BUILDING LOCAL K-D TREE FOR FLEXIBLY LABELING ARTICULATED POINT SETS

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Abstract: Optical motion capture system is widely used to acquire human motions by capturing the trajectories of markers that are attached to the body. Identifying the marker trajectories is challenging but indispensable in most of real applications. Conventional methods rely on either labor-intensive manually labeling or auto-labeling with assumption of pose similarity to the topological model. This paper presents a novel method to flexibly label markers from human motion capture sequences. The point sets in a rigid segment defined in the topological model are firstly clustered by using the spectral clustering algorithm. For each rigid segment, a local k-d tree is constructed to robustly match two point sets without pose similarity assumption. To match all rigid bodies with those in topological model for efficiently and correctly labeling, the labeling process is carefully designed using the articulated structure of acquired data. Experiments show that our method outperforms conventional methods in accuracy and is robust when labeling markers in motion capture sequences from different subjects.

1 INTRODUCTION

Marker-based motion capture (MOCAP) system has become one of the most popular methods for acquiring human motions in clinical gait analysis, sports training and computer games(Guerra-Filho, 2005)(Gleicher, 1999).It can reconstruct the motions of moving subjects by measuring the 3D trajectories of passive reflective markers attached to the subjects. To use the recorded data, information such as joint angles, skeletal parameters and the topology of the captured subjects should be extracted. Most commercial tools (e.g. Vicon(OMG, 2009)) provide an additional process called labeling to identify each marker based on the predefined topological model to get the geometric information. Unfortunately, this work is often accomplished manually which is labor-intensive, highly non-productive and error prone. Every time a new subject is to be captured, the manual identification is needed.

Generally speaking, different captured subjects will have different geometric information. Essentially, the labeling work is to generate geometric models for different captured subjects. Currently, most approaches assume that the geometric model has been identified in the first frame of motion sequences as most commercial tools do. Some approaches provide a topological model as a point set template and use the method of Point Pattern Matching (PPM) with articulated sparse feature points to generate geometric models for different captured subjects automatically. The topological model only contains marker set and the topology of the captured subjects without specific geometric information. However, this automatically labeling process requires the captured subjects to perform the same initial pose as the topological model. In situations such as clinical gait analysis for disabled people, it is difficult for them to perform special pose. It is still an open problem to label captured markers automatically using topological model without the requirement of special initial pose.

This paper describes a new method to automatically label points or generate geometric models in acquired motion data, which only requires that the acquired data has the same spatial distribution of points in each rigid body and does not need the identical scale and pose with the topological model. To serve this purpose, we assume non-interrupted marker trajectories can be obtained. To construct a local k-d tree for each rigid body (Here, ”local” means that the k-d tree is built within a local rigid body.), we first cluster these marker into different rigid groups. The labeling process is implemented using constructed local k-d tree articulately to obtain robust labeling results.

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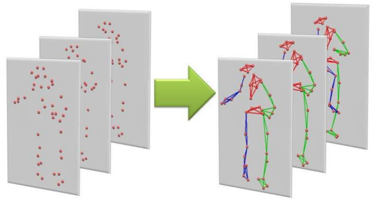


Figure 1: Input and output of our approach. Left are 3D marker trajectories acquired from MOCAP system. Right are the results of marker labeling that are also the geometric models.

Fig. 1 shows the input and output of our approach.

2 RELATED WORK

The task of labeling markers or identifying acquired data based on the topological model can be considered as the problem of Point Pattern Matching (PPM) with articulated sparse feature points.

Many researchers have efforts on the area of PPM(Cox and de Jager, 1992)(Li et al., 2003). Their work mainly focuses on geometric invariant or constrained satisfaction in affine transformation. The methods frequently used include graphics, interpretation trees(Gaede and Gnther, 1998), Hausdaorff distance(Mount et al., 1999), geometric hashing(Wolfson and Rigoutsos, 1997). The human is a high-dimensional nonrigid object that can perform various complicated motion. The geometric invariant or constrained satisfaction employed in these methods can not be easily met during human's performances. As a result, these approaches can hardly adapted for the situation of high-dimensional articulated motion.

Baihua Li et al.(Li et al., 2004)(Li et al., 2008) introduced a segment-based method for PPM and employed it to solve the problem of labeling markers. The used topological model is obtained manually from the same acquired subject at different poses. The labeling process is carried out within each rigid segment. Their method can identify acquired data that is obviously visually different from the topological model. Unfortunately, the computation cost is high, and the topological model for different subjects needs to be re-built manually. Qian Yu et al.(Yu et al., 2007) proposed a method to label markers for multiple interacting articulated targets. They learnt a motion model and a structure model for each target from calibration sequences, and used them to identify markers. Using learned models, their method can label markers for different interacting targets. In order to start the marker tracking, the markers in the first frame of calibration sequences for each target must be correctly labeled by hand. In fact, this manually process is establishing a correspondence from the topological model to the acquired motion data.

To label different subjects with a generic topological model that has similar topology to the acquired subjects, Baihua Li et al.(Li et al., 2005) designed a new similarity k-d tree and used this data structure to identify markers in only one frame. The method can identify acquired data that have non-identical scales with the topological model. It can be processed very fast, but the method requires that the pose of acquired data and topological model must be similar. Bentley(Bentley, 1975) first introduced the binary k-d tree in 1975, in which he used axis-orthogonal cutting hyper-planes through data points to partition recursively a point-set at each interior node into two subsets. Then Bentley(Bentley, 1990) introduced an optimized semi-dynamic k-d tree taking the data distribution into account, to construct this kind of k-d tree, one should first find out the dimension of the data that has the largest spread to determine the orthogonal hyper-plane axis, and then the mean-partition of data extension in that dimension is calculated to locate the hyper-plane.

Other researchers tried to extract the information such as skeletal parameters, joint angles and the topology from marker trajectories without the step of labeling. Adam G.Kirk et al.(Kirk et al., 2005) used spectral clustering algorithm(Ng et al., 2001) to identify rigid bodies from motion capture sequences and estimated the skeletal parameters. Edilson de Aguiar et al.(Aguiar et al., 2006) adopted a method that was very similar to that was proposed by Adam G.Kirk et al.(Kirk et al., 2005) to automatically extract the articulated skeletons from 3D marker trajectories. Alexander Hornung et al.(Hornung et al., 2005) also proposed a method to extracted articulated skeletons from motion sequences. They introduced a self-calibrating process to get the topology of the captured objects. The methods mentioned above can estimate skeletal parameters and topology without identifying each marker which is convenient for computer animations.

However, to get precise locations of joints in clinical analysis, the experience formulas provided by biomechanics are always used and the step of labeling is always required so these methods can not be easily applied to our problem. Inspired from their method of identifying rigid bodies, we also apply the spectral clustering algorithm(Ng et al., 2001) to get marker groups representing different rigid bodies and the details will be given in Section 3.1.

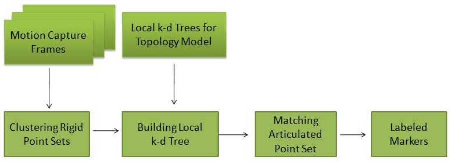


Figure 2: Pipeline overview of our algorithm.

3 OUR APPROACH

The topological model used in our method is an articulated structure composed of rigid bodies, in which the neighbor rigid bodies are linked with the shared markers. In order to label the acquired motion data with a generic topological model, we use the intrinsic rigid segment constraint of 3D marker trajectories to cluster the markers into rigid body sets. Inspired by the fact that k-d tree is a state-of-the-art method for matching point pattern with similar distributions, we build local k-d tree structure for each marker set in the topological model and the acquired data. Since the obtained rigid segment has the property of geometric invariance during affine transformation, so the proposed method can also label markers in the case of distinct pose difference.

Given the 3D marker trajectories acquired from commercial optical MOCAP system, our approach can give name for each trajectory without manual effort. There are three steps to accomplish this task.

The first step is to divide markers into different groups. Each one of them represents a rigid body part. We call this process Clustering Rigid Point Sets(See Section 3.1).

Give a list of body segments and their associated markers, we can build a local k-d tree according to the topological model for each marker group(See Section 3.2). The topological model has similar topology but non-identical scales to acquired data and a local k-d tree is also constructed for each rigid body in the topological model. Our labeling process is based on these local k-d trees to build a correspondence between the topological model to acquired data.

Finally, in order to match each rigid body with the one in topological model, we make the labeling process articulately execute for each marker group based on the topology provided by the topological model. We call this process matching articulated point sets(See Section 3.3). Fig. 2 illustrates the pipeline of our approach.

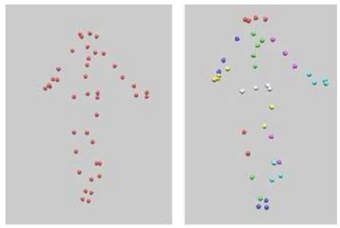


Figure 3: Rigid body clustering. Left shows the input points. Right is the result of rigid body clustering. Different color groups represent different rigid bodies.

3.1 Clustering Rigid Point Sets

In this section, we illustrate our clustering procedure, which is used to identify individual rigid bodies from the marker's 3D trajectories.

In a rigid body, the distance between any two markers keeps almost constant over time. However it varies if the markers belong to different parts. To choose markers of each rigid body, we employ spectral clustering algorithm(Ng et al., 2001) on the standard deviation of the mutual marker distances over time. To avoid manually effort we get the affinity matrix A as(Zelnik-manor and Perona, 2004) did:

$$A_{i,j} = \begin{cases} exp(-d_{i,j}^2/(2\delta_i\delta_j)), & i \neq j \\ 0, & i = j \end{cases} \quad (1)$$

where $d_{i,j}$ is the standard deviation of distance between marker i and j $\delta_i = d_{k,i}$ is the K-th neighbor of marker i in the standard deviation space. In our experiments, we use a single value of $K = 5$ because the number of markers in each group is no more than 5 and it gives good results.

We apply spectral clustering algorithm(Ng et al., 2001) hierarchically to get robust clustering result based on the topological model. First, the clustering algorithm will divide the point set into two clusters, C_1 and C_2, which represent upper and lower part of human respectively. Then the spectral clustering is employed again within each cluster to get proper marker groups that can be used to represent rigid bodies. Fig. 3 demonstrates that our approach can robustly identify all individual segments of the human body.

Because the common point shared with two segments can be clustered into any of the two segments, compared with the topological model there will be some segments having one point *lost*. This *lost* point must be added into proper segment while labeling. To solve this problem we design an articulated point set matching procedure that will be discussed in detail later.

3.2 Building Local K-D Tree

In this section, we give introduction on how to build local k-d trees for the point sets in topological model and acquired one. As there is no identical scales between topological model and acquired data, the labeling work can not be accomplished by solving the problem of absolute orientation(Arun et al., 1987) so we make use of local k-d trees to label markers in different rigid bodies.

The local k-d tree is built for each rigid body, which is identified by the procedure introduced in Section 3.1. Since single rigid will hold geometric invariance during affine transformation, constructing local k-d tree for each rigid body and labeling their associated points can be applied to acquired data that are significantly different from topological model.

As the topological model point set $T = \{t_i \in R^3\}_{i=1}^{M}$ and the acquired point set $O = \{o_i \in R^3\}_{i=1}^{M}$ are usually obtained in distinct coordinate systems and performing different pose, they need to be aligned to a consistent coordinate system by centering and rotation before constructing local k-d tree. This process can change two point set with different statues into a same pose under a consistent coordinate. Firstly, the centroids of T and Q is estimated as in Equ. 2. Second, the orientation vectors can be calculated from the weighted second distribution moments as Equ. 3 and Equ. 4 in(Li et al., 2005)

$$\vec{c_T} = \frac{\sum_{i=1}^{M} t_i}{M}, \vec{c_O} = \frac{\sum_{i=1}^{M} o_i}{M} \tag{2}$$

$$\overrightarrow{CO_T} = \frac{1}{M}\sum_i (t_i - \vec{c_T})|t_i - \vec{c_T}| \tag{3}$$

$$\overrightarrow{CO_O} = \frac{1}{M}\sum_i (o_i - \vec{c_O})|o_i - \vec{c_O}| \tag{4}$$

where $t_i \in T, o_i \in O$ and M is the number of points in the point set.

Finally, each point in T and O should be transformed with respect to their centroid and the orientation vectors by suitable translation vector t and rotation matrix R as in Equ. 5.

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = R(\begin{bmatrix} x \\ y \\ z \end{bmatrix} + t) \tag{5}$$

where x, y, z denotes the original coordinates of each point in the point set and x', y', z' represents the aligned coordinates.

For the topology model, the rotation matrix R can be set as identity matrix and the translation vector t is the negative centroid vector of $\overrightarrow{CO_T}$. For the acquired point set, R represents the rotation matrix of rotating $\overrightarrow{CO_O}$ into $\overrightarrow{CO_T}$ and t is also the negative centroid vector of $\overrightarrow{CO_O}$. In the former work of (Li et al., 2005), for the acquired data the R was defined as a rotation matrix only around the z-axis because they assumed that human objects were standing straight, parallelling to the vertical z-axis and there are no rotations around other two axis. To satisfy this assumption, the captured objects must perform the same pose with topological model and they can only change their orientations around the z-axis. As we build k-d tree for each rigid body rather than the whole body, the issue caused by pose difference can be handled by labeling each local rigid segment. Because the rigid bodies can maintain geometric invariance during affine transformation, we can define rotation matrix R for acquired data around each axis to align local different rigid bodies. This operation makes our method can apply to acquired data, which has visually different pose with the topological model.

Now, for a given aligned 3-dimension topological model points set $\tilde{T} = \{\tilde{t}_i \in R^3\}$, we can build local k-d tree for it. Firstly, we sort the points respectively along x-axis, y-axis and z-axis. Then the orthogonal-axis OA_{φ^*} will be determined as Equ. 6.

$$OA_{\varphi^*} = \frac{1}{2}\max_{\varphi\in\{x,y,z\}} \Delta\varphi \tag{6}$$

where $\Delta\varphi = \max_{\tilde{t}_{k_i}\in\tilde{T}}((\tilde{t}_{k+1})_\varphi - (\tilde{t}_k)_\varphi | (\tilde{t}_{k_{i+1}})_\varphi \geq (\tilde{t}_{k_i})_\varphi), (i = 1, \ldots, M)$ is the maximum coordinate interval in the direction φ,and i is a sorting index.

Next, we divide the point set into left subset P_l and right one P_r. The points in the left subset are smaller than OA_{φ^*} in axis φ^* while the right are bigger. Then an interior node containing the orthogonal-axis(e.g. x,y or z) and the number of points n_l split to the left tree will be constructed. In the subset P_l and P_r, the procedures of choosing orthogonal-axis and splitting point set based on the axis will be implemented recursively until they contain only one point. Then a leaf node storing this point is built.

Finally, for the aligned acquired points set, we use the information storing in each interior node of its corresponding local k-d tree in the topological model to construct the acquired one. Starting from the root node, we first split the aligned acquired point set based on the hyper-plane orthogonal-axis contained in its corresponding local k-d tree's root node in the topological model. Then the n_l smallest points along this axis in the acquired point set are stored in the left child node, and the rest are stored in the right child node. In this way, the acquired point set will be split into two parts at each interior node. In each subset, this split procedure will be implemented recursively until the leaf node contains only one point. Up to now, we have built a local k-d tree for acquired point set that has the same structure with its topological model.

3.3 Matching Articulated Point Sets

Having built local k-d tree for each rigid body of acquired data, the left-right traversal of the successive leaf nodes in the two trees for point set in topological model and acquired one serves to define the corresponding point-pair match. The label of each topological model point is assigned to its matching point in the acquired data.

However, before labeling and building the trees, there still exists a problem to be solved. Although we have divided points into different groups and each one of them represents the rigid body of human, we still cannot tell the difference between these groups. For example, which group represents the points attached to the waist? Which one corresponds with head? More importantly, the segments in the left part and the right part can not be distinguished and this left-right ambiguity will result in wrongly labeled segments. A direct method maybe enumerating each rigid body and found the right one. Employing this violent method, the computation cost could be rather high and the difference between left and right part of acquired objects can not be told. Also the *lost* point mentioned in Section 3.1 must be added into the proper segment. To solve this problem, we make the process of labeling carry out articulately based on the topology provided by the topological model.

We begin to label markers from groups that are classified as the lower part of human body. On general, the points attached to the waist can be considered belong to either lower part or upper part, but the experiments carried out by us suggest that if the captured object exercises his joint in each degree of freedom the points on the waist can be divided into upper part. Then we can find out one of the group within the upper part that has minimal standard deviation between the points pertained to the lower part and consider them as waist. Then we can build a local k-d tree for this rigid using the method mentioned in Section 3.2 and labeling its associated points by the left-right traversal of the successive leaf nodes in this local k-d tree and its corresponding one in topological model.

As the waist rigid has some common points with other groups, we can use these points combined with spectral clustering algorithm(Ng et al., 2001) to label other groups. Here we take the left thigh as an example. As shown in Fig. 4, left thigh rigid and waist rigid share the same point named ”LFWT”. This point will

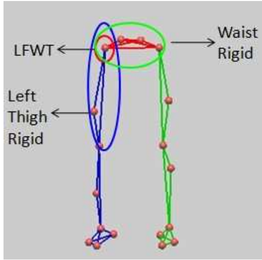


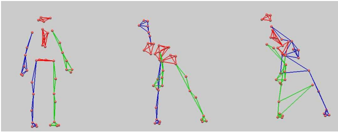
Figure 4: Example of same points. The blue ellipse represents the left thigh rigid and the green one is the waist. The point that is surrounded by a red ellipse is the common point named ”LFWT”.

be clustered into the rigid of waist because its standard deviations between other points attached to waist rigid are closer to zero than the ones belonged to left thigh rigid, which is the example of *lost* point mentioned in Section 3.1. After labeling the waist group we can identify which point being ”LFWT”. Then we add this point into the lower part group and use spectral clustering algorithm(Ng et al., 2001) to identify individual rigid bodies of the lower part. It is easy to infer that the group containing point ”LFWT” is the left thigh rigid and the matching process can be executed. Next, the points in the left calf rigid can be labeled in the same way. This procedure will be carried out articulately until all the points in each rigid body have been labeled.

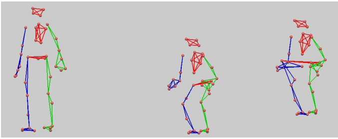
4 EXPERIMENTAL RESULTS

We tested our algorithm on motion capture sequences from the CMU motion capture database(CMU, 2009), in which 3D marker trajectories were acquired via Vicon system(OMG, 2009). The motion sequences we used for testing are comprised of 200-1500 frames and performing, for example, walking, simply exercising his joints, jumping. The frame rate is of 120 frames/sec. Each marker has a non-interrupted trajectory during the whole motion. The topological model used in our experiments is taken from a subject that have different scale and pose with the captured data. It has 41 markers, and the marker set within each rigid body of this topological model has the same spatial distribution with acquired data.

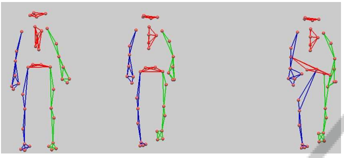
For comparison, we use some frames from each motion sequence and label them by our method and the method in(Li et al., 2005). Fig. 5 shows labeling results when these two methods applied to the same sequences of exercising, jumping and walking. As the acquired data are quite different from the topological model, our method performs much better than



a)



b)

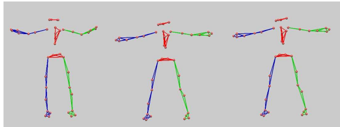


c)

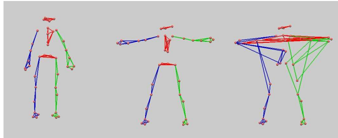
Figure 5: Labeling results for different motion sequence. a) Shows the labeling results of exercising sequences. b) Illustrates the labeling results for jumping sequences. c) Shows the labeling results of walking sequences. The topological model used in the process of labeling is shown in the left. The middle is the labeling result obtained by our methods. The labeling result employed the method proposed in(Li et al., 2005) can be found in the right.

the method proposed in(Li et al., 2005). We also label the acquired data which has different levels of differences from the topological model using these two methods. The results are given in Fig. 6. From the results, we can see that even slightly pose dissimilarity can lead wrong identification using the method in(Li et al., 2005), but our method works fairly well. The comparison suggests that our approach is well-suited for automatically labeling markers from 3D marker trajectories without the requirement of pose similarity.

To evaluate the accuracy of the method proposed in this paper, we also applied our method and the method proposed in(Li et al., 2005) to each frame of different kinds of motion sequences and the results are shown in Table 1. From the table, we can see that our approach performs much better than the method mentioned in(Li et al., 2005) for all the tested motions. Using the method proposed by us, all motion sequences are correctly labeled while the method in(Li et al., 2005) can only label some or even no frame. Experiment results show that the method proposed in(Li et al., 2005) is very sensitive to pose similarity while our method can work correctly even with visually obviously different pose.



a)



b)

Figure 6: Different topological model and labeling results. a) Shows the labeling results using the topological model that are almost the same as the acquired data. b) Shows the labeling results using the acquired data that are different from the topological model with the upper body. The topological models, labeling results obtained by our method and the results produced by the method of(Li et al., 2005) are shown from left to right.

Table 1: Labeling results on different motion sequences. BHL represents the method proposed by Baihua Li(Li et al., 2005).

Subject ID	Motion	Frames	Labeled Frames	
			Ours	BHL
14_20	Exercise	1500	1500	0
06_01	Walking	493	493	0
16_03	High Jumping	409	409	226
16_05	Long Jumping	294	294	116
14_02	Boxing	1000	1000	0
16_44	Running	215	215	0

5 CONCLUSIONS

We have proposed a fully-automatic method for labeling markers from their 3D trajectories using local k-d trees in this paper. Our approach can work properly even when the captured subject is different from topological model in pose and scale. The experiment results show that our approach performs better than the most closely related methods.

The method proposed in this paper is also very suitable for the application of clinical gait analysis for patients. First, the special pose that may be impractical for disabled people performing is not required during capturing. Second, it is convenient by removing the step of motion calibration because patients especially disabled patients always have difficult in performing motion calibration. Employing the method proposed here, the only requirement for patients is walking.

Our labeling method is based on the clustered rigid bodies, but some rigid bodies are not strictly rigid, for example, the torso of human, especially when the captured subjects perform vigorous exercises like bending their body too low. In this situation, our method may wrongly labeled markers attached to these lax rigid bodies. However, the wrongly labeled markers can be corrected using the constraint of trajectories' smoothness or solving the problem of absolute orientation(Arun et al., 1987).

Our approach relies on the assumption that each marker trajectory must be non-interrupted during the whole motion. To accomplish the clustering task, we also have to ask captured objects to exercise his joint through the full range of motion. Although these limitations are a little strict, but they can be satisfied in practice to ask the captured subject to perform calibration motion in the middle of capturing area. This requirement can effectively reduce the number of invisible markers and obtain almost non-interrupted marker trajectories. For the task of clinical gait analysis, we should let the number of invisible markers be as less as possible. In future work, we plan to reduce the limitations mentioned above in order to label markers with noisy marker trajectories.

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