

# Optimizing Operation Costs of the Heating System of a Household using Model Predictive Control Considering a Local PV Installation

Cosmin Koch-Ciobotaru<sup>1</sup>, Fridrik Rafn Isleifsson<sup>2</sup> and Oliver Gehrke<sup>2</sup>

<sup>1</sup>Automation and Applied Informatics, Politehnica University of Timisoara, Blv. Parvan 2, Timisoara, Romania

<sup>2</sup>Intelligent Energy Systems, Technical University of Denmark, Frederiksborgvej 399, Roskilde, Denmark

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**Abstract:** This paper presents a model predictive controller developed in order to minimize the cost of grid energy consumption and maximize the amount of energy consumed from a local photovoltaic (PV) installation. The usage of as much locally produced renewable energy sources (RES) as possible, diminishes the effects of their large penetration in the distribution grid and reduces overloading the grid capacity, which is an increasing problem for the power system. The controller uses 24 hour prediction data for the ambient temperature, the solar irradiance, and for the PV output power. Simulation results of a thermostatic controller, a MPC with grid price optimization, and the proposed MPC are presented and discussed.

## 1 INTRODUCTION

The main issue (Vandoorn, 2011) is that the electrical distribution grid was not designed for bi-directional power flow, i.e. that power would not only flow to the lower voltage levels where most consumer are connected, but that it could also flow “up” to the higher voltage levels.

The increased amount of PV plants in the distribution grid introduces some complications, such as the fluctuating nature of PV production which has limited predictability (Madureira, 2009). There are fast fluctuations, due to cloud transients, which cause problems with voltage regulation. There are also slower fluctuations due to the movement of the sun and changes in cloud cover, so if the PV plant generation is not coordinated with the local consumption it might be necessary to invest in more grid capacity as presented in (Ueda, 2007).

In the distribution grid there is also a foreseeable increase in new types of loads, such as heat pumps and electric vehicles, both loads that can to some degree act as flexible loads as shown in (Madureira,2009).

If loads that are flexible can be intelligently managed, it could be possible to help the distribution grids to cope with both increased renewable production and increased loads. Furthermore, this intelligent control could also reduce the need for expensive grid extensions if loads and production

are coordinated locally.

This control is seeking to incorporate predictions of weather, occupancy behaviour, renewable energy availability, and price signals from the grid. The model predictive control (MPC) presents a methodology that can use all these predicted values in order to improve the energy efficiency consumption by load shifting and peak shaving, minimize the cost of operation by using low price energy, as shown in (Nagai, 2002) and in (Ma 2011), and maximizing the use of renewable energy.

This paper proposes a MPC that minimizes the overall electrical energy cost of heating a building which also has a local PV installation. By using the buildings ten 1 kW heaters, a price signal for electrical energy, a prediction of solar irradiation, of PV output power, and of ambient temperature it is possible to coordinate the heaters consumption so that as much energy as possible is consumed from the locally produced PV.

## 2 MODEL OF THE SYSTEM

Model predictive control uses a model of the system in order to predict the process output over a future horizon of N time steps and solves a quadratic optimization problem with the control signal as the decision variable. In addition, constraints can be formulated both for manipulated and controlled

variables as formulated in (Huusom, 2010) and (Oldewurtel, 2010).

The model used in this paper is extensively presented in (Bacher, 2010) and represents a house of approximately 125 m<sup>2</sup> divided between eight rooms. Every room is equipped with heaters: two rooms have two heaters and the others have one heater each. The heaters are considered to have the output power of 1kW.

The model approximates the interior of the building to be one room with a uniform inside temperature. The state variable is the inside temperature (T<sub>i</sub>), the input is the power to the heaters (P<sub>H</sub>) and the disturbances are the solar irradiance (G) and the ambient temperature (T<sub>a</sub>).

The temperature dynamics of a given space can be modelled using a resistance-capacitance (RC) circuit analogy, see figure 1, and formulated as a linear state space model.

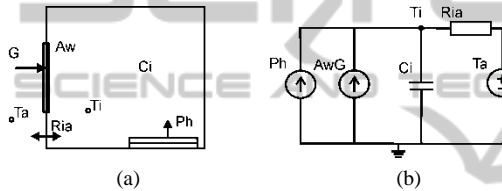


Figure 1: Thermal dynamic model of the house.

$$C_i \frac{dT_i}{dt} = \frac{1}{R_{ia}}(T_a - T_i) + A_w G + P_h \quad (1)$$

Where C<sub>i</sub> is the heat capacity of the house. This includes the indoor air and the interior objects (=3.42 [kW/°C])

R<sub>ia</sub> is the thermal resistance from the indoor to the ambient environment (=4.87 [°C/kW])

A<sub>w</sub> is the effective window area of the house with heating influence (=5.53 [m<sup>2</sup>])

### 3 OFFSET FREE MPC

The predicted disturbances values that are available to the model usually present an error compared to the real measured values. In order to eliminate the offset caused by these differences, filters have to be implemented for each of the predicted values fed into the controller. In this way, the controller will not track the predicted values, but their variations. This gives, compared to Equation 1, an extended state space model with an additional state for each filtered variable:

$$\begin{bmatrix} x_{k+1} \\ \eta_{k+1} \end{bmatrix} = \begin{bmatrix} A & E \\ 0 & A_d \end{bmatrix} \begin{bmatrix} x_k \\ \eta_k \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \Delta u_k + \begin{bmatrix} E \\ 0 \end{bmatrix} d_k \quad (2a)$$

$$y_k = [C \ 0] \begin{bmatrix} x_k \\ \eta_k \end{bmatrix} \quad (2b)$$

For simplification, we introduce the new state model on the basis of equation 3:

$$x_{p,k+1} = A_e x_{p,k} + B_e \Delta u_k + E_e d_k \quad (3a)$$

$$y_k = C_e x_{p,k} \quad (3b)$$

The usage of a Kalman filter in the algorithm consists of two stages that run cyclically:

- Time update – responsible for projecting the state ahead

$$x_{e,k+1} = A_e x_{p,k} + B_e u_k + E_e d_k \quad (4)$$

- Measurement update – which has the role to ‘correct’ the estimated values by considering the measurements taken from the system

$$x_{p,k} = x_{e,k} + K_f (z_k - C x_{e,k}) \quad (5)$$

The covariance P is a symmetric positive semidefinite solution of the discrete Riccati equation:

$$P = A P A' + Q - A P C' (C P C' + R)^{-1} C P A' \quad (6)$$

The covariance of the innovations R<sub>e</sub> and the predictive Kalman gain K<sub>f</sub> are computed using equations 7 and 8:

$$R_e = C P C' + R \quad (7)$$

$$K_f = P C' R_e^{-1} \quad (8)$$

The simulation uses the *quadprog* solver from Matlab for which the optimization problem has to be rewritten in the form of Equation 9:

$$\min_U \frac{1}{2} U' H U + g' U \quad (9)$$

Subject to

$$A_q U \leq b_q \quad (10)$$

The model output for the predicted horizon of N time steps is:

$$Y_{[N,1]} = \Phi_{[N,1]} x_0 + \Gamma_{[N,N]} U + \Gamma_{d[N,N]} D \quad (11)$$

Equation 4 has the following coefficients:

$$H = \Gamma' Q \Gamma \quad (12)$$

$$g = -\Gamma' Q (R - \Phi x_{p,k} - \Gamma_d D) \quad (13)$$

Where

$$\Phi = \begin{bmatrix} CA \\ CA^2 \\ CA^3 \\ \vdots \\ CA^N \end{bmatrix}; \Gamma = \begin{bmatrix} H_1 & 0 & 0 & \dots & 0 \\ H_2 & H_1 & 0 & \dots & 0 \\ H_3 & H_2 & H_1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H_N & H_{N-1} & H_{N-2} & \dots & H_1 \end{bmatrix} \quad (14)$$

$$\Gamma_d = \begin{bmatrix} H_{1,d} & 0 & 0 & \dots & 0 \\ H_{2,d} & H_{1,d} & 0 & \dots & 0 \\ H_{3,d} & H_{2,d} & H_{1,d} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ H_{N,d} & H_{N-1,d} & H_{N-2,d} & \dots & H_{1,d} \end{bmatrix}$$

$$H_k = C A^{k-1} B$$

$$H_{k,d} = C A^{k-1} E$$

In this case, the result of the MPC optimization problem is the difference  $\Delta u_k$  and the command to the system is  $u_k = u_{k-1} + \Delta u_k$ .

## 4 SIMULATION SCENARIOS

In all the simulations the MPC controller uses the model described in Equation 2.

These two additional state variables are used for implementing the filter in order to achieve offset free control in the presence of deviations from the predicted values of the two disturbances.

The MPC controller has hard limitations on the controlled variable – the inside temperature, that has to be inside  $[20...22]^\circ\text{C}$  interval and on the manipulated variable – power supplied to the heaters, that has to be in the  $[0...10]$  kW interval, and can have only integer as power steps.

The MPC controller starts with offline predicted values for solar irradiance, temperature, and grid price and for the third simulation case, the predicted PV power output.

The time step of the simulations is 10 minutes, and the prediction horizon is 80 time steps.

During each simulation, two different cases can be studied:

- The first, when the house does not have any PV installation – the heaters are consuming power entirely from the grid

- The second, when the house has a PV installation – the heaters are consuming power both from the PV plant and from the grid. The higher priority is to consume from the local PV plant and the remaining required power is taken from the grid. The amount of unused PV energy is sold to the grid.

### 4.1 Simulation Scenario 1

A thermostatic controller is implemented to maintain the temperature inside given limits:  $[19.2...21]$ . For comparison reasons, the limits in this simulation scenario differ from the other two scenarios in order that the average temperature in the house, for the simulation time, to be the same. This has the purpose to accurately reflect the MPC controller's effect in similar operation conditions.

### 4.2 Simulation Scenario 2

The MPC tracks the inside temperature with minimal overall energy cost. The controller is considering all the energy to be taken from the grid, at a market imposed price ( $C_G$ ).

The optimization function is represented by Equation 15:

$$\min_u \Phi = \frac{1}{2} \sum_{k=0}^N \|z_k - r_k\|_Q^2 + \frac{1}{2} \sum_{k=0}^{N-1} \|\Delta u_k\|_S^2 + \sum_{k=0}^N C_{G,k} u_k \quad (15)$$

### 4.3 Simulation Scenario 3

The MPC controller tracks the inside temperature with minimal overall energy cost, also considering the power production of the installed PV panels. The controller calculates a virtual price on which the available PV power, that has a lower cost for the user ( $C_{PV}$ ) of 0.02 Euros, is considered to alter, with a weight factor, the market imposed price.

$$U = P_{Grid} + P_{PV} \quad (16)$$

The cost minimization function would be

$$\min_{P_{Grid}} \Phi = C_G P_{Grid} \quad (17)$$

Considering  $U$  as the optimization variable and replacing 16 in 17 the equation 18 is obtained:

$$\min_U \Phi = C_G U \left(1 - \frac{P_{PV}}{U}\right) \quad (18)$$

Where  $C_G$  – is the predicted price of the grid energy

$U$  – represents the vector with the next  $N$  command values for the time horizon

$P_{PV}$  – represents the predicted output power from the PV installation

$$C_v U = C_G \left(1 - \alpha \frac{P_{PV}}{u_s}\right) U \quad (19)$$

Where additional assumptions were made:

- $\alpha = \frac{C_{pv}}{C_G}$  - a weight factor
- at each optimization step,  $u_s$  is taken as the last command value,  $u_{k-1}$ .

The optimization function is written as:

$$\min_u \Phi = \frac{1}{2} \sum_{k=0}^N \|z_k - r_k\|_Q^2 + \frac{1}{2} \sum_{k=0}^{N-1} \|\Delta u_k\|_S^2 + \sum_{k=0}^N C_{v,k} u_k \quad (20)$$

## 5 RESULTS

Results from the three simulation scenarios are presented in Figures 2 to 4 and compared in Table 1, where the following notations have been used:

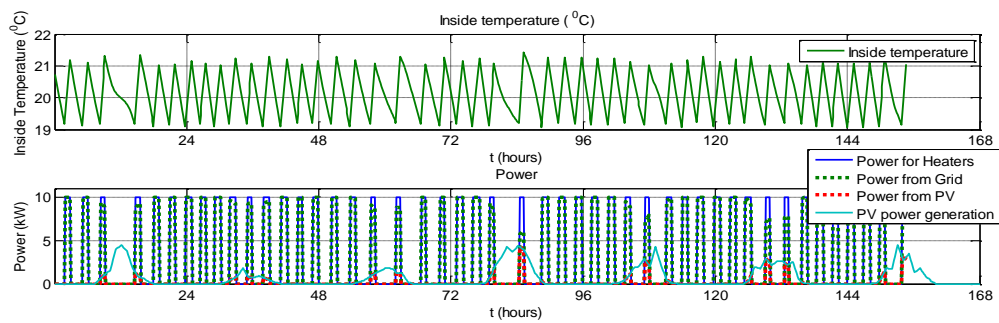


Figure 2: Thermostatic control.

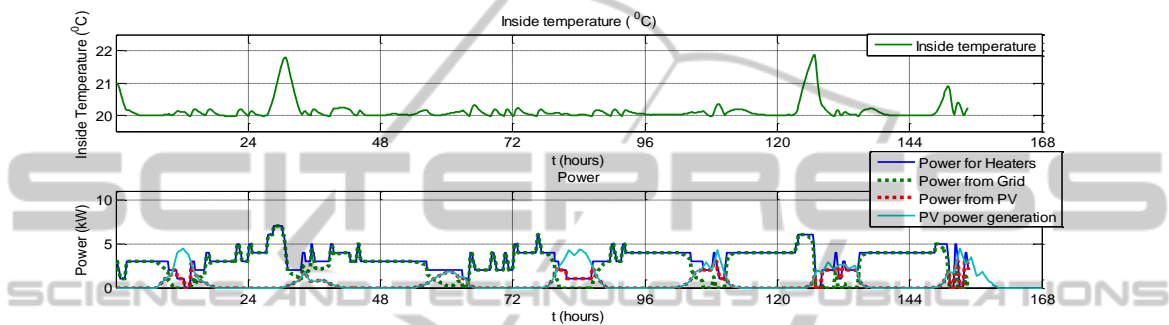


Figure 3: MPC with grid price optimization.

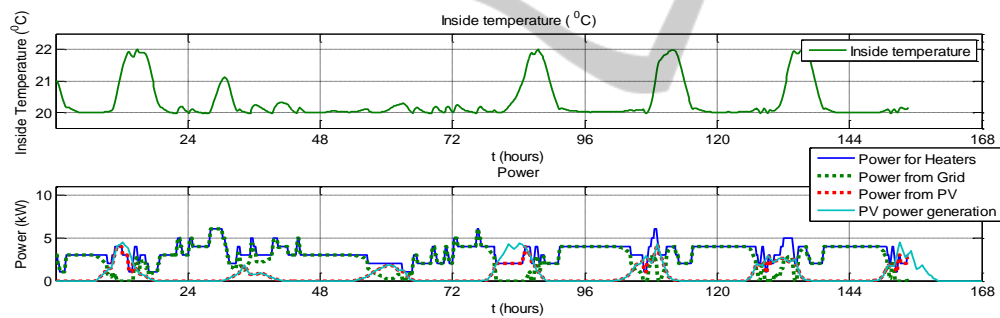


Figure 4: MPC with virtual price optimization considering PV power output prediction.

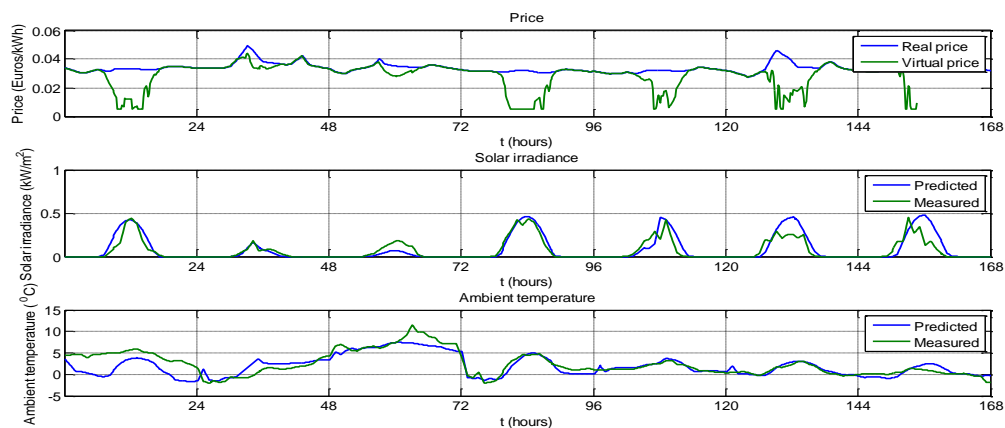


Figure 5: Data used by controllers: price values, predicted and measured ambient data.

Table 1: Energy consumption and cost results from simulations.

Sim. ID	Config. Type	$E_H$ (kWh)	$E_{PV2H}$ (kWh)	$E_{G2H}$ (kWh)	$E_{PV}$ (kWh)	$E_{PV2G}$ (kWh)	$C_{G2H}$ (Euros)	$C_{PV2H}$ (Euros)	Avg Ti (°C)
Simulation 1, with thermostatic controller around 20.14°C									
S <sub>11</sub>	No PV	496.66	-	496.66	-	-	16.75	-	20.14
S <sub>12</sub>	PV	496.66	21.68	474.98	110.58	88.90	15.98	0.43	20.14
Simulation 2, with grid price optimization									
S <sub>21</sub>	No PV	496.66	-	496.66	-	-	16.61	-	20.14
S <sub>22</sub>	PV	496.66	71.01	425.65	110.58	39.57	14.12	1.42	20.14
Simulation 3, with grid price and PV availability									
S <sub>3</sub>	PV	500.8	94.37	406.4	110.08	16.21	13.57	1.88	20.34

Sim. ID – simulation identifier

Config. Type – type of house configuration: with or without PV installation

$E_H$  – the total energy consumed by the heaters during simulation interval

$E_{PV2H}$  – the amount of energy consumed by the heaters from the local produced PV energy

$E_{G2H}$  – the amount of energy consumed by the heaters from the grid

$E_{PV}$  – the amount of energy produced by the PV installation

$E_{PV2G}$  – the amount of energy produced by the PV to be sold to the grid

$C_{G2H}$  – cost of  $E_{G2H}$  in Euros

$C_{PV2H}$  – cost of  $E_{PV2H}$  in Euros

Avg. Ti – average inside temperature over the simulated time horizon

The grid energy prices are shown in the first plot from Figure 5. It is assumed that the predicted grid energy prices coincide with the actual ones. In the same figure, the virtual price used during Simulation 3 is also plotted.

During simulations  $S_{1x}$  and  $S_{2x}$  the controller does not present information regarding the presence of an PV installation and acts according only to signals available for each case, as stated in section 3.

Achieving the same average inside temperature implies the same amount of energy is used. As the ambient temperature and the solar irradiance are the same for each simulation, the amount of electric energy used to keep the inside temperature is the same. The difference is represented by the heaters consumption shifting according to the used controller.

In  $S_{1x}$  a thermostatic controller is used, as presented in section 3. It can be seen that during clear days, with large solar irradiance values, the heaters are turned off most part of the day, the thermal energy being largely taken from the ambient factors. In  $S_{12}$  only 21.68 kWh, representing around 20% of the available PV local produced energy, is consumed from the PV.

In  $S_{2x}$  the MPC with grid price optimization is used. The same amount of electric energy is used as in  $S_{1x}$ , for achieving the same inside temperature. However, the MPC shifts the heaters' consumption to low price moments, and stores thermal energy before price peaks as it can be seen in Figure 3, before the energy price peaks at time 200 and 780, shown in Figure 5.

The MPC controller from  $S_{2x}$  achieves a cost reduction from 16.75 to 16.61 Euros in the case of  $S_{21}$  and from 15.98 to 14.12 in the case of using a PV installation of  $S_{22}$ . In  $S_{22}$ , 71.01 kWh of local PV energy is consumed, representing 64% of the PV production.

However, the local PV energy usage for  $S_{12}$  and  $S_{22}$  are unpredictable since the controller does not consider the PV production.

In  $S_3$  the MPC's objective is to consume as much locally produced energy as possible. This is realized by implementing the virtual price, presented in section 3, in the optimization function. Figure 4 depicts the operation of the MPC which uses the house's thermal capacity to store the local PV energy during large solar irradiance values.

In this case, the cost of the energy consumed from the grid is 13.57 Euros and 85% of the local PV produced energy is consumed.

## 6 CONCLUSIONS

The paper emphasises the benefits of using model predictive control for houses as dynamic thermal energy storage.

By formulating the correct optimization problems and feeding the controller with predictions on the system's variables, the MPC is able to achieve cost reduction on the electrical energy consumption from the grid.

As demonstrated through simulations in this paper, the MPC can consider the presence of an

installed PV plant maximizing the usage of locally produced renewable energy. The consumption of locally produced energy has a major benefit both for the user, by lowering the overall cost of energy and also for the operation of distribution grids with a high penetration of renewable energy generation.

This paper presented an algorithm that deals with the two problems: minimizing the operating cost of the house heating system and maximizing the use of local produced energy and lowering the burden on the distribution grid.

From the source of power consumption perspective, the algorithm can be extended to use the energy from other types of local renewable energy sources. It can be extended also from the perspective of the types of loads that are shifted, not focusing only on the heat system but also on different household appliances.

The proposed algorithm can be used to manage energy produced by other types of renewable energy generation, such as wind turbines and combined heat and power plants. The algorithm can also be modified for other types of consumption that has the ability to be shifted in time, such as water heaters, air conditioning units and refrigeration systems.

## ACKNOWLEDGEMENTS

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