CHAOTIC ANALYSIS OF METAL CUTTING WITH NONLINEAR SUSPENSION

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Abstract: This study performs a systematic analysis of dynamic behavior of cutting process of machine tool with unbalance force induced from mass eccentricity of work-piece, nonlinear cutting force and nonlinear suspension effect. Phase diagrams, power spectra, bifurcation diagrams and Poincaré section are applied to identify the dynamic motions in this study. The simulation results show that the non-periodic dynamic responses are very abundant in cutting process of machine tool. The results presented in this study provide an understanding of the operating conditions under which undesirable dynamic motion takes place in this kind of system and therefore serve as a useful source of reference for engineers in designing and controlling such systems.

1 INTRODUCTION

Since the mechanisms among tool, workpiece and chip are complicated in the cutting process, the analytical difficulties are increased in studying related academic researches and the complete studying is also hard to achieve. There are many significant and dramatic investigations are performed before. The dynamics analysis of cutting based on the model of Hastings, Oxley and Stevenson was the most popular model for many studies and many studies are based on the mathematical model proposed by them (Hastings et al., 1971). Grabec presented a series of papers discussing chaotic dynamic responses occurring in cutting machines and also found some mechanisms of chaos in the cutting process (Grabec, 1988). Altintas, Eynian and Onozuka investigated the influence of vibrations on the cutting forces (Altintas et al., 2008). Powalka, Pajor and Berczynski presented a special experiment used for cutting force identification to eliminate the regenerative phenomenon and also to improve the accuracy (Powalka et al., 2009). Hamed, Firooz, Mohammad and Mohammad proposed a single degree of freedom dynamic system including quadratic and cubic structural nonlinearities and found abundant nonlinear behaviors (Hamed et al., 2010). Therefore, we would know that cutting process is a highly nonlinear phenomenon and the linearization or simplification of analyzing cutting process may cause some simulation errors.

The related literatures are very comprehensive, some assumptions or linearization are performed in order to simplify the simulation model and economize simulation time. The assumptions or linearization may lead some dramatic errors comparing with real state. In this study, we consider the nonlinear dynamic responses in cutting process of machine tool with nonlinear suspension effect and also take the nonlinear cutting force into consideration. The nonlinear dynamic equations are solved using the fourth order Runge-Kutta method. The dynamic trajectories, power spectrum, Poincaré maps and bifurcation diagrams are applied to analyze dynamic motions.

2 MATHEMATICAL MODELING

Fig. 1 represents the model of metal cutting under nonlinear suspension. K_{1x} and K_{2x} are the first and second equivalent stiffness coefficients in the vertical direction; K_{1y} and K_{2y} are the first and second equivalent stiffness coefficients in the horizontal direction; C_x and C_y are the damping

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coefficients of the supported structure in the vertical and horizontal directions respectively; F_x and F_y are the components of external excited cutting forces; F_y is the cutting force dependence on the cutting speed and chip thickness; F_x is the thrust force which is related to the main cutting force through a related frictional coefficient μ (F_x=μF_y). The nonlinear parts of dynamic equations include nonlinear suspension term (hard spring case) and the nonlinear cutting and thrust force term.

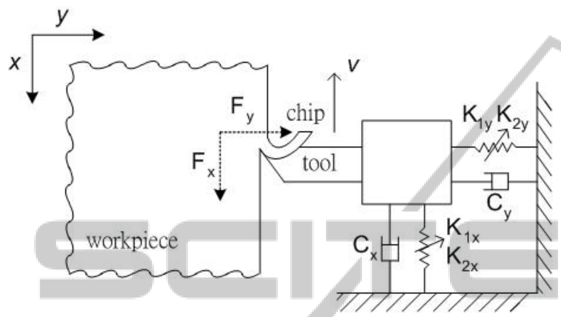

Figure 1: Model of metal cutting under nonlinear suspension.

$$M\ddot{x}+C\dot{x}+K_1x+K_2x^3=F_x \qquad (1)$$

$$M\ddot{y}+C\dot{y}+K_1y+K_2y^3=F_y \qquad (2)$$

Let $X= x/h_0$; $Y= y/h_0$; $\tau=\omega t$; $\frac{d}{dt}=\omega\frac{d}{d\tau}$; (•) denotes d/dt and (′) denotes d/dτ.

$$X''+\frac{2\xi}{s}X'+\frac{1}{s^2}X+\frac{\alpha}{s^2}X^3=\frac{F_x}{p} \qquad (3)$$

$$Y''+\frac{2\xi}{s}Y'+\frac{1}{s^2}Y+\frac{\alpha}{s^2}Y^3=\frac{F_y}{p} \qquad (4)$$

where $\xi=\frac{C}{2\sqrt{K_1M}}$, $p=Mh_0\omega^2$, $s^2=\frac{\omega^2}{\omega_n{}^2}$, $\alpha=\frac{K_2}{K_1}h_0{}^2$, $F_y=\mu F_x$, $F_x=q_0h[C_1(|V_r|-1)^2+1]H(h)$, $\mu=[C_2(v_f-1)^2+1][C_3(h-1)^2+1]H(F_x)\mathrm{sgn}(V_f)$ [2-3], $V_r=V_0-X'$, $V_f=V_0-RY'$, $h=h_0-Y$, and $R=R_0[C_4(V_r-1)^2+1]$. $H(F_x)$ may be approximated as $\frac{1}{2}[1+\tanh(\frac{F_x}{\varepsilon})]$, $H(h)$ may be approximated as $\frac{1}{2}[1+\tanh(\frac{h}{\varepsilon})]$ and $\mathrm{sgn}(V_f)$ may be approximated as $\tanh(\frac{v_f}{\varepsilon})$.

The fourth order Runge-Kutta method is applied to carry out the numerical analysis. These numerical data are then used to generate the dynamic trajectories, power spectrum, Poincaré maps and bifurcation diagrams.

3 NUMERICAL RESULTS AND DISCUSSIONS

In the present study, the nonlinear dynamics of the cutting system shown in Figure 1 are analyzed using Poincaré maps, bifurcation diagrams, the Lyapunov exponent and the fractal dimension.

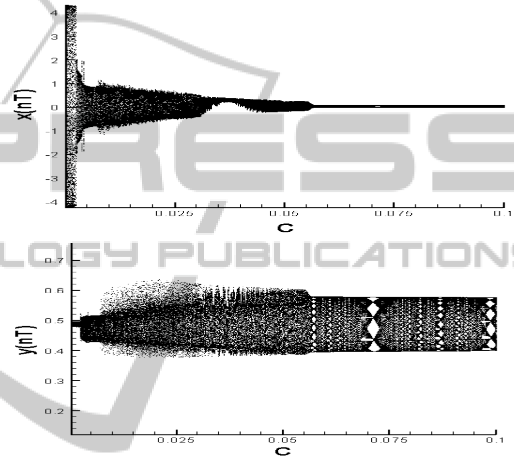

Figure 2: Bifurcation diagrams for geometric center of cutting system using dimensionless damping ratio, ξ , as bifurcation parameter.

The nonlinear dynamic equations presented in Eqs. (3) to (4) for the cutting system with nonlinear suspension effects and strongly nonlinear cutting force were solved using the fourth order Runge-Kutta method. The time step in the iterative solution procedure was assigned a value of *π/300* and the termination criterion was specified as an error tolerance of less than *0.0001*.

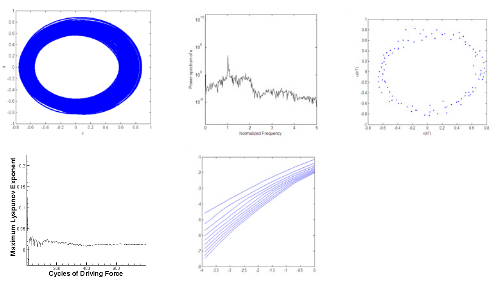

Figure 3: Simulation results obtained for cutting system with ξ =0.015 (x).

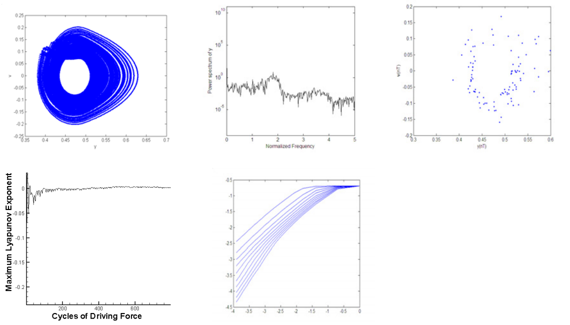

Figure 4: Simulation results obtained for cutting system with ξ=0.015 (*y*).

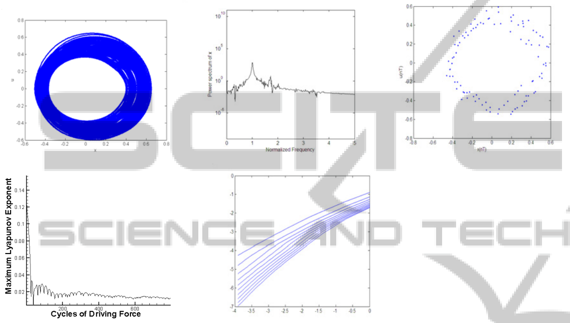

Figure 5: Simulation results obtained for cutting system with ξ=0.025 (*x*).

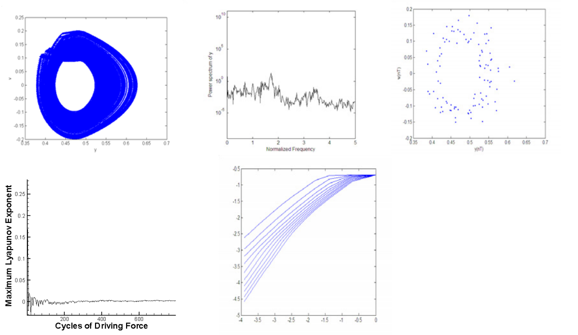

Figure 6: Simulation results obtained for cutting system with ξ=0.025 (*y*).

In practical cutting systems, dimensionless damping coefficient ξ is commonly used as a control parameter. Accordingly, the dynamic behavior of the current cutting system was examined using the dimensionless damping coefficient ξ as a bifurcation control parameter. Figure 2 presents the bifurcation diagrams for the cutting system displacement against the dimensionless damping coefficient ξ. The bifurcation diagrams show that the geometric center of cutting system performs non-synchronous motions in horizontal and vertical directions. The strongly non-periodic or even chaotic motions occurring at lower dimensionless damping coefficient and convergent its non-periodic dynamic responses to be periodic motions and the vibration amplitude also decreased at higher values in the horizontal direction, i.e. $\xi > 0.0575$. The above simulation result is seemed to be satisfied natural phenomenon. Though in the meantime, the dynamic responses of the cutting system in the vertical direction behave strongly different comparing with horizontal cases. As damping coefficient increases, the dynamic motions still perform non-periodic response and even for higher values ($\xi > 0.0575$). Thus we found very interesting non-synchronous motions in vertical and horizontal directions especially at higher damping coefficients. As we know, we may think the cutting system or other vibrating machine system would become steady at higher damping coefficient but actually the suspension of this system is highly nonlinear (Naturally or technically speaking, the suspension of those machine systems should be nonlinear case). Thus we may not seem they to be synchronous behaviors in the vertical and horizontal directions of the cutting systems and it may provide some interesting or considerable information to analyze or control these kind of systems. Figures 3 to 6 are the phase diagrams, power spectra, Poincaré Map, Lyapunov exponent and fractal dimension of the cutting system found chaotic motions at $\xi = 0.015$ *and* 0.025 in vertical and horizontal directions. It also shows that the dynamic responses are synchronous in vertical and horizontal directions from observing simulation results, firstly. Secondly, Phase diagrams show disordered dynamic behaviors; power spectra reveal numerous excitation frequencies; the return points in the Poincaré maps form geometrically fractal structures; the maximum Lyapunov exponent is positive; the fractal dimensions are found to be non-integer. Thus we may conclude that the dynamic motions perform chaotic motions at the above control parameters with the simulation results are corresponding with one another. The dimensionless rotating speed s is also an important control parameters to analyze dynamic responses of rotating machines. Figure 7 present the bifurcation diagrams for the dimensionless displacement in the vertical and horizontal direction of the cutting system using the dimensionless rotating speed s as a bifurcation parameter. It can be observed that the cutting system behaves periodic motions at low rotating speeds and exhibits non-periodic or even chaotic motions at high values of the dimensionless rotating speed. Besides, we also found that they are synchronous in the vertical and horizontal directions.

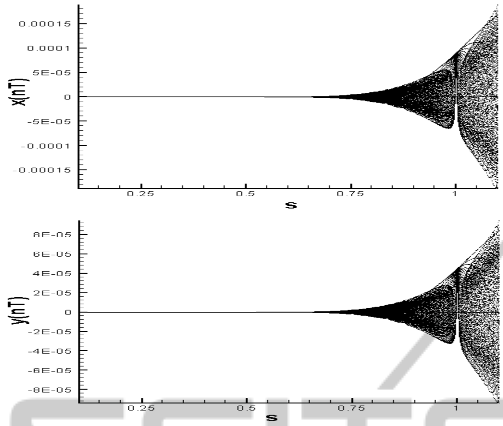

Figure 7: Bifurcation diagrams for geometric center of cutting system using dimensionless rotating speed, *s*, as bifurcation parameter.

4 CONCLUSIONS

This work shows that chaotic behavior exists in the cutting system with nonlinear suspension and nonlinear cutting force. Some interesting and useful simulation results are also found in this study. Specially, we found that dynamic responses behave non-synchronous in the vertical and horizontal directions with the increasing of the value of dimensionless damping coefficient. It is well known that if a nonlinear dynamic system behavior is chaotic, the resulting broad band vibration with comparatively large vibrational amplitude will enhance the probability of fatigue failure. In order to increase the working life of cutting system or enhance the performance of cutting system, it is important not to operate the whole system at chaotic motions. Therefore, this study may aid the theoretical understanding of nonlinear systems of cutting machine tool and escape the undesired dynamic responses for machining.

REFERENCE

Altintas, Y., Eynian, M. and Onozuka, H. (2008). Identification of dynamic cutting force coefficients and chatter stability with process damping. *Annals of CIRP*, 57, 371-374.

Grabec, I. (1988). Chaotic dynamics of the cutting process. International *Journal of Machine Tools and Manufacture,* 28, 19-32.

Grabec, I. (1988). Explanation of random vibrations in cutting on grounds of deterministic chaos. *Robotics and Computer-Integrated Manufacturing*, 4, 129-134.

Hastings, W. F., Oxley, P. L. B. and Stevenson, M. G. (1971) *Proceedings of the 12th International Machine Design and Research Conference, Manchester*, (MacMillan).

Hamed, M., Firooz, B. N., Mohammad, R. M. and Mohammad, T. A. (2010). Nonlinear behaviour of regenerative chatter in turning process with a worn tool: Forced oscillation and stability analysis. *Mechanism and Machine Theory*, 45, 1050-1066.

Powalka, B., Pajor, M. and Berczynski, S. (2009). Identification of nonlinear cutting process model in turning. *Advances in Manufacturing Science and Technology*, 33(3), 17-25.