# A NOVEL PERFORMANCE METRIC FOR GREY-SCALE EDGE DETECTION

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Abstract: This paper will discuss grey-scale edge detection evaluation techniques. It will introduce three of the most common edge comparison methods and assess their suitability for grey-scale edge detection evaluation. This suitability evaluation will include Pratt's Figure Of Merit (FOM), Bowyer's Closest Distance Metric (CDM), and Prieto and Allen's Pixel Correspondence Metric. The relative merits of each method will be discussed alongside the inconsistencies inherent to each technique. Finally, a novel performance criterion for grey-scale edge comparison, the Grey-scale Figure Of Merit (GFOM) will be introduced which overcomes some of the evaluation faults discussed. Furthermore, a new technique for assessing the relative connectivity of detected edges will be described and evaluated. Overall this will allow a robust and objective method of gauging edge detector performance.

## 1 INTRODUCTION

Regardless of the technique used for edge detection; Sobel, Prewitt (Šonka et al., 1986), Canny (Canny, 1986), or statistical, like the work of (Bowring et al., 2004) and (Fesharaki and Hellestrand, 1994), the evaluation will be the same and an effective performance measure will be suitable for all.

Many evaluation methods are subjective in their nature, usually conducted using the human perception of an edge, or with the aim of recognising any objects contained within the image. This subjective evaluation is an unsatisfactory performance test by itself, and overall objective measures like those of (Abdou and Pratt, 1979), (Bowyer et al., 2001), (Boaventura and Gonzaga, 2009) and (Prieto Segui and Allen, 2003), or a combination of both subjective and objective measures are more preferable as described in the work of (Heath et al., 1996).

Objective performance determines the accuracy of an edge detector, with restricted influence from human perception. However, although extensive study has concentrated on the development and optimisation of edge detectors there has been little development of an accurate measure to gauge their success. The most common evaluation techniques asses performance by comparing edges within the edge detected image against a pre-segmented gold standard image. This can be either user defined or automatically computed as in the work of (Fernández-García et al., 2008). However, many are gauged for binary edge detection, or on images thresholded at an appropriate but subjective value as in the technique of (Abdou and Pratt, 1979).This evaluation, while accurate in its field, makes strong assumptions about the usefulness of the detected edges. Moreover, if taken out of the application context for which it was designed, or evaluated at a different threshold level, the significance of any results can become questionable.



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## 2 EDGE DETECTION PERFORMANCE

The three main criteria for edge detection were defined by (Canny, 1986), namely the accurate detection of edges with the error rate to a minimum, the accurate location of edges with regards to the ground truth, and finally the single response to any edge ensuring any multiple edge detection is avoided.

These three criteria form the basis of any edge detection evaluation performance, which can be described as:

True Positive: the sum of true edges detected, False Positive: the sum of falsely detected edges, True Negative: the sum of true non-edges detected and False Negative: the sum of falsely detected non-edges. In addition to this we can evaluate the "localisation", which assesses the location accuracy of the actual edge to the detected edge, and finally the "single response" which evaluates if there is only a single response to any one edge testing the possibility of multiple responses to a single desired edge.

General correlation principles between the edge detected image and the associated ground truth image could be used to determine a simple performance measure of the detector. For example a simple subtraction of the detected image and the ground truth would determine the number of false positives and false negatives. However such a simple evaluation operation would not tolerate any shift in localisation of true edges in the image. Subsequently, a detector with even slight localisation errors would be unfairly awarded a poor performance value. Evaluating with reference to single pixel edges only would therefore allow no tolerance for edges that were miss-aligned by one pixel in the image. These problems were identified by (Grigorescu et al., 2003) who, through the use of a pixel region mask, allowed for slight misalignment errors and evaluated any non zero pixel within this mask to be a true positive. As described by in the work of (Joshi and Sivaswamy, 2006), this method of performance is itself subjective and using a larger mask will result in a greater number of false positives being detected and therefore reduced performance measure.

Although accurate for evaluating performance against synthetic images, both true and false positive or negative values should be approached with caution when assessing performance of real images, or images with significant levels of texture. These measures can unduly bias a noisy background area of a textured image and in the case on images where there are only a few edges in relation to the background area, can produce an inaccurately low performance measure.

In addition to this problem, many evaluation measures are gauged for object oriented edge detection, and therefore represent the result as a measure of effectiveness against final tasks, such as object recognition. This evaluation, while accurate in its field, makes strong assumptions about the usefulness of the detected edges. Moreover, if taken out of the application context, the significance of the results can become questionable. To avoid this ambiguity the work detailed in this paper performs an analysis of edge detection in its entirety without determining a final application step for the image. This allows evaluation simply on the accuracy of the "true" edges within the image and not just the object boundaries. This gives a performance indicator that is useful for a variety of post-edge detection applications.

To avoid any threshold ambiguity, and for applications which are reliant on the detection of accurate grey-level edges, as in the work of (Svoboda and Matula, 2003), a grey-scale evaluation should also be used. Grey-scale assessment is not a simple task however, and work by (Prieto Segui and Allen, 2003) has illustrated some of the difficulties. Their work evaluated the noise robustness of several grey-scale evaluation methods and highlighted inconsistencies inherent to the CDM (Closest Distance Metric)(Bowyer et al., 2001), PSNR (Peak Signal to Noise Ratio), and the FOM (Figure of Merit) evaluation methods designed by (Abdou and Pratt, 1979). To avoid these problems (Prieto Segui and Allen, 2003) developed a novel PCM (Pixel Correspondence Metric) technique, which in practise was both sensitive and consistent in its evaluation and gave more precision to the assessment of edge quality.

## 3 PERFORMANCE METRICS

### 3.1 PCM (Pixel Correspondence Metric)

The Pixel Correspondence Metric (or PCM) is the first edge evaluation measure detailed in this work. PCM evaluates all the pixel points in two given images ($PCM(a,b)$), here represented as the edge detected image (a) and the ideal ground truth image (b). For each pixel point in the edge image $a$ ($a(i,j)$) the metric checks for an appropriate match within a defined pixel neighbourhood in the ideal image $b$ ($b(i,j)$). Unlike comparison metrics which do not allow for a shift in the detected pixel points, or simply evaluate binary images, PCM evaluates the pixel match based on both the spatial distance between the pixel points and also the actual grey-level value of the detected edge point. Any non-matched pixel points are then measured as errors. This error eliminates the possibility of multiple matches to the same edge point through a process of weighted matching using bipartite graphs. This form of bipartite matching ensures that for every non-zero pixel point in the edge detected image the PCM match value is maximised using the ideal, and therefore the overall PCM for the match image is maximised. For the work presented here the implementation of PCM by (Prieto Segui and Allen, 2003) is used (See Equation 1). For a more detailed explanation and the finer workings of this technique please refer to the work of (Prieto Segui and Allen, 2003).

$$PCM_\eta(a,b) = 100\left(1 - \frac{C(M_{opt}(a,b))}{|a\bigcup b|}\right) \quad (1)$$

where: $C(M_{opt}(a,b))$ = the cost of optimal matching between each image - calculated using weighted bipartite graphs
$\eta$ = The maximum localisation error allowed between pixels
$|a\bigcup b|$ = The total number of non-zero pixels in image a or b.

### 3.2 CDM (Closest Distance Metric)

The Closest Distance Metric (CDM) is an evaluation technique based in the work of (Bowyer et al., 2001). Similar in principle to the PCM metric, the CDM technique uses a defined pixel region ($\eta$) to check every pixel in the edge detected image array ($a(i,j)$) for a corresponding match in the ideal image ($b(i,j)$) across this region. The work by (Bowyer et al., 2001) used binary edge images which allowed for the use of Receiver Operating Characteristic (ROC) curves to evaluate the trade off between true positive and false positive rates. However in this work the relative performance of grey-scale edge detection is assessed so ROC curves would not be appropriate to use with the CDM metric. This work uses the implementation of CDM as defined by (Prieto Segui and Allen, 2003) (see Equation 2). The implementation therefore allows the comparison to be based on both the distance and the grey-level strength of the actual edge against the ideal edge.

$$CDM_\eta(a,b) = 100\left(1 - \frac{C(M_c d(a,b))}{(|a\bigcup b|)}\right) \quad (2)$$

where: $\eta$ = The size of the region for matching between images.
$C(M_c d(a,b))$ = The cost of matching using a "closest-distance" metric
$|a\bigcup b|$ = The sum of non zero points in images a and b.

### 3.3 PFOM (Pratt's Figure of Merit)

The most commonly used form of edge comparison measure was defined by (Pratt, 1978), and is regarded as the standard in edge detection evaluation. The Figure Of Merit (FOM) performance is assessed on binary or threshold images against a pre-segmented ground truth edited to include only their "true" edges. Pratt's Figure Of Merit (FOM) (Pratt, 1978) is defined in Equation 3.

$$R = \frac{1}{I_{sum}} \sum_{i=1}^{I_A} \frac{1}{1+\beta d_i^2} \quad (3)$$

where: $I_{sum} = max(I, I_A)$. $I$ = The sum of the ideal edge points. $I_A$ = The sum of the detected edge points. $d_i$ = the distance of the $i^{th}$ edge point from the ideal edge point. $\beta$ = A scaling constant (typically set to $\frac{1}{9}$).

The major drawbacks of the Pratt method of evaluation is that it is designed to work with binary images only. Therefore the implementation has been adapted in this work for with grey-scale images.

### 3.4 GFOM (Grey-scale Figure of Merit)

It is often important to assess each image based on its grey-levels before any threshold or post edge detection process has been applied, to accurately gauge the effectiveness of each raw edge detector. This adapted Grey-scale Figure Of Merit (or GFOM) that addresses this requirement is now described.

The Figure Of Merit (FOM) performance is calculated for each of the 256 grey levels in both the edge detected and the ideal gold standard image independently and is shown in Figure 1. This is achieved by thresholding the image at each grey-scale level in turn. A performance value can then be calculated for that specific threshold level against the gold standard image. The sum of these 256 performance values can then be calculated, and the mean Figure of merit determined by Equation 4. This can then be assigned as the overall grey-scale performance value (GFOM) for the image.

$$GFOM = \frac{\sum_{i=1}^{L} R_i}{L} \quad (4)$$

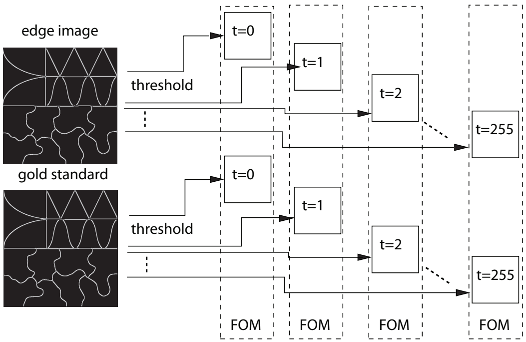



Figure 1: GFOM is Pratts FOM adapted to work with 256 grey-scale levels. Both the edge image and ground-truth image are threshold at each of the 256 grey-scale levels in turn. Pratt's Figure of merit is then calculated for the current threshold level between both images. This process is repeated for each of the 256 grey-scale levels and the sum of Pratt merit Figures is calculated. The mean of these Pratt values is then allocated as the Figure of merit for the image.

where:

L = the number of grey-scale levels (typically 256) in both the edge detected image and the gold standard image.

$R_i$ = the FOM value defined in Equation 3 with threshold value "$i = 1 \dots L$".

### 3.5 Edge Connectivity

PCM, CDM, FOM and GFOM are not designed to assess the "connectivity" of the edge. Consequently as illustrated in the results section, an accurately located edge which is fragmented or not continuous could be awarded a greater performance metric than one which is continuous but has slight localisation errors (See Table 1 and Figures 4 d,e). To gauge the accuracy of edge detectors the performance can be assessed as a combination of both edge accuracy and edge connectivity.

Here connectivity is defined as a function of the total connected points along the "ridge" of the edge without any sharp breaks. The overall connected edge value for the assessed image can be defined as a function of the total non-zero pixels (i.e the detected edge points) within the image. This technique is initially based on the work of (Zhu, 1995). Unlike Zhu, who in his work defines all the possible binary edge patterns to assess edge connectivity, this work determines the direction, and location of edges using a novel angle finding technique, which is described in the next section. Furthermore, this connectivity measure is again for grey-level images so binary pixels patterns would be of no use in this evaluation.

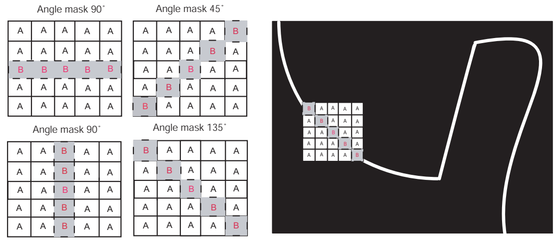



Figure 2: To assess edge connectivity the direction of the edge must first be determined. To find the edge direction, edge angle masks are applied to the image, here four edge angle masks are shown (left) but many more can be used. Each mask is applied to the image and the pixel points extracted for region A and B of each mask. The angle of an edge results from the maximum difference in the mean of the two mask regions.

### 3.6 Connectivity Edge Finding

To perform a connectivity measurement edges must first be located and their angle of orientation determined. To determine this edge orientation a set of sliding masks are used to test the location and direction of all non-zero image pixels (see Figure 2). It can be assumed that an ideal edge after detection and non-maximal suppression, should be a single pixel wide ridge within a uniform background area of the image. Using this assumption a mask can be applied to the image which can assess for any difference in the edge pixels and the background pixels. Figure 2 shows four angle masks used to detect the location and the angle of the edges. Each mask is applied to the image and the pixels covered by regions A and B of the mask extracted. All the angle masks are applied to each pixel in the image and the differences in means calculated. Where the mean of the central line in the mask (region B) differs significantly from the rest of the mask (region A), indicates the location of the edge. The angle of the mask will now indicate the edge direction (see Fig 2). This process can be applied to any edge detected image irrespective of the technique used.

### 3.7 Edge Uniformity

It can be assumed that an ideal grey-scale edge will have a uniform intensity within the pixel values running along the ridge of the edge. This will ensure a connected edge follows a uniform distribution and a fragmented or broken edge will have variability in the pixel intensities along the edge ridge.

Once the location of the edge has been determined using the "connectivity edge finding technique" the uniformity of these detected edge points can be assessed to determine the edge connectivity.

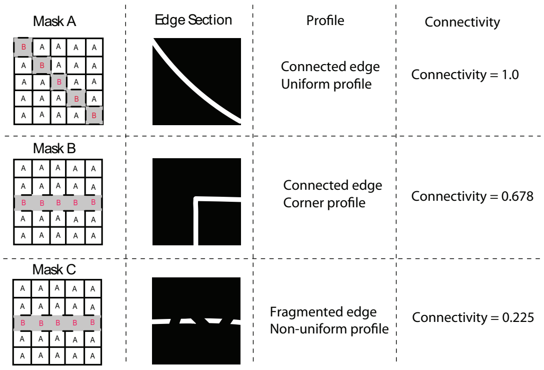



Figure 3: Three ideal connectivity masks located over different edge profiles. Mask A is located on a connected edge, mask B is located over a corner profile edge, and mask C is located over a fragmented edge. The connectivity of an edge is measured along the edge direction using second order differentiation of the co-linear edge points.

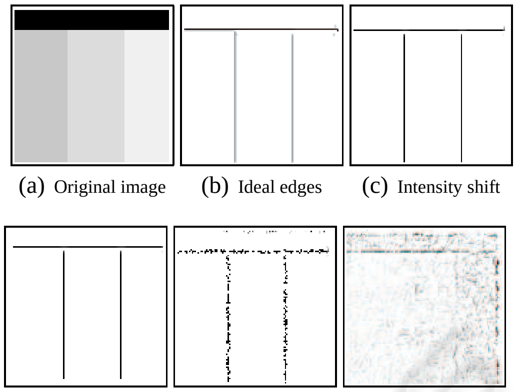
Using the same pixel mask (see Figure 2), the uniformity of the pixel values along the central mask line can be checked using second order differentiation (see Figure 3). The variability of these pixels is computed as the maximal value after second order differentiation, therefore a higher value will indicate greater variability in the points (fragmented edges) and a low value will indicate uniformity (connected edge) (see Figure 3).

The overall connectivity can be computed for every non-zero pixel in the image, with the normalised sum of this variability being the connectivity of the image. In addition the connectivity can also be computed over different pixel mask sizes, therefore allowing the connectivity of an edge to be assessed at different scales (see Table 2).

## 4 RESULTS

Comparing all the evaluation metrics discussed here shows inconsistencies common to both the PCM and CDM metrics. These inconsistencies can be partially overcome by using the GFOM and connectivity metrics.

It becomes important when objectively assessing grey-scale edge detection, to ensure the performance metric used will accurately match the observed edge detection response and reduce or increase accordingly. The images presented in Figure 4 represent four common types of edge detection faults and are used here to initially test the accuracy of the PCM, CDM and GFOM. Each image is tested against the ideal edge response shown in Figure 4b using each performance metric. The error images are therefore: Correctly located edges with an incorrect intensity level (see Figure 4c). Correctly detected edges with a location or shift error (see Figure 4d). Correctly located edges that are broken or fragmented (see Figure 4e). Noisy or spurious over-detected edges (see Figure 4).



(a) Original image (b) Ideal edges (c) Intensity shift

(d) Shifted edges (e) Fragmented (f) Spurious edges

Figure 4: Typical grey-scale edge detection images used to the show the inconsistencies inherent to edge comparison metrics. Each image represents a common output edge detection fault and is compared against the desired ideal image (b) using all the discussed performance metrics (see Table 1 for the performance results).

Table 1: The results of using comparison metrics when to common edge output images (Figure 4 c-f). Each image is tested using the discussed comparison metrics against the ground truth image (Figure 4b). PCM and CDM results are shown over a defined pixel region of PCM0/CDM0=0, PCM1/CDM1=1 and PCM2/CDM2=2. The first line of the table indicates the results of comparing the ideal image against itself with an ideal response expected from all tests. Both the PCM and CDM metrics of edge evaluation can give a falsely high response to extreme over-detection of false edges caused by noise or texture (indicated in Boldface). Connectivity results are scaled between the ideal connected edge =1 and disconnected edges = 0.

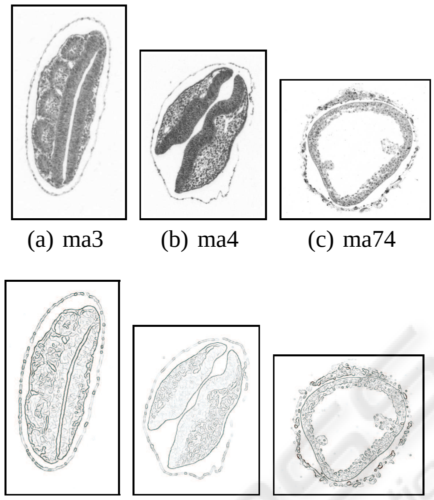
| Evaluation of Edge Comparison Metrics | | | | | | | |
|---|---|---|---|---|---|---|---|
| Image Name | PCM0 | PCM1 | PCM2 | CDM0 | CDM1 | GFOM | Connectivity |
| Ideal | 100 | 100 | 100 | 100 | 100 | 100 | 0.975 |
| Intensity Shift | 38.68 | 37.29 | 36.16 | 38.68 | 38.68 | 38.70 | 0.972 |
| Shifted edges | 33.19 | 33.81 | 34.32 | 33.91 | 33.81 | 22.70 | 0.956 |
| Fragmented | 36.66 | 53.5 | 53.93 | 36.66 | 36.74 | 35.50 | 0.075 |
| Spurious edges | **87.83** | **66.32** | **88.43** | **87.83** | **88.11** | 13.40 | 0.124 |

Table 1 shows how all metrics perform accurately if the edge height is comparable to the desired edge height (see Figure 4b). Moreover if the detected edge intensity is greater than the desired edge intensity, as is common with some ranking statistical edge detectors as in the work of (Svoboda et al., 2006), all performance metrics reduce accordingly. This is not a gauge of any inaccurate edge detection, but represents a true correspondence of the actual grey-scale edge height compared to the output detected edge intensity. If a small pixel shift is now introduced to the results, as shown in Figure 4d then any results should decrease with reference to this shift. Both PCM and CDM allow for small shifts in pixel positions using a user defined pixel mask to check for an optimal match, and as such decrease as expected. GFOM uses the distance transform to also allow for slight edge localisation errors and furthermore the performance is seen to decrease in response to a shift (see Table 1).

If this detected edge becomes broken or fragmented as shown in Figure 4e, then a correct performance metric should reduce accordingly. This is true for both the CDM and GFOM metrics where the observed performance is shown to reduce, however the results clearly illustrate an inconsistency in the PCM metric. With broken or fragmented edges the PCM metric gives a higher performance value than it does for an edge that is continuous (see Table 1). In addition to this, if the edge height in the image is a similar "strength" value to any spurious edges or noise in the image, which can be common when assessing textured images, both the PCM and CDM metrics will assume this to be a correctly detected edge. Therefore with extreme over-detection of edges, as shown in Figure 4f, both the PCM and CDM metrics give a falsely high response (see Table 1). This problem of incorrect performance for this over-detected (or noisy) edge image is not matched by the new GFOM which accurately measures this as the poorest result. However, GFOM is not without its own faults.

GFOM takes every threshold level in the edge detected image and checks the ground truth for an exact match at that same threshold level, therefore it will not tolerate edge intensity changes. The GFOM technique can be adapted to avoid this problem by allowing a ranged threshold comparison between both images, however this would result in GFOM having some of the same inconsistencies as highlighted for both the PCM and CDM metrics. To avoid these problems all three performance metrics can be used in conjunction allowing a consistency in the results unavailable with a single objective performance measure.

The results further showed how all the comparison metrics were found to favour accurately located edges over edges that, although accurate, have slight localisation errors (see Figure 4d). Moreover, a greater edge performance measure could be awarded to an edge that is accurately located but is badly fragmented (see Figure 4e), against a poor response for an edge that is continuous but has slight localisation errors (see Table 1). In this situation it becomes important to determine what is required by the edge detector. If it can detect an accurate edge which is fragmented or not continuous, then is this preferred over an edge



(a) ma3 (b) ma4 (c) ma74

(d) (e) (f)

Figure 5: A sample set of histological test images used in the connectivity measurement, and the results after applying the Canny edge detection with a $\sigma = 2$. Images courtesy of the Edinburgh Mouse Atlas Project (EMAP).

shifted somewhat but is continuous and therefore unbroken?

Many segmentation techniques, including those of (Svoboda and Matula, 2003) and (Bowring et al., 2004), require a continuous contour to accurately segment the objects. In this situation the localisation of edges becomes less important and the connectivity of edges is paramount. Therefore an accurate gauge of the connectivity of the edge detection results can be important as the metric for evaluation.

If we analyse the results for the connectivity of edges (see Figure 3, Table 1 and Table 2) it is clear to see the reduction in the connectivity measure awarded to edges that are fragmented or broken. As previously discussed the connectivity measure can be computed over different sized pixel masks (see Table 2), therefore giving a different measure for each mask applied. A larger connectivity mask will detect more variability in the pixel points and give a greater sensitivity to the results, however this can award sharp corners or curvature in the image a lower connectivity value (see Figure 3). Smaller connectivity masks will allow better performance for curvature and corners but will be less sensitive to the changes (or breaks) in the edge pixels. However, We can see that with a fixed size pixel mask there is a consistent pattern in the connectivity results for a uniform edge (connectivity 1.0), and non-uniform edge (connectivity 0.225) and a corner or shape based edge (connectivity 0.678), see Figure 3.

Table 2: Connectivity results for the Canny edge detector across a selection of test images shown in Figure 5. Each result is computed using a connectivity region mask of 5 (con.5),11 (con.11),15 (con.15) and 21 (con.21) pixels.

| **Connectivity Measurement** | | | | |
|---|---|---|---|---|
| Image | Canny (Con.5) | Canny (Con.11) | Canny (Con.15) | Canny (con.21) |
| ma3 | 0.820 | 0.663 | 0.604 | 0.532 |
| ma4 | 0.823 | 0.673 | 0.618 | 0.553 |
| ma74 | 0.804 | 0.642 | 0.588 | 0.529 |

## 5 CONCLUSIONS

This paper discussed existing methods for evaluation of greyscale edge detection and introduced new methods for edge detector evaluation. Initial results which compared the common objective methods, showed ambiguities in the evaluation. The Pixel Comparison Metric (PCM) and Closest Distance Metric (CDM) both showed inconsistencies when the edge detected height is comparable to noise or false edges in the image. It was further shown that if the detected edge height is the same or a similar value to noise in the image both metrics will give a falsely high performance measure. Moreover, both the PCM and CDM metrics were seen to give a greater response to over-detected edges, than accurately located edges of different grey-levels. This shows a bias in the results towards the location of edges over the accuracy of edges.

The Greyscale Figure of Merit (GFOM) (an adapted form of Pratt's Figure of Merit) was then introduced. The new GFOM measure can overcome some of the inconsistencies of the PCM and CDM metrics therefore allowing a more robust evaluation of grey-scale edge detection against an ideal ground truth. All the comparison metrics were found to favour accurately located edges which were broken of fragmented over edges that, although accurate, have slight localisation errors. In this case a greater edge performance measure could be awarded to an edge that is accurately located but is badly fragmented, against a poor response for an edge that is continuous but has slight localisation errors.

To overcome this problem and aid in the evaluation, a novel edge continuity measure was developed and tested. This measure uses a unique pixel mask applied to the edge image and assess the angle and location of edges. The uniformity of the detected edge pixels is then assessed and the connectivity of the edge defined. This connectivity measure can be used independently or in conjunction with the previously discussed metrics to give a robustness to the results currently unavailable with any single grey-scale performance measure.

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