Evaluating Learning Algorithms for Stochastic Finite Automata
Comparative Empirical Analyses on Learning Models for Technical Systems

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Abstract: Finite automata are used to model a large variety of technical systems and form the basis of important tasks such as model-based development, early simulations and model-based diagnosis. However, such models are today still mostly derived manually, in an expensive and time-consuming manner.
Therefore in the past twenty years, several successful algorithms have been developed for learning various types of finite automata. These algorithms use measurements of the technical systems to automatically derive the underlying automata models.
However, today users face a serious problem when looking for such model learning algorithm: Which algorithm to choose for which problem and which technical system? This papers closes this gap by comparative empirical analyses of the most popular algorithms (i) using two real-world production facilities and (ii) using artificial datasets to analyze the algorithms' convergence and scalability. Finally, based on these results, several observations for choosing an appropriate automaton learning algorithm for a specific problem are given.

1 INTRODUCTION

In computer science, the maturing of a field of research happens normally in two phases: First, a number of algorithms are developed. Then in a second phase, these algorithms are evaluated and compared. Only after this second phase, their broad application by non-experts becomes possible. For several reasons, the second phase is often neglected, leaving non-experts insecure and uneasy about the application of newly developed algorithms.

The field of learning finite automata, where the learning includes states and transitions, is an example for this. Several algorithms have been developed (see section 3 for an overview), but there is a lack of comparative studies. Different algorithms are often evaluated in different application areas, using datasets that are not always publicly available. Moreover, various algorithms learn somewhat specific finite automata for which the common comparison criteria have to be established. This paper will help to close this gap by introducing several important comparison criteria and by evaluating algorithms on the same datasets.

In the rest of this section, a motivation for the learning of automata is given. Section 2 then gives an overview of stochastic finite automata formalisms. Section 3 outlines the four algorithms ALERGIA, MDI, BUTLA, and HyBUTLA for learning these automata. Criteria for comparing the algorithms are outlined in section 4. In section 5, comparative empirical analyses are conducted using two real-world production facilities. Furthermore, due to the size and structure limitations of the real datasets, we also use artificial data in section 6 for further analyses on the algorithms' convergence and scalability. The results are analyzed in section 7 and observations for the usage of these algorithms are given. Section 8 gives a conclusion.

The learning of automata is a key technology in a variety of fields such as model-based development, verification, testing, and model-based diagnosis (Niggemann and Stroop, 2008). The importance stems from the facts that (i) complex dynamical technical systems (such as production systems) can be modeled using different types of finite automata and (ii) a manual creation of these models is often too expensive and labor intensive.

A typical application scenario is the model-based anomaly detection for technical systems: The more complex technical system becomes, the more impor-

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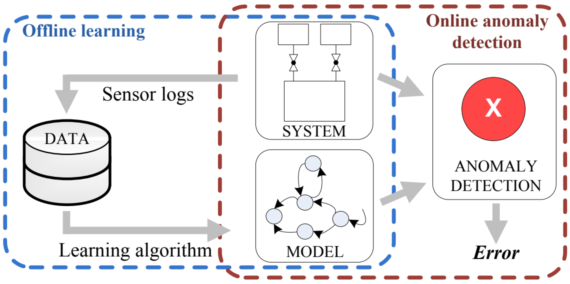


Figure 1: Model-based anomaly detection.

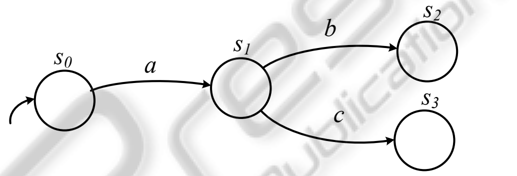
tant are automatic and adaptive anomaly detection and diagnosis systems. This increasing complexity is mainly due to the high number of software-based components and the usage of distributed architectures in modern embedded systems. Ensuring proper functioning of these technical systems has led to the development of various monitoring, anomaly detection and diagnosis techniques. Model-based approaches have established themselves among the most successful ones. However, they require a behavior model of a system, which in most cases is still derived manually. Manual modeling of systems that exhibit state-based, probabilistic, temporal, and/or continuous behavior is a hard task that requires a lot of efforts and resources. Therefore, researchers have investigated the possibilities to learn behavior models automatically from logged data (see section 3).

The general approach to model-based anomaly detection, which uses a learned behavior model, is illustrated in figure 1. In the first phase, based on logs of the system, a model of the normal behavior is learned. Then, in the second phase, this normal behavior model is used during a system's operation to detect anomalies. For this, the predictions of the model are compared to the actual measurements from the running system. If a significant discrepancy is detected, the user is informed.

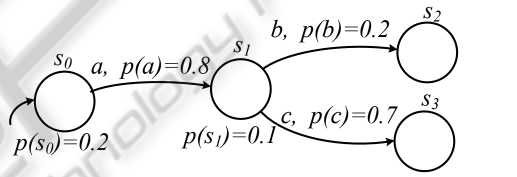
2 FINITE AUTOMATA FORMALISMS

This paper deals with learning models for the three types of technical systems: non-timed discrete event systems, timed discrete event systems, and hybrid systems. There are many mathematical formalisms that can model their behavior, e.g. Bond graphs (Narasimhan and Biswas, 2007), Petri nets (Cabasino et al., 2010), continuous Petri nets (David and Alla, 1987), hybrid Petri nets (David and Alla, 2001), Particle filters (Wang and Dearden, 2009), Kalman filters (Hofbaur and Williams, 2002) and Bayesian networks (Zhao et al., 2005). Due to a number of positive results for the learning of stochastic finite automata from data, they are in the focus of this paper.

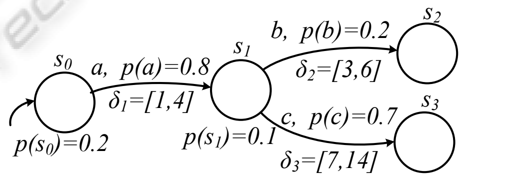
Non-timed Discrete Event Systems (nDES) show a state-based behavior, i.e. they are represented by a set of discrete states (modes of operation) and a finite set of events that trigger transitions between those states (mode switches) (Cassandras and Lafortune, 2008). Such systems can be easily modeled as the well-known Deterministic Finite Automaton (DFA) as illustrated in figure 2(a): States are denoted by s_0, s_1, s_2, and s_3, while letters a, b, and c denote events that trigger transitions. The automaton is deterministic in a sense that one event can trigger only one transition for each state.



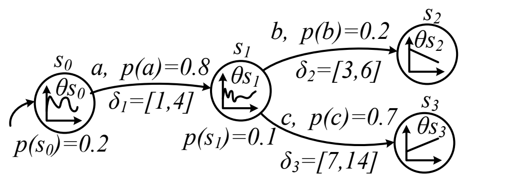
(a) Deterministic finite automaton.



(b) Stochastic deterministic finite automaton.



(c) Stochastic deterministic timed automaton.



(d) Stochastic deterministic hybrid automaton.

Figure 2: Deterministic finite automata for modeling technical systems.

Since the behavior of technical systems is always subjected to statistical fluctuations (e.g. because of noise or external disturbances), a model should take this into account. Therefore, a stochastic version of the DFA was developed (Carrasco and Oncina, 1994). Stochastic Deterministic Finite Automaton[1] (SDFA)

[1]Some authors denote such automata *probabilistic*, rather than stochastic. E.g. see (Thollard et al., 2000). This

is illustrated in figure 2(b). In contrast to DFA, it models the probabilities of staying in a state or transiting to another state given a specific event.

Technical systems also show a behavior over time, i.e. timing information must be modeled also. These systems are called timed Discrete Event Systems (tDES). In such systems, it is very important at what particular point in time an event happens. Clearly, SDFA cannot be used to model tDES. For that reason, Stochastic Deterministic Timed Automata (SDTA) are used (stochastic version of timed automaton presented in (Alur and Dill, 1994)). An example is shown in figure 2(c) and, unlike SDFAs, it contains time intervals δ during which transitions must occur.

In addition to state-based, probabilistic and timed behavior, real-world technical systems also exhibit a mixture of (value-)discrete and (value-)continuous behavior over time. So far, the presented formalisms can only deal with discrete signals (i.e. events). But within one discrete state, continuous signals often change their values over time. An example is a fluidic system which has two states: First of all, values such as pressure and flow change over time according to one set of differential equations. But if a valve is opened (the opening corresponds to an event), the system moves to another state which is described by a different set of differential equations.

Such systems, where discrete and continuous dynamics interact, are called hybrid systems (Alur et al., 1995; Henzinger, 1996; Branicky, 2005). For modeling hybrid systems, the formalism of Stochastic Deterministic Hybrid Automaton (SDHA) (a stochastic version of hybrid automaton given in (Alur et al., 1995)) can be used. A SDHA is illustrated in figure 2(d): The difference compared to SDTA are the θ functions associated with discrete states. These functions describe the change of continuous values over time. A more detailed overview of finite automata formalisms can be found in (Kumar et al., 2010).

3 LEARNING STOCHASTIC FINITE AUTOMATA

3.1 State Merging Approach

In this paper, four well-known automata learning algorithms are presented and evaluated: ALERGIA, MDI, BUTLA, and HyBUTLA. In general, all these algorithms use the state merging approach for learning that is illustrated in figure 3.

also applies to other types of stochastic automata.

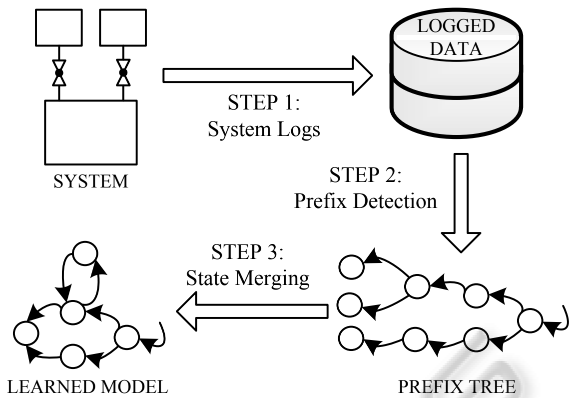


Figure 3: State merging approach for learning automata.

In **step 1**, all relevant signals are measured over multiple production cycles of system's normal operation. Measurements are logged in a database. For timed systems, logs also include time stamps. For hybrid systems, both discrete and continuous signals are recorded. The underlying data acquisition technologies are described e.g. in (Pethig et al., 2012).

An initial automaton called Prefix Tree Acceptor (PTA) is then built in **step 2**. Each logged cycle of a system represents one automaton learning example. Each such example comprises multiple events of a system which are defined as changes in discrete (typically binary) signals. These changes trigger transitions between automaton states. A PTA is obtained when common initial sequences of events of different examples (i.e. example prefixes) are combined together. A PTA represents different examples as paths from the initial state to one of the leaf states. Examples share the prefix parts of the paths. So a PTA is just a smart way to store all examples.

In **step 3**, the learning takes place: compatible pairs of PTA states are merged until the underlying automata is identified. State merging makes the automaton smaller and more general. Different algorithms use different compatibility tests.

3.2 Classification of Algorithms

In general, the learning algorithms can use learning examples that can be both positive (coming from a normal operation) and negative (coming from an abnormal operation) (Angluin, 1988). However, in real technical systems the number of negative examples is typically very small. Therefore, for the modeling of technical systems (and in this paper) the focus is on algorithms relying only on positive examples.

Automata learning algorithms can work either in an online or an offline manner (Vodenčarević et al., 2011). Online algorithms can request additional examples during the learning process. Offline algorithms use only a given, static dataset, previously logged in the system. All four given algorithms described in this paper work in an offline manner.

The order of merging PTA states has a significant influence on the learning performance, especially the algorithm runtime (Niggemann et al., 2012). Learning algorithms normally use either a top-down or a bottom-up merging order. In the top-down order, states are checked for compatibility starting from the initial state and progressing towards the leaf states. When two states are found to be compatible using some compatibility measure, their respective large subtrees have to be recursively checked for compatibility also. This is illustrated in figure 4 (left), where subtrees t_1 and t_2 have to be compared, before the two compatible states s_1 and s_2 are merged. In the bottom-up approach (figure 4 (right)), such recursive checks are minimized, as the merging process starts at leaf states and moves towards the initial state. Algorithms ALERGIA and MDI use the top-down, while BUTLA and HyBUTLA use the bottom-up merging order.

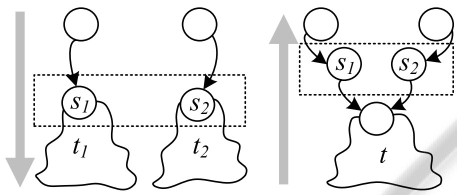


Figure 4: Top-down (left) and bottom-up (right) merging orders.

3.3 Learning Algorithms in a Nutshell

Stochastic deterministic finite automata (SDFAs) can be learned automatically from data using the algorithms ALERGIA and MDI. Stochastic deterministic timed automata (SDTAs) are learned using the BUTLA algorithm, while HyBUTLA learns stochastic deterministic hybrid automata (SDHAs). Both BUTLA and HyBUTLA learn corresponding automata classes with only one clock for tracking time. In this section, these algorithms are briefly explained.

The ALERGIA Algorithm. After building a PTA (see section 3.1), the ALERGIA (Carrasco and Oncina, 1994) algorithm proceeds with checking the compatibility of states in a top-down order. For every single state, the probabilities of stopping in that state or taking a specific transition to another state are computed based on the number of its arriving, ending and outgoing learning examples. Let the number of examples that arrive to state s_k be g_k, the number of examples that end in s_k be $f_k(\#)$, and the number of outgoing examples with the event a be $f_k(a)$. The outgoing probability for s_k with the event a is then $f_k(a)/g_k$, while the ending probability is $f_k(\#)/g_k$. Once these probabilities are computed, the compatibility between any two states s_0 and s_1 is evaluated using the Hoeffding bound (Hoeffding, 1963):

$$\left|\frac{f_0}{g_0} - \frac{f_1}{g_1}\right| > \sqrt{\frac{1}{2}\log\left(\frac{2}{\alpha}\right)}\left(\frac{1}{\sqrt{g_o}} + \frac{1}{\sqrt{g_1}}\right). \quad (1)$$

where $(1-\alpha)^2, \alpha \in \mathbf{R}, \alpha > 0$ is the probability that the inequality is true. Here, f_0 and f_1 denote either the number of outgoing or ending examples for the states s_0 and s_1 respectively, as the Hoeffding bound is computed for both probabilities. If the inequality is true, the difference between estimated probabilities (left side) is larger than a threshold which depends on α (right side) and the states will not be merged. Otherwise the states are declared as compatible, and then their corresponding subtrees are also checked. This is done by recursively evaluating Hoeffding bound for all states in both subtrees. When states are finally merged, the probabilities are recomputed for a new state. In addition, due to the possible appearance of non-determinism in a resulting automaton, it is made deterministic by merging non-deterministic states and transitions. Reported time complexity of ALERGIA is $O(n^3)$, where n is the size of the input data. Formal proof of convergence is given for ALERGIA's version called RLIPS in (Carrasco and Oncina, 1999).

The MDI Algorithm. In the ALERGIA algorithm, the compatibility check based on Hoeffding bound represents the local merging criterion, i.e. the probabilities of two states and their subtrees are compared. There is no global information that tells how different is the whole newly obtained automaton from the previous one (before merging), or from the initial PTA. Conversely, the algorithm Minimal Divergence Inference (MDI) takes this information into account. The MDI algorithm (Thollard et al., 2000) trades off the minimal divergence of the automaton from the learning examples and the minimal automaton size. The states of the PTA A_0 are checked for compatibility in the top-down order like in ALERGIA, but the merging criterion is based on the Kullback-Leibler (K-L) divergence between two automata (that also represent probability distributions of learning examples), rather than on the Hoeffding bound. The K-L divergence $D(A||A')$ between automata A and A' is calculated according to:

$$D(A||A') = \sum_{x_i} p(x_i|A) log \frac{p(x_i|A)}{p(x_i|A')}, \quad (2)$$

where x_i represents one example used for learning. Let A_1 be the temporary automaton obtained by successfully merging states of the PTA A_0. Further let

A_2 be a potential new automaton obtained by merging the states of A_1. The divergence increment while going from A_1 to A_2 is defined as:

$$\Delta(A_1, A_2) = D(A_0||A_2) - D(A_0||A_1). \quad (3)$$

The merge of states of A_1 will be kept if the divergence increment between obtained automaton A_2 and A_1 is small enough relative to the size reduction. Let α_M denote the threshold, and $|A_1|$ and $|A_2|$ the sizes of automata A_1 and A_2, respectively (the number of states). Then the compatibility criterion is:

$$\frac{\Delta(A_1, A_2)}{|A_1| - |A_2|} < \alpha_M. \quad (4)$$

If this inequality is true, the states are similar and will stay permanently merged. Reported time complexity of MDI is $O(n^2)$. Although some experiments indicate that MDI significantly outperforms ALERGIA (see e.g. (Vidal et al., 2005)), MDI lacks the proof of convergence.

Please note that the MDI algorithm needs to do the merge first in order to obtain the new automaton, which is then compared with the previous one using the criterion given by (4). In case the criterion is not satisfied, the merge is discarded and the search for the next potentially compatible state pair proceeds in the previous automaton. The number of merges reported in the results in the following sections represents only those merges that were not discarded.

The BUTLA Algorithm. ALERGIA and MDI proceed top-down while searching for compatible states. The BUTLA algorithm (Bottom-Up Timing Learning Algorithm) firstly introduces the bottom-up strategy (Maier et al., 2011). The criterion for the compatibility check uses the Hoeffding bound (see equation (1)) similar to the ALERGIA algorithm.

Additionally, BUTLA learns the timing of the system. In a preprocessing step, for each available event a probability density function (PDF)—probability over time—is calculated. If the PDF is the sum of several Gaussian distributions, separate events are created for each Gaussian distribution. In the prefix tree creation and in the merging step, these events are handled as different events.

The HyBUTLA Algorithm. The BUTLA algorithm merging approach and the learning of timing information is also applied in the HyBUTLA algorithm (Hybrid Bottom-Up Timing Learning Algorithm). Both of them have the worst case runtime of $O(n^3)$ (sub-quadratic in the average case). They differ only in two aspects. First, HyBUTLA learns the behavior of continuous output signals (based on continuous input signals). For each state of the prefix tree, functions that describe this behavior are approximated, e.g. using regression (Hastie et al., 2008). When two states are merged, their portions of continuous data are combined and functions for the newly created state are learned. The HyBUTLA algorithm is the first hybrid automata learning algorithm (Vodenčarević et al., 2011). In the experiments presented in this paper, the regression method used for learning continuous output functions was multiple linear regression with linear terms.

The second difference between HyBUTLA and BUTLA is that in BUTLA only the status of the changing discrete signals defines a transition, while the status of all other discrete signals is not preserved. In HyBUTLA, the changing discrete signals also trigger a transition, but the transition event includes the status of all other discrete signals. This difference is illustrated in figure 5. If the system has three discrete signals: d_1, d_2, and d_3, the change in d_2 (from 0 to 1) triggers the transition in both algorithms, but the transition event for BUTLA is defined only as $d_2 = 1$, while for HyBUTLA it contains also values of other signals: $\{d_1, d_2, d_3\} = \{0, 1, 0\}$

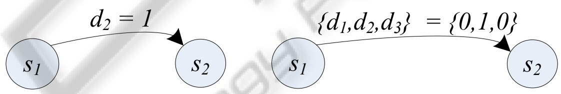


Figure 5: Different event definitions for BUTLA (left) and HyBUTLA (right).

4 CRITERIA FOR ALGORITHM'S EVALUATION

Sections 5 and 6 evaluate the performance of the algorithms using real-world and artificial data. In this section, the used criteria and their importance for the evaluation are outlined.

#states. The number of states is the primary measure of the automaton size. In general, the goal is to obtain the smallest possible model of a system, with the highest possible accuracy.

#merges. The number of successful merges tells how many pairs of states have been merged during learning. It is closely related to the number of states, as the sum: (*#states* + *#merges*) equals to the number of states in the prefix tree. State merging is used to reduce the automaton size, but also to make it more general. Intuitively, the more successful merges, the higher is the generalization ability of the algorithm.

#comparisons. During a merging step, a search procedure is performed in order to find as many compatible states as possible. The more comparisons are made, the higher is the chance that compatible states will be found and merged.

#determinizations. As stated earlier, some algorithms use the top-down, while others use the bottom-up merging order. While merging top-down the large subtrees are encountered, thus the occurrence of non-determinism in the automaton is more frequent. Number of determinizations indicates the portion of non-determinism created during the merging step, that needed to be resolved by the learning algorithm.

size reduction (%). Size reduction is the measure of relative difference between the sizes of the final automaton and the prefix tree. It is calculated as *#merges* / (*#states* + *#merges*). Successful algorithms can achieve high rates of size reduction.

R^2 (%): Averaged coefficient of determination $R^2(\%)$ over the automaton states shows the portion of variability in the continuous data that is accounted for by the regression function used for approximation. Average R^2 can be measured only for the HyBUTLA algorithm and it shows its ability to model the continuous dynamics of the system. In the following experiments, multiple linear regression with linear terms was used as the regression method.

5 EXPERIMENTS USING REAL-WORLD SYSTEMS

5.1 The Lemgo Smart Factory

A first real-world case study was conducted using an exemplary plant called the Lemgo Smart Factory at the Institute Industrial IT in Lemgo, Germany. The Lemgo Smart Factory is a hybrid technical system that is used for storing, transporting, processing and packing bulk materials (e.g. corn). It has a modular design, uses both centralized and distributed automation concepts, and comprises around 250 measurable signals.

Here, the corn processing unit was used as an example. In total, logs from 15 production cycles are available for learning the models. Logs contain the time stamps, 9 discrete (binary) control signals, 9 continuous input signals, and machine's active power as the monitored continuous output variable.

The results are summarized in Table 1. It shows the values for various comparison criteria outlined in section 4. On this dataset, the algorithms had similar performance in the sense of size reduction, number of states and merges. It can be seen that BUTLA and HyBUTLA performed significantly more comparisons than ALERGIA and MDI. This is due to comparing the timing of transitions, in addition to comparing their probabilities. Even with simple method such as multiple linear regression, high R^2 value is achieved. Since ALERGIA and MDI use the top-down merging order, they had to perform more determinizations.

Table 1: Algorithm comparison for Lemgo Smart Factory data.

	ALE	MDI	BUT	HyBUT
#states	9	13	9	15
#merges	80	76	77	88
#comparisons	130	183	429	733
#determiniz.	44	171	21	14
reduction(%)	89.89	85.39	89.53	85.44
$R^2(\%)$	-	-	-	94.54

5.2 The Jowat AG

The Jowat AG with headquarters in Detmold is one of the leading suppliers of industrial adhesives. These are mainly used in woodworking and furniture manufacture, in the paper and packaging industry, the textile industry, the graphic arts, and the automotive industry. The company was founded in 1919 and has manufacturing sites in Germany in Detmold and Zeitz, plus three other producing subsidiaries, the Jowat Corporation in the USA, the Jowat Swiss AG, and the Jowat Manufacturing in Malaysia. The supplier of all adhesive groups is manufacturing approx. 70,000 tons of adhesives per year, with around 790 employees. A global sales structure with 16 Jowat sales organizations plus partner companies is guaranteeing local service with close customer contact.

The data was logged in one of the plants, during production of one product. In total 14 production cycles were logged. The modeled part of the system is the input raw material subsystem, which contains 6 material supply units (smaller containers) connected to a large container where materials are mixed. Recorded discrete variables are 15 valve open signals and their feedbacks (in total 30 discrete variables). The continuous output variable whose dynamics was learned is the large container weight. Continuous input variables are weights of 6 smaller containers and the pressure of the raw material pump. The results of the algorithms' comparison are given in Table 2.

Table 2: Algorithm comparison for Jowat AG data.

	ALE	MDI	BUT	HyBUT
#states	27	16	17	13
#merges	418	429	473	507
#comparisons	1025	605	4526	3576
#determiniz.	348	578	150	111
reduction(%)	93.93	96.4	96.5	97.5
$R^2(\%)$	-	-	-	89.8

The trends are here similar as in Table 1. High number of merges and high size reductions are evident for all algorithms. Again, BUTLA and HyBUTLA do more comparisons, while ALERGIA and MDI perform more determinizations. Also in this moderately larger system, relatively high R^2 of around 90 % is achieved.

6 EXPERIMENTS USING ARTIFICIAL DATA

In this section, learning examples were generated artificially from a given input model. This allows (i) for the generation of arbitrarily complex examples and (ii) for the assessment of the learned model by comparing it to the given input model.

6.1 Convergence Experiments

The experiments given here are conducted to examine how close the learning algorithms can converge to the number of states in the predefined model that generated the learning data. Algorithms use an artificial dataset randomly drawn according to a predefined Reber-like model. It is illustrated in figure 6. Events that trigger transitions are given together with their corresponding transition probabilities. The original Reber model (Reber, 1967) was changed in a way that equal subsequent events do not occur (including transitions that have equal source and destination state), because this cannot appear in production plants.

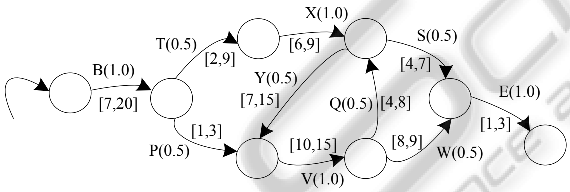


Figure 6: Reber-like automaton.

The number of generated learning examples was 1000. Example length was around 40 samples. In addition to discrete variables that trigger transitions (see figure 6), one continuous output and five continuous input signals were randomly generated according to normal distribution for HyBUTLA experiments. Input and output signals have the mean value and standard deviation 220 ± 22 and 1206 ± 120.6, respectively. Moreover, a randomly generated time interval has been associated with every transition of the automaton. Summarized results for the four algorithms are given in Table 3.
indent Due to the different way the HyBUTLA algorithm defines the transition (see section 3.3), it generated significantly more states in the prefix tree, thus

Table 3: Algorithm comparison for artificial data.

	ALE	MDI	BUT	HyBUT
#states	13	5	11	8
#merges	14	22	16	3733
#comparisons	157	43	82	12081
#determiniz.	7	35	1	249
reduction(%)	51.85	81.48	59.26	99.79

having the high number of merges, comparisons, as well as the high size reduction. However, it converged to the automaton with the exact number of states of the Reber-like automaton. ALERGIA had the worst performance on this task, while MDI and BUTLA had the same deviation from the target automaton.

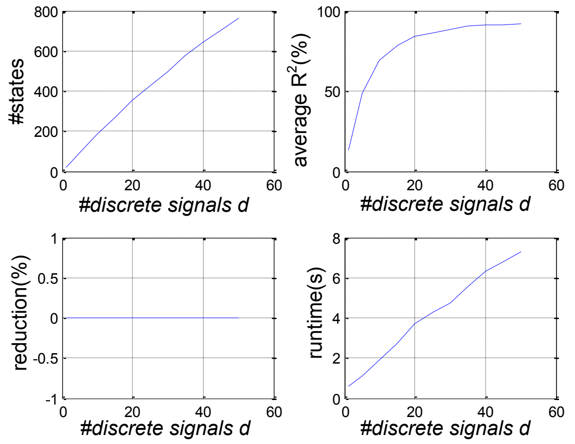
6.2 HyBUTLA Scalability Experiments

The goal of scalability experiments is to evaluate the HyBUTLA algorithm performance, in the presence of the increasing number of two types of signals in the system: discrete and continuous. A series of experiments were conducted by increasing the number of one type of signals, while keeping the number of the other type constant. Results are given graphically for created prefix tree acceptor (PTA) and learned hybrid automaton. Given performance metrics include the number of PTA states, the number of merges, average model coefficient of determination (R^2), size reduction (in relation to PTA size), and learning time.

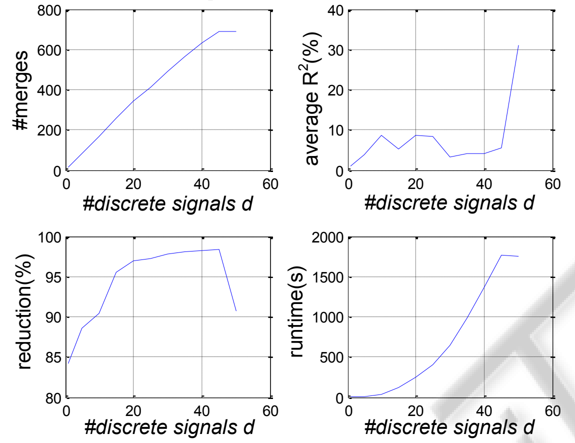
In total, 22 artificial datasets were generated. Each dataset comprises 10 learning examples. The size of each learning example was picked from a range of $[150, 250]$ samples with a random number generator that uses an uniform distribution. Normal distribution was used for generating independent continuous input signals, as well as the output signal. The mean value and standard deviation for input signals is 220 ± 22, and for the output it is 1206 ± 120.6. Discrete signals were generated following an uniform distribution. They represent independent binary variables. Locations and lengths of bit-switches are picked randomly for every signal. Each discrete signal changes two times per learning example. For easier reading, the number of continuous signals will be denoted by c, and the number of discrete signals by d.

6.2.1 Analysis with Constant Number of Continuous Signals

In these experiments, d was increased from 1 to 50, while c was kept constant at value 5. Figure 7(a) shows the results for PTA. Its number of states increases linearly with d. This makes the portions of continuous data in each state smaller and easier to approximate. Therefore, R^2 rises with d. The re-

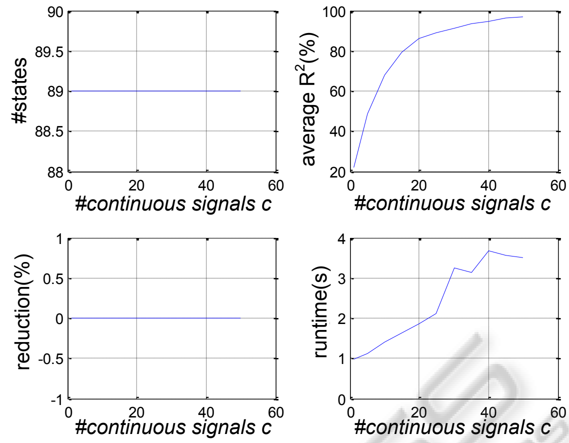


(a) PTA performance metrics for $c = 5$.

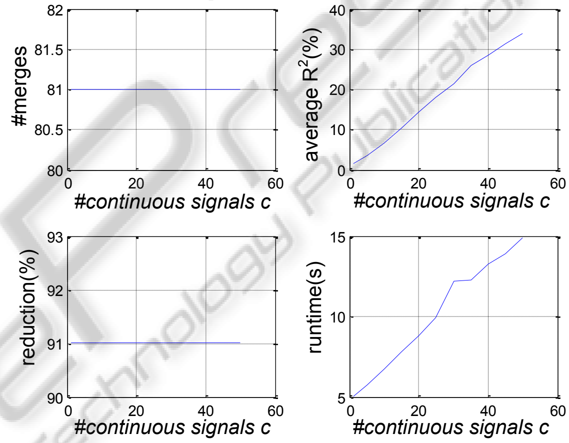


(b) Learned automaton performance metrics for $c = 5$.

Figure 7: Results for constant c.



(a) PTA performance metrics for $d = 5$.



(b) Learned automaton performance metrics for $d = 5$.

Figure 8: Results for constant d.

duction is always zero for PTA experiments. PTA construction time is approximately linear in d. Figure 7(b) shows results for merged PTAs (learned automata). The number of merges grows (approximately linearly) with d. As in the case of PTA, for higher d, higher R^2 is obtained. In general, both size reduction and learning time grow with d. The measured learning time is subquadratic. Since both PTA size and the number of merges grow linearly with d (i.e. with the size of dominantly discrete system), the size of the final learned model will also grow. Experiments are done for more values of constant c, and the results are similar (not shown due to space restrictions).

6.2.2 Analysis with Constant Number of Discrete Signals

Here c was increased from 1 to 50, while d was kept at value 5. Figure 8(a) gives the results for PTA. Since states are derived based on changes in discrete signals only, the increase in c does not generate the new ones. However, the growth of c increases the average R^2. Like before, reduction is zero, while learning time grows with c. Results for the learned automata are shown in figure 8(b). Merging criteria do not include continuous signals, thus the number of merges is constant in c (and so is the reduction). Both R^2 and learning time grow approximately linearly with c. Since the model size remains unchanged, one should always try to log and use as many input signals as possible since more predictors approximate the output signal better. More experiments were conducted for different values of constant d and similar trends are observed (not shown due to space restrictions).

7 DISCUSSION

Based on the analysis presented in this paper, several general observations for learning behavior models for technical systems are given. These observations could be used in practice by non-experts. Table 4 gives the overview of stochastic automata formalisms suitable for modeling the three types of technical systems: non-timed Discrete Event System (nDES), timed Discrete Event System (tDES), and a Hybrid System (HS). Table also gives the algorithms that can successfully learn such automata.

Table 4: Systems, models and learning algorithms.

System	nDES	tDES	HS
Model	SDFA	SDTA	SDHA
Algorithm	ALERGIA MDI	BUTLA	HyBUTLA

Analyses in real-world systems have shown similar trends for smaller exemplary Lemgo Smart Factory and moderately bigger Jowat AG datasets. It can be observed that all four algorithms achieved similar and relatively high size reduction rates in both cases, which demonstrates their ability to produce small and more general models. Obtained models that have 9–27 states can be easily visualized, understood and interpreted by humans, thus they provide a good insight in the system's modes of operation and its behavior in general. Getting such insight by using the prefix trees with hundreds of states would not be possible. It should be noted that the HyBUTLA algorithm was able to create models with average R^2 of around 90% for both systems using relatively simple regression method such as multiple linear regression. These models represent the continuous dynamics of the systems quite well. Furthermore, it can be seen that the bottom-up algorithms (BUTLA and HyBUTLA) perform more thorough search for compatible states, as they make more comparisons of the states. Intuitively, this advantage is payed with their increased runtime. Top-down algorithms create more non-determinism in the model that they need to resolve.

The convergence experiments on artificial data given in section 6.1 have demonstrated that the algorithms can converge either to the exact or close to the number of states of the predefined model that generated the learning data. Since most real-world production systems are hybrid systems, a special attention was devoted to the HyBUTLA algorithm in section 6.2. Based on the conducted experiments with changing number of discrete (d), and continuous (c) signals, further observations are derived and summarized in table 5. For dominantly discrete systems, one can expect to obtain large PTAs, but at the same time to benefit from merging in the sense of size reduction. Very small models (high reduction rates) could be obtained. Unfortunately, this typically produces lower accuracy of approximating continuous output signals (low average R^2). For dominantly continuous systems, the situation is converse. With smaller d, PTAs of the small size are obtained. Larger c does not influence neither the PTA size, nor the number of merges. Small d enables very few or no merges, thus merging does not bring significant benefit in modeling such systems in the sense of size reduction. However, typically higher average R^2 values can be obtained.

Table 5: Observations for modeling hybrid systems with HyBUTLA algorithm.

Dominantly discrete system (larger d, smaller c)	Dominantly continuous system (smaller d, larger c)
larger PTA many merges higher size reduction lower average R^2	smaller PTA fewer or no merges lower size reduction higher average R^2

8 CONCLUSIONS

In order to tackle the drawbacks of manual modeling of technical systems, several algorithms can be used for learning behavior models automatically from logged data. This paper focused on four such algorithms which learn models using the formalism of stochastic finite automata. Automata can represent non-timed and timed discrete event systems, as well as the hybrid systems. The usability of the algorithms ALERGIA, MDI, BUTLA and HyBUTLA has been evaluated and compared in real-world as well as in an artificial data settings. In general, all four algorithms have produced small and tractable models, which provide an easy and good insight in the underlying behavior of the corresponding technical systems. The paper also gives several general observations for applying such algorithms to various types of technical systems.

This paper provided a comparative empirical analyses of the aforementioned algorithms. The future work will mainly include theoretical comparisons.

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