

How to Use Information Theory for Image Inpainting and Blind Spot Filling-in?

J. M. Berthommé, T. Chateau and M. Dhome

Institut Pascal - UMR 6602, Université Blaise Pascal, 24 Avenue des Landais, 63177 Aubière Cedex, France

Keywords: Image Inpainting, Information Theory, Blind Spot.

Abstract: This paper shows how information theory can both drive the digital image inpainting process and the optical illusion due to the blind spot. The defended position is that the missing information is padded by the “most probable information around” via a simple filling-in scheme. Thus the proposed algorithm aims to keep the entropy constant. It cares not to create too much novelty as well as not to destroy too much information. For this, the image is broken down into regular squares in order to build a dictionary of unique words and to estimate the entropy. Then the occluded region is completed, word by word and layer by layer, by picking the element which respects the existing image, which minimizes the entropy deviation if there are several candidates, and which limits its potential increase in the case where no compatible word exists and where a new one must be introduced.

1 INTRODUCTION

The eye’s blind spot has been discovered in 1660 by Edme Mariotte, a French physicist, whose experiment seemed magical when it was first presented to Louis XIV’s court. Today, despite three-and-a-half centuries of progress, this demonstration still resists interpretation.

From the 19th century until now naturalists then neuroscientists have remarkably well investigated the visual system. However, despite tons of observations they do not fully understand yet its functioning. Brain modelling remains a real challenge.

On the other hand, the relatively recent community of computer vision, clearly based on hard science, has a lot to do to solve its own problems, *e.g.* segmentation, 3D reconstruction or tracking. It looks for good algorithms, not laws. As a result it has regularly claimed that it has nothing to do with medicine, perceived as too experimental.

Whatever these clichés, some pieces seem to match. Image Inpainting experiments can precisely simulate the illusion of the blind spot and more fundamentally Information Theory can *simply explain* the principle underlying the phenomenon.

2 THE BLIND SPOT

2.1 Demonstration

Let us start by showing an experiment that ophthalmologists know well. Look at the top part (a) of figure 1 with the big dot and the cross. Close your right eye and force your left eye to stare at the cross slightly sidelongly. Slowly move your head closer to the screen. When the image of the dot hits your blind spot it disappears. Note that as soon as you let your left eye directly look at the dot it immediately reappears. Then use the same distance in case (b). The hole of the horizontal line is now completed so that it appears continuous. These two examples have demonstrated the existence of your blind spot.

2.2 Partial explanation

Neuroscientists half understand the phenomenon. They see why information is *missing* but not why it is *completed*. As shown in figure 2 it occurs where the optic nerve leaves the eye. The axons of the retinal ganglion cells concentrate at one point and go through the retina, preventing any presence of photoreceptors. In human beings the blind spot is large, about 4° of the view field. It is located at slightly different angles in each eye, probably to facilitate their mutual filling. Finally, some invertebrates like cephalopods do not



Figure 1: Two experiments that demonstrate the existence of the blind spot. To make the black dot in (a) or the white hole in (b) disappear, close your right eye and force your left eye to look at the cross with a slight angle, then slowly move your head back and forth at about 25 cm from the screen.

have a blind spot. Their nerve fibers route *behind* the retina and do not block light.

at: <http://www.lasmea.univ-bpclermont.fr/Personnel/Jean-Marc.Berthomme/>.

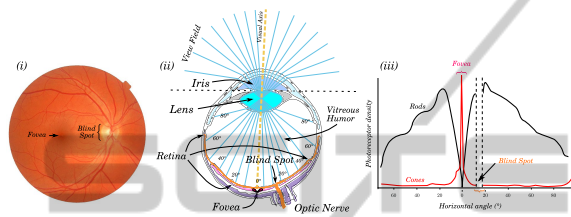


Figure 2: In the eyes of the vertebrates, the nerve fibers route *before* the retina, blocking some light and creating a blind spot where the fibers pass through the retina and out of the eye. (i) human retinography, (ii) diagram of human eye, and (iii) photoreceptor density highlight the blind spot.

3 ENTROPY INPAINTING

Inpainting is the process that replaces undesired information by contextual information without altering the global consistency of the signal. It is not necessarily restricted to image restoration as it can also apply to sounds or videos. In all cases it relies on digital signals.

2.3 Emulation

We are almost always unaware of our natural blind spots. So how are they naturally filled in? One radical solution would be to *experiment* with a conscious patient full of instruments in his eye and his cortex. To the best of our knowledge, this is not yet feasible. Beyond ethics, there are still technical issues.

3.1 Formulation

Inpainting should not make unexpected things arise or disappear. So, in the Information Theory framework, we state that: “Inpainting must *neither* create *nor* destroy information“. By defining X_u , the *unknown* part of an image, and X_k , its *known* part, the ideal goal is to get: $H(X_u, X_k) = H(X_k)$, which is equivalent to (MacKay, 2003): $H(X_u/X_k) = 0$. Thus the proposed algorithm aims to keep the entropy of the growing known region constant. As long as there are n_u unknown bits there are 2^{n_u} possible subimages so $H(X_u) = n_u$ bits. Concerning $H(X_k)$ its estimation requires to divide X_k into pieces in order to specify a dictionary \mathcal{D} and therefore propagate the filling-in process. Anyway, inpainting can be reformulated as an optimization problem looking for an unknown signal X_u^* and an unknown dictionary \mathcal{D}^* so that:

Other people have made the parallel with Image Processing and tried to *model* the phenomenon in order to emulate the filling-in process. That is the perspective we have adopted. But, from what we read, and whatever the community, they have talked about “*visual interpolation*” (Durgin, 1995) and have applied variational approaches (Liu et al., 2007) (Arias et al., 2011) to constrained Partial Differential Equation (Sato, 2011) (PDE) problems. We contest this interpretation. Why? To make it short, simply because interpolation can fast create unexpected things, *i.e.* disorder, in other words *entropy*.

$$\begin{aligned} & \text{minimize} && |H_{\mathcal{D}}(X_u, X_k) - H_{\mathcal{D}}(X_k)| \\ & \text{subject to} && X_k \end{aligned}$$

In the following section this idea is reformulated within the Information Theory framework. Based on that principle we describe an inpainting algorithm that aims to keep the entropy constant. It can not only reproduce the blind spot experiments (a) and (b) but also complete any damaged image. We have provided MATLAB implementations able to inpaint binary, grayscale or RGB images of any bit depth

3.2 Building the Dictionary & Estimating the Entropy

When the unwanted information is removed from the damaged image the method first breaks down the remaining known region. The partition possibilities are huge, from blocks of 1 x 1 pixels to the size of the image itself. Whatever the choice, the goal is to fetch

the redundant patterns, so all the shapes are not relevant. Note that 1 x 1 cutting makes an exception because it loses the spatial consistency, so it is never used. Similarly, a sliding windowing is applied to the image in order not to miss any stitching between the patches. Though precise, this exhaustive enumeration is expensive. So, to shorten it, exclusive or even random windowing strategies could be considered. The counting of the broken blocks leads to the building of a dictionary \mathcal{D} . It is composed of unique words associated with their frequencies p_i . These latter allow to estimate the entropy H of the image.

3.3 Filling in the Selection

Based on such a dictionary, any missing pixel can be replaced by looking for the patch - or the word following the point of view - that best fits the selection around. Three cases can occur. There can be one, several or no compatible words in the dictionary. Note that the compatibility is checked on the known pixels of the selection. It is calculated with the logic functions NOT, XOR and AND as each piece of image is stored as a set of Boolean. So, if there is only one word, it is always taken. If there are several, the selected one must minimize the absolute entropy deviation. Finally, if no compatible word exists a new one is created. It does not challenge the known part of the selection. It only retains the consensus within the dictionary concerning the unknown part. To conclude, the whole process is summarized in algorithm 1 below.

4 EXPERIMENTAL RESULTS

Our implementation has investigated dictionaries of square patches. It can emulate the blind spot completion described in section 2 with an image of size 25x100 and patches of any size between 2x2 to 25x25 in this case. The results, grouped in figure 3 below, correspond to a filling in with words of size 3x3.

We have also explored characteristic patterns like crosses in order to see how to constrain the novelty creation. This work has started with binary images and was then extended to grayscale and RGB images of any bit depth. Random images have equally procured a deep reflection. We first wanted to minimize the entropy, not its absolute deviation. So we encoded redundancy and thus destroyed information. As random images are already at maximum entropy, regardless of their breaking down, we must maintain their entropy constant to be able to reproduce their pattern.

Algorithm 1: Entropy Inpainting Algorithm.

```

input : unknown signal  $X_u$  and known signal
          $X_k$  such that  $X_u \cap X_k = \emptyset$ 
output:  $X_u$  completed so that
          $H_{\mathcal{D}}(X_u, X_k) \approx H_{\mathcal{D}}(X_k)$ 

build a dictionary  $\mathcal{D}$  from  $X_k$  and calculate
 $H(X_u)$  and  $H(X_k)$ ;
while  $X_u$  is not completed do
  define an overlapping layer  $\mathcal{L}$  between  $X_u$ 
  and  $X_k$ ;
  while  $\mathcal{L}$  is not completed do
    define a selection  $\mathcal{S}$  inside  $\mathcal{L}$ ;
    find the compatible words between  $\mathcal{S}$ 
    and  $\mathcal{D}$ ;
    if there is one candidate then
      select this word;
    else if there are several candidates then
      select the word which minimizes
      the absolute entropy deviation;
    else // there is no candidate
      create a new word compatible with
       $\mathcal{S}$  and  $\mathcal{D}$ ;
    end
    fill in  $\mathcal{S}$  with the returned word;
    update  $X_u$ ,  $X_k$ ,  $\mathcal{D}$  and recalculate  $H(X_u)$ 
    and  $H(X_k)$ ;
  end
end

```

Finally we have tried to inpaint natural images taken from the benchmark dataset proposed by (Kawai et al., 2009) at: <http://yokoya.naist.jp/research/inpainting>. Our RMSE values compare the inpainted image to the original one within the completed region only. They are not relevant to highlight the image global consistency but they are good to compare the inpainting methods. We readily acknowledge that our implementation could highly be improved compared to the others. Clearly we do not manage edges and complex textures.

5 CONCLUSIONS

This paper has underscored the fact that Information Theory can *simply formulate* the inpainting process and *precisely emulate* the blind spot filling-in. It has emphasized that the goal of inpainting is *neither* to create *nor* to destroy information. Thus the inpainting process was reformulated within the Information Theory framework under the form of an optimization problem looking for both a dictionary and an unknown signal. It aims to keep the entropy of the grow-

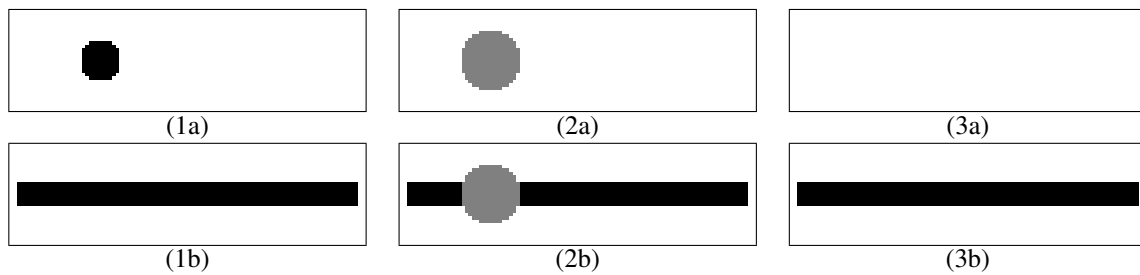


Figure 3: Entropy inpainting with 3x3 patches emulating the blind spot filling-in. (1) denotes the original images, (2) the damaged images with the same gray cache, and (3) the final inpainted images. (a) and (b) refer to the experiments described in section 2.

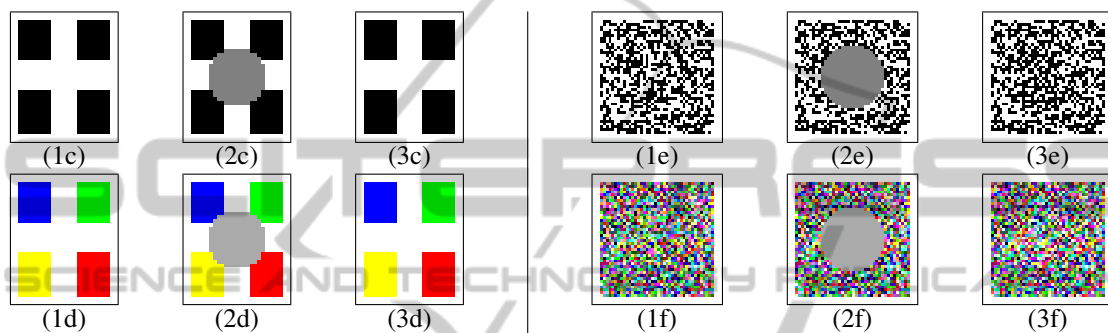


Figure 4: Entropy inpainting with 3x3 patches recovering characteristic patterns on binary and RGB images. (1) denotes the original images, (2) the damaged images with the gray cache, and (3) the final inpainted images. (c) and (d) cases are sensitive to the creation of information. (e) and (f) cases are sensitive to the destruction of information.

ing known signal constant. For that, an example of *entropy inpainting* algorithm has been proposed.

The provided implementation has simulated the described optical illusions due to the blind spot experiments. It can equally inpaint binary, grayscale or RGB images of any bit depth. However it is far from being optimal. This is mainly due to the exclusive use of fixed size square patches in the dictionary. This can be greatly improved by removing almost all the constraints on the shape of the words.

Last but not least, we are convinced that it is worth modelling simple optical illusions to push the theory to its limits and to better understand the perception process. Many things remain unclear and there are still a lot of things to do and to learn.

REFERENCES

- Arias, P., Facciolo, G., Caselles, V., and Sapiro, G. (2011). A variational framework for exemplar-based image inpainting. *International journal of computer vision*, 93(3):319–347.
- Durgin, F. H. (1995). On the filling in of the visual blind spot: some rules of thumb. *Perception*, 24:827–840.
- Kawai, N., Sato, T., and Yokoya, N. (2009). Image inpainting considering brightness change and spatial locality

of textures and its evaluation. *Advances in Image and Video Technology*, pages 271–282.

- Liu, D., Sun, X., Wu, F., Li, S., and Zhang, Y. (2007). Image compression with edge-based inpainting. *Circuits and Systems for Video Technology, IEEE Transactions on*, 17(10):1273–1287.
- MacKay, D. (2003). *Information Theory, Inference, and Learning Algorithms*. Cambridge University Press.
- Satoh, S. (2011). Computational identity between digital image inpainting and filling-in process at the blind spot. *Neural Computing & Applications*, pages 1–9.