

# IMPROVED 2D MAXIMUM ENTROPY THRESHOLD SEGMENTATION METHOD BASED ON PSO

Liping Zheng

*School of Computer Science, Liaocheng University, Liaocheng, Shandong, China*

Guangyao Li

*CAD Research Center, Tongji University, Shanghai, China*

Jing Liang

*School of Electrical Engineering, Zhengzhou University, Zhengzhou, Henan, China*

Quanke Pan

*School of Computer Science, Liaocheng University, Liaocheng, Shandong, China*

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**Abstract:** Image segmentation plays an important role in the field of image processing. Threshold segmentation is a simple and important method in image segmentation. Maximum Entropy is a common threshold segmentation method. In order to adequately utilize gray information and spatial information of image, an improved 2D entropy computation method is proposed. Otherwise, Particle Swarm Optimization(PSO) algorithm is used to solve maximum of improved entropy. Maximum takes as the optimal image segmentation threshold. In this paper, two CT images were segmented in experiment. Experimental results show that this method can quickly and accurately obtain segmentation threshold. Otherwise, this method has strong anti-noise capability and save computation time.

## 1 INTRODUCTION

Image segmentation is to separate pixel of the image into some non-intersecting regions(Pal, 1993). Image segmentation is the base of image processing. Because of the complexity and diversity of image, there isn't a common segmentation algorithm. In application, different algorithms are used according to features of image and application purposes.

Threshold segmentation method is a widely used and effective method for segmentation gray level image. Segmentation results rely on the segmentation threshold value. Therefore it is very important to choose an optimal threshold value.

### 1.1 Threshold Segmentation Method

Threshold segmentation method is one of the most essential methods in image segmentation. This method utilizes features of pixels. That is said, object pixels and background pixels of image can be

distinguished by their gray level values. In process of segmentation, the histogram of the image is usually used, other image information such as the spatial information is not utilized(Glasbey, 1993).By choosing an adequate threshold value and segmentation image, object can be extracted from their background. Selection criterion of segmentation threshold value plays an important role in threshold segmentation method.

In past years, many schemes have appeared in literature. Many different selection criterions were proposed, such as Otsu method(Otsu, 1979), minimum error threshold value method(Kittler, 1986). In 1980, entropy-based selecting approach was proposed(PUN, 1980). Kapur proposed the one-dimension entropy threshold value method(Kapur, 1985). When segment an image, one-dimension threshold value method solely depended on the gray-level distribution. While the Signal-to-Noise of image is low and the background of image is complex, segmentation results with

one-dimensional entropy segmentation method are poor. Therefore, Abutaleb proposed two-dimension entropy method in 1989(Abutaleb, 1989).

The two-dimension entropy is obtained from the two-dimension histogram which is constructed with using the gray value of image and the average gray value of image. Two-dimension entropy method is a region-dependent method. When computing optimal threshold value, two-dimension entropy method needs cost more time than one-dimension entropy method.

In order to reduce the computation time and improve accuracy of segmentation threshold value, an improved 2D maximum entropy threshold segmentation method is proposed. This method narrows down the search space. And PSO algorithm is used to solve the optimal segmentation threshold value. PSO algorithm has advantages of quick convergence. Therefore it can reduce computation time. Improved 2D entropy is called spatial difference attribute information value entropy (SDAIVE). This improved threshold segmentation method based on PSO is called PSO-SDAIVE algorithm.

## 2 IMPROVEMENT OF 2D GRAY HISTOGRAM

While segment image with 2D threshold value method, associated entropy need to be computed and obtain the optimal segmentation threshold value(Sahoo, 1988). In order to reduce the solving time and improve the searching efficiency, the 2D gray histogram is improved in this paper.

### 2.1 2D Gray Histogram

In 2D gray histogram, the horizontal axis is the gray-level value of pixel. It's range is  $[0, L-1]$ ; the vertical axis is the average gray-level value of pixel. It's range is  $[0, L-1]$ . Let  $f(m,n)$  denotes the gray-level value of pixel which located at point(m,n). In image,  $\{f(m,n) | m \in \{1, 2, \dots, M\}, n \in \{1, 2, \dots, N\}\}$ . Let  $g(m,n)$  is the average gray-level value of the neighborhood of pixel(m,n). The whole number of probability value of  $(f(m,n), g(m,n))$  is  $L \times L$ .

In the plan of 2D gray histogram, object pixels and background pixels locate the diagonal neighborhood. Noise points and edge pixels are far from diagonal.

### 2.2 2D D-value Attribute Gray Histogram

In traditional 2D gray histogram, the search space is big and the solving time of the optimal threshold value is long. In order to narrow down the search space, 2D gray histogram is improved. According to pixel attributes, the searching region is reduced.

This improved 2D gray histogram is called 2D D-value attribute gray histogram. The horizontal axis of 2D D-value attribute gray histogram is gray value. The vertical axis is the D-value between gray value and average gray value. It denotes absolute value of the difference between  $f(m,n)$  and  $g(m,n)$ .

The histogram with a given attribute is called attribute histogram. In this paper, the associated attribute condition is set. Suppose  $L_1 < f(m,n) < L_2$  and  $|f(m,n) - g(m,n)| < \mu$ . The attribute condition restricts the range of search space. That is the gray value of pixel is between  $L_1$  and  $L_2$ . The average gray value is also in given range. Only pixels which satisfy attribute condition can be searched.

Figure1 shows the plan of 2D D-value attribute gray histogram. According to attribute condition, pixels in region G are searched. A pair of value (s,w) represents segmentation threshold value. Impact of noise points are reduced by using this improved gray histogram.

Comparing with the traditional 2D gray histogram, pixels in region G should satisfy the following conditions:  $L_1 \leq f(m,n) \leq s$  and

$\max\{0, L_1 - w\} \leq g(m,n) \leq \min\{s + w, L - 1\}$ . Pixels in region H should satisfy  $s + 1 \leq f(m,n) \leq L_2$  and  $\max\{0, s + 1 - w\} \leq g(m,n) \leq \min\{L_2 + w, L - 1\}$ .

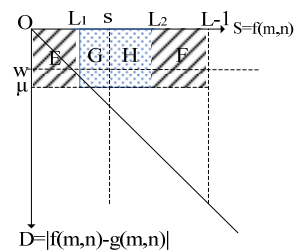


Figure 1: The plan of 2D D-value attribute gray histogram

## 3 THE IMPROVED 2D ENTROPY

The essence of image segmentation with entropy is that utilize the gray probability of image. The gray information of image includes the gray probability

and associated gray value. Spatial information value includes gray value, average gray value and other information. In order to better utilize the gray and spatial information of image, the improved computation method of 2D entropy is proposed in this paper.

### 3.1 Spatial Information Function

According to gray value  $i$  and average gray value  $j$ , spatial information function is defined. When computing image 2D entropy, pixel spatial information value substitutes for gray probability. Formula (1) shows computation of spatial information function. In formula,  $IV$  denotes other spatial gray value.

$$I(i, j) = I_{ij} = \{c_1(i+1)^2 + c_2(j+1)^2 + c_3IV\} p_{ij} \quad (1)$$

$$c_1 + c_2 + c_3 = 1, 0 \leq c_1 \leq 1, 0 \leq c_2 \leq 1, 0 \leq c_3 \leq 1$$

In this paper, only consider gray value and average gray. Suppose weights of gray and average gray are same. Therefore, spatial information function formula is reduced as formula (2).

$$I(i, j) = I_{ij} = 0.5(i+1)^2 + 0.5(j+1)^2 p_{ij} \quad (2)$$

Suppose  $m_{ij}$  is the total number of occurrence  $(i, j)$ . The joint probability function  $p_{ij}$  is defined as formula(3).

$$M = \sum_{i=0}^{L-1} \sum_{j=0}^{L-1} m_{ij} \quad p_{ij} = \frac{m_{ij}}{M} \quad (3)$$

$$i, j = 0, 1, \dots, L-1$$

Refer the 2D D-value attribute gray histogram and compute the information value entropy. This entropy is called spatial difference attribute information value entropy (SDAIVE).

### 3.2 Computation of SDAIVE

Suppose threshold value pair  $(s, w)$ , that is said, the gray value is  $s$ , and the gray D-value is  $w$ . According to figure 2, the gray value range and the gray D-value range are confirmed.

Formula (4) defines the SDAIVE of object pixels and background pixels.  $H'(O)$  is the SDAIVE of object pixels.  $H'(B)$  is the SDAIVE of background pixels.

$$H'(O) = \ln P_O + \frac{H_O}{P_O} \quad (4)$$

$$H'(B) = \ln P_B + \frac{H_B}{P_B}$$

Formula (5) defines computation of  $P_O$ . Formula (6) defines  $P_B$ .  $P_O$  denotes the gray information

quantity while gray value range is  $[L_1, s]$ .  $P_B$  denotes the gray information quantity while gray value range is  $[s+1, L_2]$ . Formula (7) defines  $H_O$  and  $H_B$ .

$$P_O = I_O = \sum_{i=L_1}^s \sum_{j=L_1-w}^{w+s} I_{ij} \quad (5)$$

$$= \sum_{i=L_1}^s \sum_{j=L_1-w}^{w+s} \frac{(i+1)^2 + (j+1)^2}{2} p_{ij}$$

$$P_B = I_B = \sum_{i=s+1}^{L_2} \sum_{j=s+1-w}^{w+L_2} I_{ij} \quad (6)$$

$$= \sum_{i=s+1}^{L_2} \sum_{j=s+1-w}^{w+L_2} \frac{(i+1)^2 + (j+1)^2}{2} p_{ij}$$

$$H_O = - \sum_{i=L_1}^s \sum_{j=L_1-w}^{w+s} I_{ij} \ln I_{ij} \quad (7)$$

$$H_B = - \sum_{i=s+1}^{L_2} \sum_{j=s+1-w}^{w+L_2} I_{ij} \ln I_{ij}$$

Formula (8) defines discriminant function of SDAIVE. In this paper, the maximum of SDAIVE is taken as the selection criterion of threshold value. Formula (9) defines the maximum function.

Solve the maximum of SDAIVE with some optimize algorithm and obtain the optimal segmentation threshold. Then segment image with the optimal threshold value  $(s, w)^*$ .

$$\psi(s, w) = H'(O) + H'(B)$$

$$= \ln \sum_{i=L_1}^s \sum_{j=L_1-w}^{w+s} I_{ij} + \frac{(- \sum_{i=L_1}^s \sum_{j=L_1-w}^{w+s} I_{ij} \ln I_{ij})}{\sum_{i=L_1}^s \sum_{j=L_1-w}^{w+s} I_{ij}} +$$

$$\ln \sum_{i=s+1}^{L_2} \sum_{j=s+1-w}^{w+L_2} I_{ij} + \frac{(- \sum_{i=s+1}^{L_2} \sum_{j=s+1-w}^{w+L_2} I_{ij} \ln I_{ij})}{\sum_{i=s+1}^{L_2} \sum_{j=s+1-w}^{w+L_2} I_{ij}} \quad (8)$$

$$= \ln \left( \sum_{i=L_1}^s \sum_{j=L_1-w}^{w+s} I_{ij} \times \sum_{i=s+1}^{L_2} \sum_{j=s+1-w}^{w+L_2} I_{ij} \right) -$$

$$\frac{(\sum_{i=L_1}^s \sum_{j=L_1-w}^{w+s} I_{ij} \ln I_{ij})}{\sum_{i=L_1}^s \sum_{j=L_1-w}^{w+s} I_{ij}} - \frac{(\sum_{i=s+1}^{L_2} \sum_{j=s+1-w}^{w+L_2} I_{ij} \ln I_{ij})}{\sum_{i=s+1}^{L_2} \sum_{j=s+1-w}^{w+L_2} I_{ij}}$$

$$= \ln(P_O \times P_B) + \frac{H_O}{P_O} + \frac{H_B}{P_B}$$

$$(s, w)^* = \text{Arg}_{L_1 < s < L_2, 0 < w < u} \text{Max} (\psi(s, w)) \quad (9)$$

## 4 SEARCHING SEGMENTATION THRESHOLD WITH PSO

In this paper, PSO algorithm is used to solve the

SDAIVE maximum. PSO algorithm was jointly proposed by the American sociologist and psychologist of James Kennedy and electrical engineer Russell Eberhart in 1995. The basic idea of PSO algorithm was inspired by researching results of behavior about bird groups and makes use of a biological communities model which was proposed by biologist Frank Heppner.

#### 4.1 Basic Concepts of PSO Algorithm

PSO algorithm is a Swarm Intelligence Algorithm. It takes particle as individual and flight with a certain speed in the search space. These particles haven't quality and volume. Every particle has the simple rules of conduct. According to the flying experience of individuals and groups, particles can dynamically adjust the flying speed.

Since 1995, researchers proposed different algorithm models in different fields. Kenney constructed simple PSO model. Eberhart constructed PSO model with inertia weight factor. YuhuiShi and Clerc constructed PSO model with the shrinkage factor. Through a large number of experimentations, the significance and role of different control parameters in models were detailed analysis. Corresponding reference values were identified (Carlisle, 2001).

In PSO algorithm model, choices of parameters are the focus of researching. There are six important control parameters in PSO algorithm. They are population size, cognitive learning rate  $c_1$ , social learning rate  $c_2$ , the maximum of particle flying speed  $V_{max}$ , the inertia weight factor  $\omega$ , constriction factor  $K$ . The population size of particles refers the number of particles in iterative process.

In these algorithm models, the basic and earliest PSO model was defined by Eberhart and Kennedy (Kennedy, 1995). It was called simple PSO model. The specific model as formula (10):

$$\begin{aligned} v_{id}(t+1) &= v_{id}(t) + c_1 r_1 (P_{id}(t) - x_{id}(t)) \\ &\quad + c_2 r_2 (P_{gd}(t) - x_{id}(t)) \\ x_{id}(t+1) &= x_{id}(t) + v_{id}(t+1) \end{aligned} \quad (10)$$

#### 4.2 Computation of Optimal Threshold with simple PSO

In this paper, the simple PSO algorithm was used. Because the sum of  $c_2$  and  $c_1$  should be 4, the parameters  $c_2=c_1=2$ ,  $r_1=r_2=0.5$ . In experiment, the iterative formula of speed is simplified as formula (11):

$$v_{id}(t+1) = v_{id}(t) + P_{id}(t) + P_{gd}(t) - 2x_{id}(t) \quad (11)$$

Because gray value of pixel is integer, the solving of SDAIVE maximum can take as integer planning problem. Pixel particles move along the gray value (V-direction) and the gray D-value ( $\mu$ -direction) at the same time. Speed and location of particles need iterate in two directions at the same time.

In this paper,  $t$  denotes the particle generation.  $i$  denotes serial number of particle.  $v_{iv}$  denotes the speed change in gray value direction.  $v_{i\mu}$  denotes the speed change in gray D-value direction.  $x_{iv}$  denotes the location change in gray value direction.  $x_{i\mu}$  denotes the location change in gray D-value direction.  $P_{iv}(t)$  and  $P_{i\mu}(t)$  are the corresponding particle locations of the SDAIVE maximum in  $t$  generation.

Iterative formulas of speed and location as following:

$$v_{iv}(t+1) = v_{iv}(t) + P_{iv}(t) + P_{gv} - 2x_{iv}(t) \quad (12)$$

$$x_{iv}(t+1) = x_{iv}(t) + v_{iv}(t+1) \quad (13)$$

$$v_{i\mu}(t+1) = v_{i\mu}(t) + P_{i\mu}(t) + P_{g\mu} - 2x_{i\mu}(t) \quad (14)$$

$$x_{i\mu}(t+1) = x_{i\mu}(t) + v_{i\mu}(t+1) \quad (15)$$

By iteration, the location of SDAIVE maximum is confirmed. That is said, the location of optimal threshold is confirmed.

## 5 EXPERIMENT AND RESULTS

In order to validate the capability of algorithm which is proposed in this paper, gray image needs be segmented in experiment. CT image is a kind of gray image. Therefore, two CT images were segmented in experiment. Experiment results were analyzed.

Input two head CT image IM10 and IM10'. The type of image is uint16. The gray range is [0, 65535]. Image IM10 hasn't noise. IM10' has 'salt & pepper' noise. Figure2 shows two head CT images.

In this experiment, bones need to be separated from CT images. Therefore, bones were taken as object and other tissues were taken as background. Figure3 shows the 2D D-value attribute gray histogram of IM10 and IM10'. According to improved gray histogram, computed their  $H'(O)$  and  $H'(B)$  of two images. The SDAIVE maximum of image was solved with PSO. At last, optimal

segmentation threshold values were obtained.

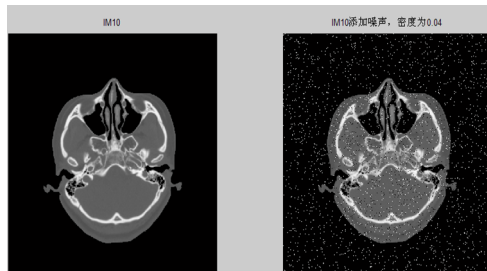


Figure 2: Input IM10 and IM10'.

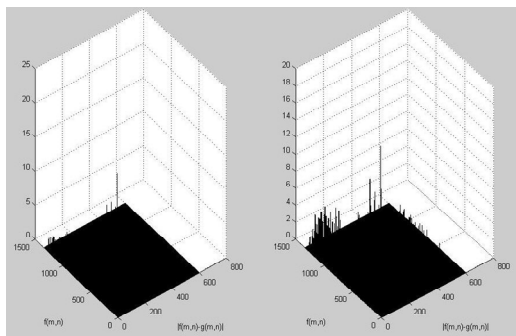


Figure 3: The 2D D-value attribute gray histogram.

The optimal threshold value of IM10 is (2253, 500). The optimal threshold value of IM10' is (2275, 525). Figure 4 shows the segmentation results of two images.

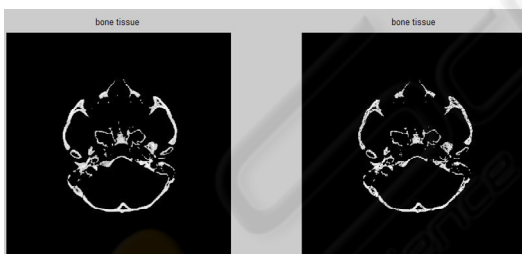


Figure 4: The segmentation results of IM10 and IM10'.

Experiment results show that this improved segmentation method has good anti-noise capability. Otherwise, the search space is small. The time of solving optimal threshold value is decreased. If using genetic algorithm, the computation quantity is also reduced, but the genetic algorithm is complex.

## 6 CONCLUSIONS

The one-dimension entropy method only considers gray information. Otherwise, it has poor anti-noise capability. Two-dimension entropy method considers

the spatial information, but the computing quantity is large.

Improved 2D maximum entropy threshold segmentation method based on PSO is called PSO-SDAIVE algorithm. This algorithm not only considers the spatial information, but also considers the gray information and decreases the computing quantity. Otherwise, the neighboring pixel control parameter is set. The overly-smoothness of images can be avoided.

In this paper, simple PSO model is used. In future, we will research the solving method of SDAIVE with different PSO models and search an optimal PSO model with shortest time.

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