons are still equal to zero. If there is no specific input layer then the definition must be understood that some neurons are externally activated and the others are not.

The external activation of the input neurons causes via the different functions the "spread of information", determined by the respective weight values. In the case of simple feed forward networks the final activation values of an output layer are immediately generated; in the case of feed back networks or recurrent ones the output is generated in a more complex manner; yet in the end in all cases a certain output vector is generated, i.e., each neuron of the output layer, if there is any, has obtained a certain activation value. If there is no distinction between different layers as for example it is the case with a Hopfield network or an interactive network the output vector will consist of the final activation values of all neurons. Note that except in the case of feed forward networks the output vector may be an attractor with a period $p > 1$. The network will then oscillate between different vectors, i.e. between different states of the attractor. For theoretical and practical purposes neural networks are mainly analyzed with respect to the input-output relation. Therefore, we define the final state S_f of a neural network as $S_f = ((A_i), (A_f))$, if (A_i) is again the input vector and (A_f) the final output vector. If (A_f) is an attractor with period $p > 1$, then the components of (A_f) consists of ordered sets, i.e. the set of all different activation values the output neurons obtain in the attractor.

Because in the experiments described below we investigate only the behavior of feed forward networks with respect to different MC-values, for practical purposes we just define the final state as the values of the output vector after the external activation via the input vector. Hence we speak of a large basin of attraction if many different input vectors generate the same output vector and vice versa. The limiting case MC = 1 for example defines a network where each different input vector generates a different output vector. Accordingly the case M = 1/n defines a network where practically all n different input vectors generate the same output vector.

With these definitions it is easy to explain and measure in a formal manner the characteristics of neural networks with respect to robustness. A robust network, i.e. a network that is tolerant of faulty inputs, has necessarily a MC-value significantly smaller than 1. Robustness means that different inputs, i.e. inputs that differ from the correct one, still will generate the "correct" output, i.e. that output that is generated by the correct input. That is possible only if some faulty inputs belong to the same basin of attraction as the correct input; these and only these inputs from this basin of attraction will generate the correct output. All other faulty inputs transcend the limits of tolerance with respect to the correct output and will accordingly generate another output. If MC = 1 or near 1 then the network will not be robust at all for the respective reasons.

The same explanation can be given for the also frequently quoted capability of neural networks to "generalize": In a formal sense the generalizing capability is just the same as robustness, only looked upon from another perspective. A new input can be perceived as "similar" or as "nearly the same" as an input that the net has already learned if and only if the similar input belongs to the same basin of attraction as the input the network has been trained to remember. In other words, the training process with respect to a certain vector automatically is also a training process with respect to the elements of the according basin of attraction. The capability of generalization, hence, can be understood as the result of the construction of a certain basin of attraction. Accordingly the generalization capability is again dependent on the MC-values: if these are small, i.e. if the basins of attraction are rather large, then the network has a comparatively great generalizing capability and vice versa. Because one network can have only one MC-value it is obvious that systems like the human brain must have for one and the same perceiving task at least two different networks, namely one with a great generalization capability, i.e., a small MC-value, and one with a large MC-value to perceive different inputs as different.

Robustness and generalizing capability of a network, hence, can be "globally" defined by the according MC-value. Yet there is a *caveat*: it is always possible to generate networks via according training methods that are characterized by different basins of attractions with different sizes. Therefore, the MC-value is not necessarily a unique measure for the size of all basins of attraction of a particular network. The term "basin of attraction" refers always only to a certain "equivalence class" of input vectors, namely a set of input vectors that are equivalent in the sense that they generate the same attractor. The size of these sets may be quite different for specific attractors. Hence, the MC-value gives just an average measure with respect to the different basins of attraction. With respect to some attractors and their generating inputs the networks may be robust and with respect to others not. Considering that possibility the concept of MC could also be defined as the difference in size of all basins of attraction of the networks. Fortunately the results of our present experiments hint at the fact that in most cases the basins of attraction of a certain networks differ not much in size. The *caveat* is necessary for theoretical and methodical reasons but seems not to be very important in practical contexts.

Concepts like "size of basins of attraction" and "values of meaning generation capacity" obviously are very useful for the explanation of important characteristics like robustness or generalizing capability. Yet in a strict sense they are too general concepts because they only explain the behavior of certain neural networks from very general characteristics of complex dynamical systems. They do not explain, which *structural* characteristics of neural networks may be the reason for specific MC-values. Hence, these concepts remain, so to speak, on a phenomenological level.

In the beginning of our article we mentioned the fact that frequently certain topological characte-ristics of complex dynamical explain the behavior of such systems. The topology of a neural network is mainly expressed in the weight matrix. Hence the thought suggests itself to look for features of the weight matrix that could explain the size of basins of attraction and MC-values. In anticipation of our results we may say that we were successful in the sense that we found some general trends although no deterministic relations.

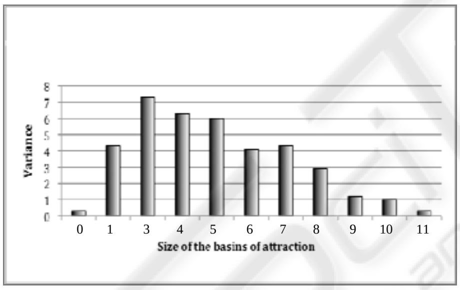
2 Two Experimental Series

In the first experimental analysis we used a standard three-layered feed forward network; we chose this type because it is very frequently used for tasks of pattern recognition and related problems. Because, as is well known, two layers are not enough to solve problems of non-linear separableness we took three layers in order to get results for networks with sufficient efficiency. The input layer consists of 10 units, the hidden layer of 5 and the output layer of 10 units. Input and output neurons are binary coded, which results in 2^{10} = 1024 possible input patterns. To keep the experiments as clearly as possible we defined "equivalence classes" of input patterns:

all input patterns with the same number of zeroes are members of the same class. By choosing at random one pattern from each class we obtained 11 different input patterns. The activation function respectively is the sigmoid function; because of the three layers we chose as learning rule the standard Back Propagation rule.

The training design was the following: In each step the network was trained to associate different input patterns with one target pattern; the target pattern was again chosen at random from the 11 input patterns. In the first step the tasks was to associate each input pattern with one different target pattern; the according basins of attraction all were of size one and the MC-value of this network after the training process is 1:1. In the next steps the sizes of the basins of attraction were gradually increased to 2, 3, 4, 5, and 6; in the last step the size of the only basin of attraction finally was 11, i.e. all input layers had to be associated with one and the same target pattern and the according MC-value is MC = 1/11. We did not investigate basins of attraction with sizes 7 or 10 because in the according experiments the other basins would become too small; for example, one basin of attraction with the size of 8 would force the network to take into regard also at least one basin of attraction of size 3. Hence we only investigated networks with basins of attraction of maximum size 5, 6, and 11. By taking into regard different combinations of basins of attraction we obtained 11 different networks.

The according weight matrices were analyzed with respect to the variance of their weight values. This variance analysis was separately performed for the weight matrix between the input layer and the hidden layer and the matrix between the hidden layer and the output one. The results are shown in figure 1:



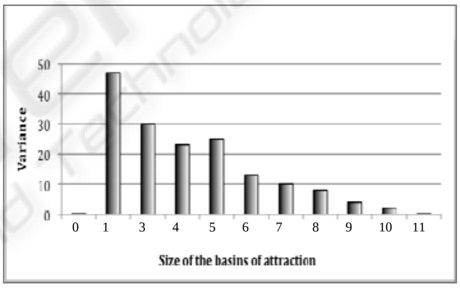


Fig. 1. Variance of the first part of matrix (left figure) and the second part (right figure) in relation to the size of the basins of attraction.

The order of the different networks in both figures is according to increasing size of the basins of attraction. No 1 is the case with MC = 1:1, no 11 is the network with MC = 1:11.

The left figure obviously gives no unambiguous result with respect to possible relations between variance values and the size of basins of attraction but it suggest a certain trend, namely the decreasing of the variance by increasing the basins sizes. The right figure confirms this and even shows an unambiguous result: The variance values indeed gradually decrease with the increasing of the size of the basins of attraction. We then combined the two matrices by summing up the variance values of both matrices and obtained the final result shown in figure 2:

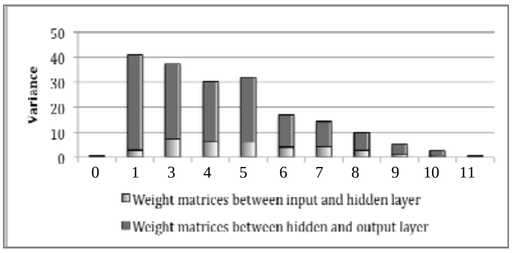


Fig. 2. Variance and size of basins of attraction for the whole network; the networks are ordered as in figure 1.

This final result completely confirms the trend shown in figure 1, left side, and the clear results of figure 1, right side: the larger the size of the basins of attraction are, i.e., the smaller the MC-values are, the smaller are the variance values and vice versa. By the way, the difference between the variance of the upper matrix and that of the lower one is probably due to the fact that the Back Propagation rule does not operate in exactly the same way on both matrices: The lower half of the whole matrix is changed by directly taking into account the error, i.e. the distance between the output neurons and those of the target vector. The changing of the upper half of the matrix is done by computing a certain proportion of the error and thus "dispersing" the changed weight values with respect to those of the lower half. Yet these just force the variance of the whole matrix to the shown result. If our networks had contained only two layers the whole result would have been like that of figure 2. We shall come back to this effect of a certain learning rule in the next section.

We did not expect such unambiguous results yet on hindsight they are quite plausible and comprehensible: Low variance values mean dispersion of information or of the differences between different information respectively because of the near equality of the weight values. If on the other hand the weight values are significantly different, i.e. a high variance, then differences between different messages can be preserved. As small or large sizes respectively of basins of attraction have exactly that effect on the performing of messages it is no wonder that we obtained that clear and unambiguous relation between variance and size of basins of attraction.

Yet although these clear results are quite satisfactory we knew very well that they must be treated with a great methodical *caveat*: the behaviour of neural networks, as that of practically all complex dynamical systems, depends on many parameters, in this case for example on specific propagation, activation and output functions, number of layers, special learning rules and so on. The results shown above were obtained with a specific type of neural network, although a standard and frequently used one with a standard learning rule. To make sure that our results are not only valid for this special methodical procedure we undertook another experimental series.

In these experiments we did not use one of the standard learning rules for neural networks but a Genetic Algorithm (GA). The combination of a GA with neural networks has frequently been done since the systematic analysis of neural networks in the eighties. Usually a GA or another evolutionary algorithm is used in addition to a certain learning rule in order to improve structural aspects of a network that are not changed by the learning rule, e.g. number of layers, number of neurons in a particular layer, threshold values and so on. In our experiments we used the GA as a substitute for a learning rule like the Back Propagation rule in the first experimental series. The according weight matrices of the different networks are, when using a GA, written as a vector and the GA operates on these vectors by the usual "genetic operators", i.e. mutation and recombination (crossover).

We chose this procedure for two reasons: On the one hand the operational logic of a GA or any other evolutionary algorithm is very different from that of the standard learning rules. A learning rule modifies usually just one network; in this sense it is a simulation of ontogenetic learning. In contrast an evolutionary algorithm always operates on a certain *population* of objects and optimizes the single objects by selecting the best ones from this population at time t. This is a model of phylogenetic evolution. In addition learning rules like the Back Propagation rule or its simpler form, namely the Delta Rule, represent the type of supervised learning. Evolutionary algorithms represent another type of learning, i.e. the enforcing learning. In contrast to supervised learning enforcing learning systems get no feed back in form of numerical values that represent the size of the error. The systems just get the information if new results after an optimization step are better or worse than the old ones or if there is no change at all in the improvement process. Therefore, the training procedure in the second series is as different from that of the first one as one can imagine.

We assumed that by choosing such different procedures similar results from both experiments would be a very strong indicator for our working hypothesis, namely the relation between MC-values or size of the basins of attraction respectively and the mentioned characteristics of the according weight matrices. To be sure, that would not be a final proof but at least a "circumstantial evidence" that the results of the first series are no artifacts, i.e., that they are not only effects from the chosen procedure.

On the other hand we were in addition interested in the question if networks with certain MC-values are better or worse suited to adapt to changing environmental conditions. It is evident that *per se* high or low MC-values are not good or bad. It always depends on the situation if a network performs better with high or low capabilities to generate different meanings. Sometimes it is better to process a message in a rather general fashion and sometimes it is necessary to perceive even small differences. Yet from a perspective of evolutionary adaptation it is quite sensible to ask if systems with higher or lower MC can adjust better. That is why we used an evolutionary algorithm to investigate this problem although it is another question than that of a relation between the variance of the weight matrix and the according MC-values.

Because a GA can be constructed with using many different parameters like size of the mutation rate, size of the sub vectors in crossover, selection schemas, schemas of crossover ("wedding schemas"), keeping the best "parents" or not and so on it is rather difficult to obtain results that are representative for all possible GA-versions. We used a standard GA with a mutation rate of 10%, a population of 20 networks, initially generated at random, crossover segments of 5, and a selection schema according to the fitness of the respective networks. Because the networks were optimized with respect to the same association tasks as in the first series those networks are "fitter" than others that successfully have learned more association tasks than others. If for example a network is optimized with respect to the task to operate according to two basins of attraction of size 8 then a network is better that correctly associates 6 vectors of each basin to the target vector than a network that does this only for 5 vectors.

The population consists again of three-layered feed forward networks with binary coding for input and output layers; the input and output vectors consist of four neurons and the hidden layer of three. As in the first series the networks operate with the sigmoid activation function. We simplified the networks a bit because, as mentioned, a GA has not one network to operate with but a whole population. The target vectors were chosen at random; the vectors for the respective basins of attraction were chosen according to their Hamming distance to those output vectors that define the basins of attraction. It is no surprise that the GA came faster to satisfactory results, i.e. the generation of networks that are able to solve the respective association tasks, if the MC-values of the networks should be large than in the cases when the MC should be small. The main results are shown in figure 3:

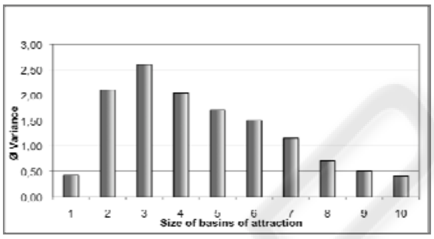


Fig. 3. Variance and size of basins of attraction in networks generated by a GA.

The figure obviously expresses a striking similarity to figure 1 of the first series. The trend is the same, namely a clear relation between the size of the variance and the increasing size of the basins of attraction or the decreasing size of the MC-values respectively. Like in figure 1 the exceptions from this trend occur in the cases of rather small basins of attraction, but only there. As we remarked in the preceding section these exceptions may be due to the fact that the GA even more disperses the weight values than does the Back Propagation rule for the upper half of the weight matrix. This fact clearly demonstrates that the relation between variance values and the sizes of the basins of attraction is "only" a statistical one, although the correlation is very clear. We omit for the sake of brevity he results of the evolutionary analysis.

As we mentioned in the beginning of this section, the fact that such totally different optimization algorithms like Back Propagation rule and GA, including the different types of learning, generate the same trend with respect to our working hypothesis is important evidence that the hypothesis may be valid in a general sense. Yet in both series we just considered "case studies", i.e. we concentrated in both cases on one single network type and in the case of the GA-training on populations of the same type of networks. That is why we started a third series.

3 Third Series: Statistical Analysis of Large Samples

Experiments with large samples of neural networks are always difficult because of the large number of variables or parameters respectively that have to be taken into account. Besides the influence of different learning rules, activation and propagation functions and such parameters like learning rates and momentum the main problem is a “combinatorial explosion”: if one takes into account the many different possible combinations of neurons in the different layers and in addition the possible variations of the number of layers one quickly gets such large samples that it is seldom possible to obtain meaningful results. That is why we chose another way in the preceding sections, namely the analysis of the two case studies in order to get a meaningful hypothesis at all.

Yet despite the great difference between our two case studies it is always rather problematic to draw general consequences from only several case studies. That is why we studied a larger sample of two-layered neural nets, i.e., ca. 400.000 different networks. We restricted the experiment to networks of two layers in order to keep the experiments as clear as possible. The number of neurons in the input and output vector are in all experiments the same and ranged from 3 to 10. The restriction to equal dimensions of the two vectors was introduced because networks with different sizes of the two vectors do not generate all MC-values with the same probability: If the input vector is larger than the output one then MC = 1 would not be possible at all because always more than one input vector will generate the same output vector. For example, a simple network that is trained to learn a certain Boolean function has an input vector of size 2 and an output vector of size 1. Its MC-value is 0.5. If conversely the output vector is larger than the input vector the probability for large MC-values will be greater than in networks with the same number of neurons in both vectors. To avoid such distortions we used only vectors of equal size.

The networks were, as in the two case studies, binary coded and operated with the sigmoid function. Thus we obtained ca. 400.000 pairs (MC, v), v being the variance. The general results are the following:

As we supposed from the results of the two case studies the relation between variance and MC-values is “only” a statistical one in the sense that there are always exceptions from the general rule. Yet we discovered very clearly that indeed there is a significant probability: the larger the variance is the smaller is the probability to obtain networks with small MC-values, that is with large basins of attraction, and vice versa. This result is in particular valid for variance values significantly large or small. Only in the “middle regions” of variance values the probability to obtain MC-values as a deviation from the general rule is a bit larger but not very much. This probability distribution is shown in figure 4:

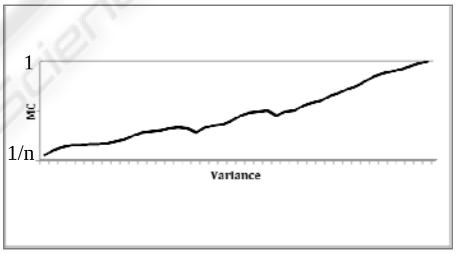


Fig. 4. Statistical relation between variance (x-axis) and MC-values (y-axis).

By the way, the deviations in the middle regions from the general trend may be a first explanation for the mentioned results from section 2 with respect to the evolutionary capability of networks with different MC-values. These networks adapt more easily to changing environments than those with very large or very small values.

The hypothesis of the relation between MC-values and variance values seems to be a valid one, at last as a statistical relation, Hence it is possible to predict the meaning generating capacity of a network and thus its practical behaviour for certain purposes with sufficient probability from a variance analysis of its weight matrix. Yet a *caveat* is in order: We analyzed only one type of networks and further experiments are necessary if our results are also valid for different types like, e.g. recurrent nets of Self Organized Maps.

4 Interpretations and Conclusions

The behavior of complex dynamical systems can practically never be explained or predicted by using only one numerical value (a scalar) as the decisive parameter. In a mathematical sense the problem for such a system is always the task of solving equations with a lot of variables, that is more variables than equations. It is well known that for such tasks there is always more than one solution. When considering neural networks by investigating the according weight matrix it is rather evident that for example large basins of attraction may be constructed by very different matrices. Hence, it is no wonder that the variance value, considered as a structural measure for the occurrence of certain MC-values and the according sizes of the basins of attraction, always allows for exceptions.

The knowledge about parameters that could predict the meaning generation capacity would not only give important insights into the logic of neural network operations; that would be an improvement of our theoretical understanding of these complex dynamical systems. It could also give practical advantages if one needs neural networks with certain capabilities – either if one needs robust networks with great generalizing capacity or if there is need for sensitive networks that react in a different manner to different inputs. To be sure, the relations we have discovered so far are of only statistical validity. Yet to know that with a high probability one gets the desired characteristics of a certain network if one constructs a weight matrix with a specific variance is frequently a practical advantage: one has not to train the networks and look afterwards if the network has the desired behavior but can construct the desired matrix and if necessary make the additionally needed improvements via learning rules and/or additional optimization algorithms like for example a GA. For these theoretical and practical reasons it will be worthwhile to investigate such relations as we have shown in this article further and deeper.

References

1. Klüver, J. and Klüver, C., 2007: On Communication. An Interdisciplinary and Mathematical Approach. Dordrecht (NL): Springer.
2. Sacks, O., 1985: The Man Who Mistook His Wife for a Hat. New York: Summit Books.
3. Wolfram, S., 2001: A new Kind of Science. Champagne (Ill.): Wolfram Media.