

# Distributed Control of Dangerous Goods Flows

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**Keywords:** Dangerous Good Transport, Optimisation Problem, Decentralised Control, Linear Quadratic Regulator.

**Abstract:** A risk-based approach to managing dangerous goods (DG) transport flows by road is proposed, solving a real-time flow assignment problem. The model assumes the planned scheduling of the fleets and the medium planned speed for vehicles known a priori. The objective is to plan in the vehicle tour schedules in base on DG and general vehicle flows data on the infrastructures acquired in real time. The model minimises both the total risk on the road network and the gap between the real delivery times with respect to the planned ones. The first objective is a social intent of a National Authority and the second one represents the main important cost minimisation for DG carriers. The proposed model is formulated according to an original distributed control approach, based on the decomposition of the original centralised linear quadratic problem.

## 1 INTRODUCTION

Currently, the main important Dangerous Goods (DG) transportation companies use Intelligent Transport Systems (ITS) to implement DG information systems in order to monitor and manage their fleet during the tours (Benza et al., 2010)

From a legislative viewpoint, recently, the European Commission emanated directives to impose to the DG transportation companies the adoption of new ITS aiming to improve safety and security on road infrastructure. The Directive 2010/40/EU of the European Parliament, on the framework for the deployment of Intelligent Transport Systems in the field of road transport and for interfaces with other modes of transport, has entered into force on August 28, 2010. The EU Commission has recognised that ITS would significantly help traffic management and enable various users to be better informed and make safer, more coordinated and "smarter" use of transport networks. Besides, it asserts also that ITS should integrate telecommunications, electronics, and information technologies with transport engineering in order to plan, design, operate, maintain and manage transport systems.

In this paper a risk-based approach to managing DG transport flows by road is proposed, solving a real-time flow assignment problem. The model assumes that the planned scheduling of deliveries

and the average planned speed for vehicles are known a priori. The objective is to plan the vehicle tour schedules depending on DG and general vehicle flows data on the infrastructures, acquired in real time. The innovative aspect of the proposed approach is to balance two different objectives which usually are referred to different subjects involved in DG transportation: the model minimises both the total risk on the road network and the gap between the real delivery times with respect to the planned ones. The first objective is a social intent of a National Authority and the second one represents the most important cost for DG carriers.

A similar approach has already been presented by Roncoli et al. (2012) assuming that a central DM takes his decisions minimising both the risk due to eventual accidents, and the cost due to delay in deliveries. In this paper, a model with similar targets is presented, however assuming a set of decentralised DMs, allowing a significant reduction of the information exchanged in the network, which is limited to neighbouring nodes only.

This paper introduces a model, formulated following a game theory framework, presenting the mathematical formulation, and a functional approach to solve it. Moreover, a case study is presented, illustrating the feasibility and the effectiveness of the solution.

## 2 MODEL FORMULATION

### 2.1 Mathematical Description

The model is formulated considering a logistic network represented by a graph  $G$  made up of nodes (set  $N$ ) and directed links (set  $M$ ). Each node, that represents a region involved in DG shipment, is considered as an autonomous DM, and it may be an origin node, a destination node, or simply a transition node. Links does not have a physical meaning, but simply represent the movement of product from a node to another. The topology of the graph is described by two subsets, defined for each node:

$E \subseteq M$  including the links entering the node;

$L \subseteq M$  including the links leaving the node.

The model is defined in a discrete time domain, considering a time window  $k = 1, \dots, K$ .

The following variables associated to each node are defined:

- $d_n$  dimension of the node;
- $i^d(k)$  state variables related to the DG mass quantity present at the node at instant  $k$  directed to destination  $d$  ;
- $r(k)$  input variable related to a time-dependent value of risk;
- $\hat{i}^d(k)$  input variable related to the planned quantity present at the node directed to destination  $d$  at time  $k$ .

The variables  $r(k)$  and  $\hat{i}^d(k)$  are assumed to be positive only at the origin and at the destination nodes, being zero at transition nodes.

Links are characterised by:

- $\hat{v}_m^d(k)$  input variable related to the forecasted reference (maximum) speed (expressed as the space covered in a unitary time interval) for the DG transported along link  $m$  directed to destination  $d$  during time interval  $(k, k + 1]$ . These values may represent the speed limit over the link, as well as the speed due to potential traffic congestion during a specific time interval.

The control variables are  $q_m^d(k)$  and  $s_m^d(k)$ , where:

- $q_m^d(k)$  control variables related to the DG flow leaving the node through link  $m$  directed to destination  $d$  during the unitary time interval  $(k, k + 1]$ ;
- $s_m^d(k)$  control variables related to the DG flow

entering in the node through link  $m$  directed to destination  $d$  during the unitary time interval  $(k, k + 1]$ .

Multipliers  $p_m^d(k)$  are also introduced in order to guarantee a global convergence of the problem.

Variables introduced above permit to specify the linear dynamic of the system as:

$$i^d(k + 1) = i^d(k) + \sum_{m \in E} s_m^d(k) - \sum_{m \in L} q_m^d(k) \tag{1}$$

$$k = 0..K - 1, \forall d \in D$$

Where values  $i^d(k)$  are initialised as:

$$i^d(0) = \hat{i}^d(0) \quad \forall d \in D \tag{2}$$

The cost function to be minimised at each node is represented by a weighted sum of quadratic functions:

$$\begin{aligned} \min_{i,q} \alpha & \sum_{k=0}^{K-1} \sum_{d \in D} (i^d(k) - \hat{i}^d(k))^2 \frac{(r(k))^2}{d^2} \\ & + \beta \sum_{d \in D} (i^d(K) - \hat{i}^d(K))^2 \frac{(r(K))^2}{d^2} \\ & + \gamma \sum_{k=0}^{K-1} \sum_{d \in D} \sum_{m \in L} \left( q_m^d(k) - \frac{i^d(k) \hat{v}_m^d(k)}{d} \right)^2 \\ & + \gamma \sum_{k=0}^{K-1} \sum_{d \in D} \sum_{m \in E} \left( s_m^d(k) - \frac{i^d(k) \hat{v}_m^d(k)}{d} \right)^2 \\ & + \delta \sum_{k=0}^{K-1} \sum_{d \in D} \sum_{m \in L} q_m^d(k) p_m^d(k) \\ & - \delta \sum_{k=0}^{K-1} \sum_{d \in D} \sum_{m \in E} s_m^d(k) p_m^d(k) \end{aligned} \tag{3}$$

The equation (3) could be divided in three main parts: the inventory-risk part, the flow-inventory part, and the flow-multipliers part. The first one, weighted by coefficient  $\alpha$  when  $k > K$  and  $\beta$  when  $k = K$ , has a double purpose, depending on the typology of node. In fact, in an intermediate node, according to the definition of  $\hat{i}^d(k)$ , it raises that  $i^d(k) - \hat{i}^d(k) = i^d(k)$ , thus the risk is weighted only by the density of DG product at the node. Making the assumption that the product is to be at origin and destination nodes, it is clear that, in these circumstances, only the difference between the real product and the planned one is to be considered. The second part, weighted by coefficient  $\gamma$ , aims to minimise the gap between the computed speed and

the planned one, according to:

$$q_m^d(k) = \frac{i^d(k)}{d_n} v_m^d(k) \quad (4)$$

$$k = 0..K-1, \forall m \in L, \forall d \in D$$

Equation (4) is valid also for  $s_m^d(k)$ , considering  $m \in E$ . This part of the cost function have also the purpose to avoid possible ‘‘jumps’’ of nodes during the dispatching of DG products. The last part, weighted by coefficient  $\delta$ , represents the core of the decentralised model: the multipliers  $p_m^d(k)$  are adjusted between neighbouring nodes in order to achieve a global convergence.

The solutions of this minimisation problem, performed at each node, when put together may not constitute a feasible schedule since coupling constraints have been relaxed by the multipliers. In fact, the problem that arises is a non-cooperative game among several players, and a further optimisation part has to be added. Introducing the notation  $q_{n,m}^d(k)$  and  $s_{n,m}^d(k)$  for variables  $q_m^d(k)$  and  $s_m^d(k)$  computed at node  $n$ , the problem that must be performed for each link shared by neighbouring nodes is:

$$\max_p p_m^d(k) [q_{t,m}^d(k) - s_{w,m}^d(k)] \quad (5)$$

$$k = 0..K-1, \forall t, w \in N, m \in L_t \cap E_w$$

The multipliers are thus iteratively adjusted based on the degree of constraint violations: hence, the links represent the ‘‘market makers’’, who adjust values  $p_m^d(k)$  taking advantage on the gap between  $q_t^d(k)$  and  $s_w^d(k)$ . Under these assumption, the Nash equilibrium of this game represents therefore the solution of the complete problem (Rantzer, 2009).

## 2.2 Solving Approaches

The minimisation problem at nodes could be rewritten in matrix form, considering  $I(k)$  as the vector of state variables and  $Q(k)$  as the vector of control variables, generating a quadratic cost function:

$$\min_{I, Q} \sum_{k=0}^{K-1} [I'(k)F(k)I(k) + Q'(k)G(k)Q(k) + 2I'(k)R(k)Q(k)] + I'(K)F_K I(K) \quad (6)$$

Subject to a linear state equation:

$$I(k+1) = AI(k) + BQ(k) \quad (7)$$

$$I(0) = \hat{I}(0)$$

This problem is assumed to be a Linear Quadratic Regulator (LQR), and it could be easily

solved finding the closed loop optimal control, given by following equations, as described by Shaiju and Petersen (2008):

$$Q_k^* = K_k I_k \quad (8)$$

$$K_k = -(G + B'P_{k+1}B)^{-1}(R'_k + B'P_{k+1}A)$$

Values for matrices  $P_k$  in (8) are calculated via the Riccati recursion (DARE):

$$P_k = F_k + A'P_{k+1}A - (R'_k + B'P_{k+1}A)'(G + B'P_{k+1}B)^{-1}(R'_k + B'P_{k+1}A) \quad (9)$$

$$P_T = F_T$$

The maximisation problem for links could be handled applying a gradient ascent method: defining an appropriate step  $g$ , the values of  $p_m^d(k)$  are updated at each step according to:

$$p_m^d(k) = p_m^d(k) + g[q_{t,m}^d(k) - s_{w,m}^d(k)] \quad (10)$$

$$k = 0..K-1, \forall t, w \in N, m \in L_t \cap E_w$$

## 3 CASE STUDY

In order to show the effectiveness of the proposed methodology, a case study has been realised considering a network made up of 8 nodes and 10 links, as shown in Figure 1. The time window considered is  $k = 1, \dots, 24$ , simulating an entire day of planning, and thus assuming a time interval of one hour. Planned speeds are set considering that some links could be affected by variation of traffic during the day causing a slowdown, whereas risk values are set assuming that some areas could be more crowded in some time intervals. Moreover, a set of 5 deliveries of 20 units each is planned.

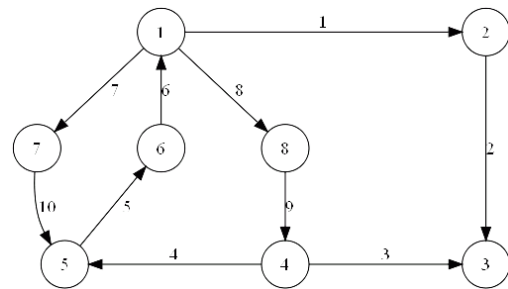


Figure 1: The network considered for the case study.

A calibration process was necessary to determine proper values for coefficients  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  in order to respect problem purposes and to guarantee a convergence of the algorithm. An important aspect of the model is the computation of the speed over

links, which guarantees the physical feasibility of the solution. In fact, if the speed computed by the model is higher than the planned one, the solution may not be applicable to a real network. Figure 2 shows that the speed computed by this model is always lower than the planned one, and moreover their behaviour is very similar.

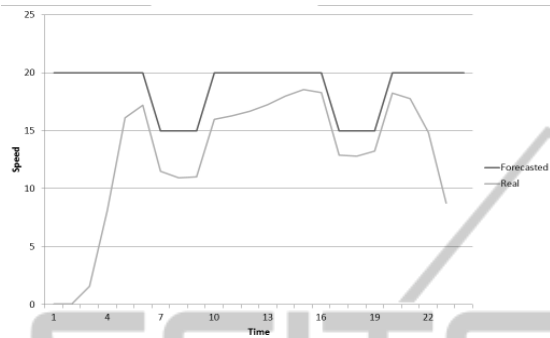


Figure 2: A comparison between the forecasted speed and the real one over a link.

The success in the dispatch is another important aspect to be pointed out. Results show a complete delivery of product at the destinations at least at the last time instant, as illustrated in Figure 3.

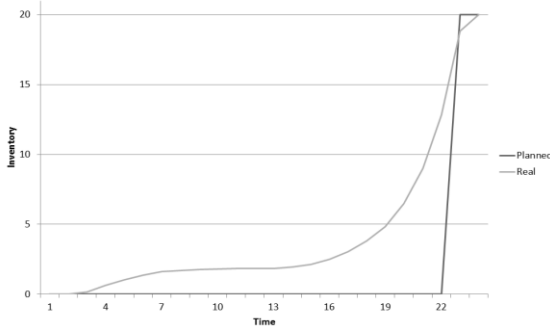


Figure 3: The trend of the planned inventory and the computed one at a destination node.

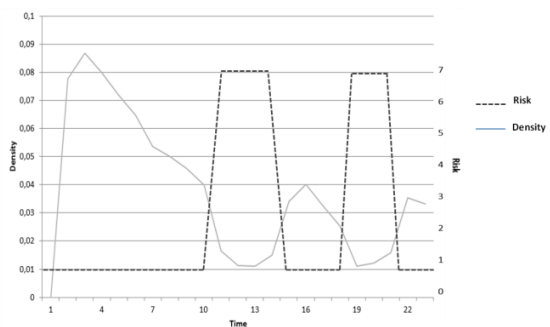


Figure 4: The density computed for an intermediate node according to a time-varying value of risk.

Assuming that the previous illustrated targets have

been achieved, the main part of this methodology is thus the minimisation of risk. As a matter of facts, this model aims to avoid, or at least decrease, the quantity of products in a specific area (node) during time intervals characterised by an high risk value. The Figure 4 highlights that the value of density at a node decreases when risk value at the same node raises and, on the contrary, computed density raises when risk is lower.

## 4 CONCLUSIONS

This paper presents an innovative methodology for a time-dependent dispatching of hazardous materials. This method considers decentralised Decision Makers, allowing a moderate exchange of information in the whole network. This assumption make this methodology attractive for very large scale networks. Besides, the problem to be performed by each DM is formulated as a LQR, making it computationally efficient. A case study applied on a medium size network provided results showing a proper behaviour with respect to initial goals. A possible critical aspect of this methodology is the elevated number of iterations that could be necessary to achieve a full convergence of the model. Further investigations could be the increase of the information exchanged between neighbouring nodes, for example transferring also the planned inventory values.

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