Digital Self-tuning Control for Pressure Process

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Abstract: Two digital control systems - Self-tuning PID (Proportional-Integral-Derivative) Control and Predictor-based self-tuning control with constraints - for the continuous-time pressure process control are presented in this paper. The digital self-tuning PID control with optimization of closed-loop parameters and sampling period is proposed. The multidimensional optimization problem of closed-loop parameters and sampling period is solved by subcomponent search method that enables dividing the problem to one-dimensional optimization problems. The golden section search is adjusted to solve those – one-dimensional - optimization problems. The predictor-based self-tuning control with constraints is adapted for both minimum-phase and nonminimum-phase process models. The control quality of pressure process of both control systems has been experimentally investigated. The results of experimental analysis demonstrate that the digital self-tuning PID control with optimization is more efficient as compared to predictive-based self-tuning control with constraints for pressure process.

1 INTRODUCTION

At present, various physical nature processes are still continuous-time processes, but are frequently controlled by digital controllers (Isermann, 1991; Åström and Wittenmark, 1997; Bobál, et al, 2005). The digital PID (proportional-integral-derivative) control laws are the most common for such processes (Åström and Hagglund, 1995; 2001; Levine 1999). The PID controllers are so widely used for its easiness to apply and generally provides sufficient control quality if it is properly tuned.

For the digital PID control based on digital self-tuning PID controllers the selection of suitable closed-loop parameters (Vu, et al, 2007; Kosorus, et al, 2012) and proper sampling period (Boucher, et al, 1989; Isermann, 1991; Åström and Wittenmark, 1997; Levine, 2011) is substantial since directly influences the control quality of the process. Furthermore, the determination of closed-loop parameters and sampling period is not straightforward, at the design stage of the control.

The digital self-tuning PID control system with on-line identification and optimization of closed-loop parameters and sampling period is developed for pressure process control (Liaucius, et al, 2011; Liaucius and Kaminskas, 2012a) in this paper.

As an alternative to self-tuning PID control for pressure process, the predictor-based self-tuning control with constraints (Kaminskas, 2007) is analysed. This control method has been modified for both minimum-phase and nonminimum-phase process models. The results of experimental analysis of both control approaches are presented.

2 THE PRESSURE PROCESS

The scheme of pressure process is depicted in Figure 1.

The process consists of four main components: combined air inlet and outlet tanks, two air chambers and two tubes with balls in them. The air from the inlet tank flows to air channels through air chambers and leaves the equipment through the upper outlet tank. The distance to balls is measured using ultrasound distance sensors. The fans are used to create pressure in the air channels in order to lift the balls in tubes. The air chambers are utilized for the purpose to stabilize oscillations of the pressure in each tube. The momentum of the fan, the inductance of the fan motor, air turbulence in the tube leads to complex physics governing ball behaviour. Slightly different weights of the balls and the location of the

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air feeding vent additionally impact the behaviour of ball in the tubes.

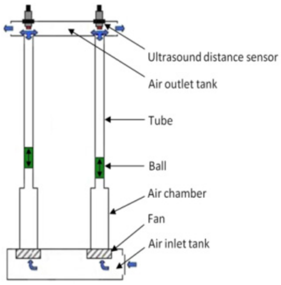

Figure 1: The scheme of the pressure process.

The control signals (inputs) of the process are the voltage values for each fan from the range 0 to 10V. The intermediate values of voltage affect the power of the fan proportionately. The control responses (outputs) are the distances between the balls and the bottom of their tubes from the range 20 to 90 in centimetres. The control problem is to regulate the speed of the fan supplying the air into the tube so as to keep the ball suspended at some pre-determined level in the tube.

3 SELF-TUNING PID CONTROL WITH OPTIMIZATION

The mathematical model of the process is necessary in order to design the digital PID control system with on-line identification. Each tube of the process is defined by discrete linear second order difference equations, i.e.

$$A^{(i)}\left(z^{-1}\right)y_t^{(i)} = B^{(i)}\left(z^{-1}\right)u_t^{(i)} + \xi_t^{(i)}, \tag{1}$$

$$\begin{aligned} A^{(i)}\left(z^{-1}\right) &= 1 + a_1^{(i)} z^{-1} + a_2^{(i)} z^{-2}, \\ B^{(i)}\left(z^{-1}\right) &= b_1^{(i)} z^{-1} + b_2^{(i)} z^{-2}, \end{aligned} \tag{2}$$

where $A^{(i)}(z^{-1})$, $B^{(i)}(z^{-1})$ are the model polynomials, $i = 1, 2$ is the number of the tube of pressure process, $y_t^{(i)} = y^{(i)}(tT_0)$, $u_t^{(i)} = u^{(i)}(tT_0)$ - output and input signals with sampling period T_0 respectively, $\xi_t^{(i)}$ - a white noise of the ith tube with a zero mean and finite variance and z^{-1} is the backward-shift operator ($z^{-1} y_t^{(i)} = y_{t-1}^{(i)}$).

Unknown model parameters of the ith tube

$$\Theta^{(i)T} = \left[a_1^{(i)}, a_2^{(i)}, b_1^{(i)}, b_2^{(i)}\right], \tag{3}$$

are estimated by recursive least squares algorithm with forgetting factor (Liaucius and Kaminskas, 2012a)

$$\hat{\Theta}_t^{(i)} = \begin{cases} \hat{\Theta}_{t-1}^{(i)}, & if \left|e_t^{(i)}\right| \le \delta_e^{(i)} \text{or} \left|z_j^{(i)}\right| < 1 \\ \hat{\Theta}_{t-1}^{(i)} + \dfrac{C_t^{(i)} \varphi_{t-1}^{(i)}}{1 + \eta_t^{(i)}} \hat{\varepsilon}_t^{(i)}, & otherwise \end{cases} \tag{4}$$

$$\begin{aligned} \varphi_t^{(i)T} &= \left[-y_{t-1}^{(i)}, -y_{t-2}^{(i)}, u_{t-1}^{(i)}, u_{t-2}^{(i)}\right], \\ \eta_t^{(i)} &= \varphi_{t-1}^{(i)T} C_t^{(i)} \varphi_{t-1}^{(i)}, \\ \hat{\varepsilon}_t^{(i)} &= y_t^{(i)} - \hat{\Theta}_{t-1}^{(i)T} \varphi_{t-1}^{(i)}, \end{aligned} \tag{5}$$

$$C_t^{(i)} = \begin{cases} C_{t-1}^{(i)} + \dfrac{C_{t-1}^{(i)} \varphi_{t-1}^{(i)} \varphi_{t-1}^{(i)T} C_{t-1}^{(i)}}{\lambda^{-1}{}_t^{(i)} + \eta_t^{(i)}} \hat{\varepsilon}_t^{(i)}, & if\ \eta_t^{(i)} > 0 \\ C_{t-1}^{(i)}, & if\ \eta_t^{(i)} = 0 \end{cases} \tag{6}$$

$$\lambda_t^{(i)} = \varphi_t^{(i)} - \frac{1 - \varphi_t^{(i)}}{\eta_{t-1}^{(i)}}, \tag{7}$$

$$e_t^{(i)} = y_t^{(i)*} - y_t^{(i)}, \tag{8}$$

where $e_t^{(i)}$ is the control error, $\delta_e^{(i)}$ - a constant that defines the admissible interval of control error $e_t^{(i)}$ and $z_j^{(i)}$, $j = 1, 2$ are the roots of polynomial $\hat{A}_t^{(i)}(z^{-1})$ and $y_t^{(i)*}$ is a reference signal of the ith tube.

Applying on-line identification algorithm (4), the estimates of model parameters are updated only if the value of $e_t^{(i)}$ is outside of the admissible interval defined by $\delta_e^{(i)}$ and the current on-line model is stable.

The results of on-line identification of models are used to the digital self-tuning PID controllers (Ortega and Kelly, 1984; Bobál, et al, 2005), which are defined as follows:

$$S_t^{(i)}\left(z^{-1}\right)u_t^{(i)} = \beta_t^{(i)} e_t^{(i)} - R_t^{(i)}\left(z^{-1}\right)y_t^{(i)}, \tag{9}$$

$$\begin{aligned} S_t^{(i)}\left(z^{-1}\right) &= \left(1 - z^{-1}\right)\left(1 + \gamma_t^{(i)}\right), \\ R_t^{(i)}\left(z^{-1}\right) &= r_{0t}^{(i)} - \left(r_{0t}^{(i)} + r_{2t}^{(i)}\right)z^{-1} + r_{2t}^{(i)} z^{-2}, \end{aligned} \tag{10}$$

where $e_t^{(i)}$ is the control error (8), $S_t^{(i)}$, $R_t^{(i)}$ are the polynomials and $\beta_t^{(i)}, \gamma_t^{(i)}, r_{0t}^{(i)}, r_{2t}^{(i)}$ are the parameters of the controller of the ith tube that are calculated by expressions

$$\beta_t^{(i)} = \frac{1}{\hat{b}_{1t}^{(i)}}\left(d_1^{(i)} + 1 - \hat{a}_{1t}^{(i)} - \gamma_t^{(i)} - \hat{b}_{1t}^{(i)} r_{0t}^{(i)}\right), \tag{11}$$

$$\gamma_t^{(i)} = r_{2t}^{(i)} \frac{\hat{b}_{2t}^{(i)}}{\hat{a}_{2t}^{(i)}}, \; r_{0t}^{(i)} = r_{2t}^{(i)}\left(\frac{\hat{b}_{1t}^{(i)}}{\hat{b}_{2t}^{(i)}} - \frac{\hat{a}_{1t}^{(i)}}{\hat{a}_{2t}^{(i)}}\right) - \frac{\hat{a}_{2t}^{(i)}}{\hat{b}_{2t}^{(i)}}, \tag{12}$$

$$\begin{aligned} r_{2t}'^{(i)} &= \hat{a}_{2t}^{(i)} \hat{b}_{2t}^{(i)}\left[\hat{a}_{1t}^{(i)}\left(\hat{b}_{1t}^{(i)} + \hat{b}_{2t}^{(i)}\right) + \hat{b}_{1t}^{(i)}\left(d_{2t}^{(i)} - \hat{a}_{2t}^{(i)}\right)\right], \\ r_{2t}''^{(i)} &= -\hat{a}_{2t}^{(i)}\left[\left(\hat{b}_{2t}^{(i)}\right)^2\left(d_{1t}^{(i)} + 1\right) + \hat{a}_{2t}^{(i)}\left(\hat{b}_{1t}^{(i)}\right)^2\right], \end{aligned} \tag{13}$$

$$r_{2t}^{(i)} = \frac{r_{2t}'^{(i)} + r_{2t}''^{(i)}}{\left[\hat{b}_{1t}^{(i)} + \hat{b}_{2t}^{(i)}\right]\left[\hat{a}_{1t}^{(i)}\hat{b}_{1t}^{(i)}\hat{b}_{2t}^{(i)} - \hat{a}_{2t}^{(i)}\left(\hat{b}_{1t}^{(i)}\right)^2 - \left(\hat{b}_{2t}^{(i)}\right)^2\right]}, \tag{14}$$

which are obtained by solving the system

$$\begin{bmatrix} \hat{b}_{1t}^{(i)} & 0 & \hat{b}_{1t}^{(i)} & 1 \\ \hat{b}_{2t}^{(i)} - \hat{b}_{1t}^{(i)} & -\hat{b}_{1t}^{(i)} & \hat{b}_{2t}^{(i)} & \hat{a}_{1t}^{(i)} - 1 \\ \hat{b}_{2t}^{(i)} & \hat{b}_{2t}^{(i)} - \hat{b}_{1t}^{(i)} & 0 & \hat{a}_{2t}^{(i)} - \hat{a}_{1t}^{(i)} \\ 0 & \hat{b}_{2t}^{(i)} & 0 & -\hat{a}_{2t}^{(i)} \end{bmatrix} \begin{bmatrix} r_{0t}^{(i)} \\ r_{2t}^{(i)} \\ \beta_t^{(i)} \\ \gamma_t^{(i)} \end{bmatrix} = \begin{bmatrix} d_1^{(i)} + 1 - \hat{a}_{1t}^{(i)} \\ d_2^{(i)} + \hat{a}_{1t}^{(i)} - \hat{a}_{2t}^{(i)} \\ d_3^{(i)} - \hat{a}_{2t}^{(i)} \\ d_4^{(i)} \end{bmatrix} \tag{15}$$

where

$$d_1^{(i)} = \begin{cases} -2\exp(-\zeta\omega T_0)\cos\left(\omega T_0\sqrt{1-\zeta^2}\right), & if\ \zeta \le 1 \\ -2\exp(-\zeta\omega T_0)\cos\left(\omega T_0\sqrt{\zeta^2-1}\right), & if\ \zeta > 1 \end{cases}, \tag{16}$$

$$d_2^{(i)} = \exp(-2\zeta\omega T_0),\ d_3^{(i)} = d_4^{(i)} = 0,$$

ω is the natural frequency of oscillation, ζ is the damping factor of characteristic equation of continuous-time closed-loop system

$$s^2 + 2\zeta\omega s + \omega^2 = 0. \tag{17}$$

The scheme of the digital PID controller is depicted in Figure 2. Such structure of PID controller is more effective as compared to the structure of conventional PID controller for pressure process control (Liaucius, et al, 2011).

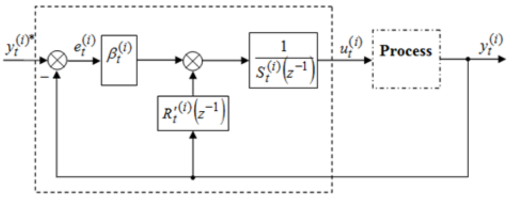

Figure 2: The scheme of the digital PID controller.

The required control response of control system with digital self-tuning PID controllers can be achieved by the selection of proper closed-loop parameters (ω, ζ) and sampling period (T_0). Therefore, is reasonable to find such values (ω^*, ζ^*, T_0^*) of these parameters that minimize control quality criterion

$$\omega^*, \zeta^*, T_0^*:\ Q(\omega, \zeta, T_0) \rightarrow \min_{\omega, \zeta, T_0}, \tag{18}$$

$$Q(\omega,\zeta,T_0) = \frac{1}{N}\sum_{t=1}^{N}\left\{\left[\left(y_t^{(1)*} - y_t^{(1)}\right)^2 + \left(y_t^{(2)*} - y_t^{(2)}\right)^2\right] + \right.$$
$$\left. + \rho\left[\left(u_t^{(1)} - u_{t-1}^{(1)}\right)^2 + \left(u_t^{(2)*} - u_{t-1}^{(2)}\right)^2\right]\right\}, \tag{19}$$

where N is the number of observations, $y_t^{(i)*}$ is the reference signal of the ith tube, $\rho \ge 0$ is a weight coefficient. The criterion consists of two parts: the first part estimates the variance of control error of each tube, the second - characterizes the variance of control signal change of each tube.

The scheme of the digital self-tuning control of the pressure process is depicted in Figure 3.

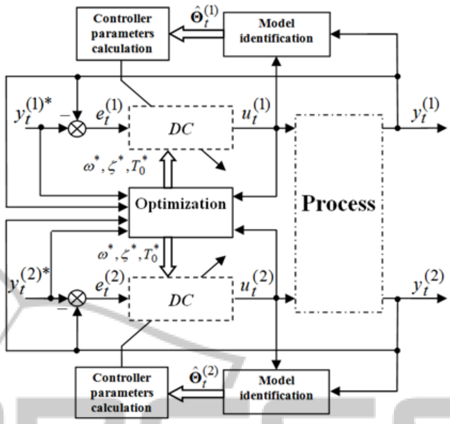

Figure 3: The scheme of the digital self-tuning control of the pressure process.

The optimization problem (18) is solved as follows. The optimal sampling period T_0^* is obtained by

$$T_0^*:\ J_T(T_0) \rightarrow \min_{\omega, \zeta, T_0}, \tag{20}$$

$$J_T(T_0) = \min_{\omega, \zeta}\left(Q(\omega, \zeta, T_0)\right). \tag{21}$$

The optimal closed-loop parameters ω^*, ζ^* are obtained by

$$\omega^*, \zeta^*:\ Q\left(\omega, \zeta, T_0^*\right) \rightarrow \min_{\omega, \zeta}. \tag{22}$$

In order to solve optimization problem of (20), a technique of one dimensional search is used. The most popular algorithms of this technique are golden section and quadratic interpolation (Kaminskas, 1982). The results of experimental analysis (Liaucius and Kaminskas, 2012b) showed that golden section algorithm for pressure process is more effective.

Golden section algorithm is related with an initial uncertainty interval

$$[T_{01}, T_{04}] \subseteq [0, T_{0\max}], \tag{23}$$

reduction to the interval

$$\left[T_{01}^{(L)}, T_{04}^{(L)}\right],\quad if\ \left(T_{04}^{(L)} - T_{01}^{(L)}\right) \le \Delta T_0, \tag{24}$$

where its length is not longer than desired ΔT_0 and with a function (21) minimum inside. For this purpose, two new values of sampling period T_0 are chosen by

$$
\begin{aligned}
T_{02}^{(l)} &= 0.382 \times \left(T_{04}^{(l)} - T_{01}^{(l)}\right) + T_{01}^{(l)} \\
T_{03}^{(l)} &= 0.618 \times \left(T_{04}^{(l)} - T_{01}^{(l)}\right) + T_{01}^{(l)}
\end{aligned}, \ l = 1, 2, \cdots, L \tag{25}
$$

in the search procedure and a new uncertainty interval is then defined by the rule

$$
\begin{cases}
\left[T_{01}^{(l+1)}, T_{04}^{(l+1)}\right] = \left[T_{01}^{(l)}, T_{03}^{(l)}\right], & \text{if } J_T\left(T_{02}^{(l)}\right) < J_T\left(T_{03}^{(l)}\right) \\
\left[T_{01}^{(l+1)}, T_{04}^{(l+1)}\right] = \left[T_{02}^{(l)}, T_{04}^{(l)}\right], & \text{otherwise}
\end{cases}. \tag{26}
$$

Then the optimal sampling period T_0^* is obtained by

$$
T_0^* = \begin{cases} T_{02}^{(L-1)}, & \text{if } J_T\left(T_{02}^{(L-1)}\right) < J_T\left(T_{03}^{(L-1)}\right) \\ T_{03}^{(L-1)}, & \text{otherwise} \end{cases}. \tag{27}
$$

The subcomponent optimization method (Kaminskas, 1982) is applied to solve the optimization problem of (21):

$$
\begin{aligned}
\omega^{(j)}: &\quad J_\omega(\omega) \to \min_\omega \\
\zeta^{(j)}: &\quad J_\zeta(\zeta) \to \min_\zeta
\end{aligned}, \ j = 1, 2, \cdots \tag{28}
$$

where

$$
\begin{aligned}
J_\omega(\omega) &= Q\left(\omega, \zeta^{(j-1)}, T_0^{(l+1)}\right) \\
J_\zeta(\zeta) &= Q\left(\omega^{(j)}, \zeta, T_0^{(l+1)}\right)
\end{aligned}, \tag{29}
$$

$T_0^{(l+1)}$ is a T_0 value, obtained by golden section algorithm, where the new value of (21) must be calculated, i.e.:

$$
T_0^{(l+1)} = \begin{cases} T_{02}^{(l+1)}, & \text{if } J_T\left(T_{02}^{(l)}\right) < J_T\left(T_{03}^{(l)}\right) \\ T_{03}^{(l+1)}, & \text{otherwise} \end{cases}. \tag{30}
$$

Each of the optimization problems (28) are solved by golden section search analogously to (23)-(27).

4 PREDICTOR-BASED SELF-TUNING CONTROL WITH CONSTRAINTS

Since the pressure process is defined by the model (1)-(2), the control law of predictor-based self-tuning controller with constraints (Kaminskas, 2007) for the ith tube is described by equations

$$
u_{t+1}^{(i)} = \begin{cases} \min\left\{u_{\max}^{(i)}, u_t^{(i)} + \delta_t^{(i)}, \tilde{u}_{t+1}^{(i)}\right\}, & \text{if } \tilde{u}_{t+1}^{(i)} \ge u_t^{(i)} \\ \max\left\{u_{\min}^{(i)}, u_t^{(i)} - \delta_t^{(i)}, \tilde{u}_{t+1}^{(i)}\right\}, & \text{otherwise} \end{cases}, \tag{31}
$$

$$
\tilde{B}_t^{(i)}\left(z^{-1}\right)\tilde{u}_{t+1}^{(i)} = \left[-L_t^{(i)}\left(z^{-1}\right)y_t^{(i)} + zy_{t+1}^{(i)*}\right], \tag{32}
$$

$$
\tilde{B}_t^{(i)}\left(z^{-1}\right) = \tilde{b}_{0t}^{(i)} + \tilde{b}_{1t}^{(i)} z^{-1} + \tilde{b}_{2t}^{(i)} z^{-2}, \tag{33}
$$

$$
\begin{aligned}
L_t^{(i)}\left(z^{-1}\right) &= l_{0t}^{(i)} + l_{1t}^{(i)} z^{-1} = \\
&= \left[\left(\hat{a}_{1t}^{(i)}\right)^2 - \hat{a}_{2t}^{(i)}\right] + \left[\hat{a}_{1t}^{(i)}\hat{a}_{2t}^{(i)}\right] z^{-1},
\end{aligned} \tag{34}
$$

where

$$
\tilde{b}_{0t}^{(i)} = \begin{cases}
\hat{b}_{1t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1 \\
\hat{b}_{2t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1 \\
-\hat{a}_{1t}^{(i)}\hat{b}_{1t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1 \\
-\hat{a}_{1t}^{(i)}\hat{b}_{2t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1
\end{cases}, \tag{35}
$$

$$
\tilde{b}_{1t}^{(i)} = \begin{cases}
\hat{b}_{2t}^{(i)} - \hat{a}_{1t}^{(i)}\hat{b}_{1t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1 \\
\hat{b}_{1t}^{(i)} - \hat{a}_{1t}^{(i)}\hat{b}_{2t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1 \\
\hat{b}_{1t}^{(i)} - \hat{a}_{1t}^{(i)}\hat{b}_{2t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1 \\
\hat{b}_{2t}^{(i)} - \hat{a}_{1t}^{(i)}\hat{b}_{1t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1
\end{cases}, \tag{36}
$$

$$
\tilde{b}_{2t}^{(i)} = \begin{cases}
-\hat{a}_{1t}^{(i)}\hat{b}_{2t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1 \\
-\hat{a}_{1t}^{(i)}\hat{b}_{1t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| \le 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1 \\
\hat{b}_{2t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| \le 1 \\
\hat{b}_{1t}^{(i)}, & \text{if } \left|\hat{a}_{1t}^{(i)}\right| > 1 \text{ and } \left|\hat{b}_{2t}^{(i)} / \hat{b}_{1t}^{(i)}\right| > 1
\end{cases}, \tag{37}
$$

$u_{\min}^{(i)}$, $u_{\max}^{(i)}$ are the control signal boundaries of the ith tube, $\delta_t^{(i)} > 0$ is the restriction value for the change rate of the control signal, z is a forward-shift operator ($zy_t^{(i)*} = y_{t+1}^{(i)*}$). The coefficients of polynomial $L_t^{(i)}\left(z^{-1}\right)$ are found from equation

$$
1 = \hat{A}_t^{(i)}\left(z^{-1}\right)F_t^{(i)}\left(z^{-1}\right) + z^{-2}L_t^{(i)}\left(z^{-1}\right), \tag{38}
$$

where

$$
F_t^{(i)}\left(z^{-1}\right) = 1 + f_{1t}^{(i)} z^{-1} = 1 - \hat{a}_{1t}^{(i)} z^{-1}. \tag{39}
$$

The coefficients (35)-(37) are obtained by applying factorization method (Åström and Wittenmark, 1980) to polynomial

$$
\tilde{B}_t^{(i)}\left(z^{-1}\right) = \bar{B}_t^{(i)}\left(z^{-1}\right)F_t^{(i)}\left(z^{-1}\right). \tag{40}
$$

where

$$
\bar{B}_t^{(i)}\left(z^{-1}\right) = \hat{b}_{1t}^{(i)} + \hat{b}_{2t}^{(i)} z^{-1}. \tag{41}
$$

In each expression of the coefficients (35)-(37), the first and the third conditions correspond to minimum-phase model, while the second and the fourth - to nonminimum-phase.

The scheme of predictor-based self-tuning controller with constraints is illustrated in Figure 4.

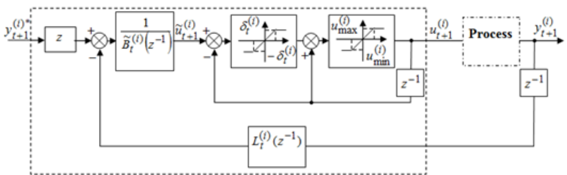

Figure 4: The scheme of predictor-based self-tuning controller with constraints.

In on-line identification algorithm (4) for control (31)-(37), $z_j^{(i)}$ are the characteristic polynomial of closed-loop

$$D_t^{(i)}\left(z^{-1}\right)=\overline{B}_t^{(i)}\left(z^{-1}\right)+\hat{A}_t^{(i)}\left(z^{-1}\right)F_t^{(i)}\left(z^{-1}\right)\times \times\left[\overline{B}{}^{\bullet}{}_t^{(i)}\left(z^{-1}\right)-\overline{B}_t^{(i)}\left(z^{-1}\right)\right], \quad if\left|\hat{a}_{1t}^{(i)}\right|\le 1 \; and \; \left|\hat{b}_{2t}^{(i)}/\hat{b}_{1t}^{(i)}\right|>1, \tag{42}$$

$$D_t^{(i)}\left(z^{-1}\right)=1+\hat{A}_t^{(i)}\left(z^{-1}\right)\times\left[F^{\bullet}{}_t^{(i)}\left(z^{-1}\right)-F_t^{(i)}\left(z^{-1}\right)\right], \quad if\left|\hat{a}_{1t}^{(i)}\right|>1 \; and \; \left|\hat{b}_{2t}^{(i)}/\hat{b}_{1t}^{(i)}\right|\le 1, \tag{43}$$

$$D_t^{(i)}\left(z^{-1}\right)=\overline{B}_t^{(i)}\left(z^{-1}\right)+\hat{A}_t^{(i)}\left(z^{-1}\right)\times \times\left[\overline{B}{}^{\bullet}{}_t^{(i)}\left(z^{-1}\right)F^{\bullet}{}_t^{(i)}\left(z^{-1}\right)-\overline{B}_t^{(i)}\left(z^{-1}\right)F_t^{(i)}\left(z^{-1}\right)\right], \quad if\left|\hat{a}_{1t}^{(i)}\right|>1 \; and \; \left|\hat{b}_{2t}^{(i)}/\hat{b}_{1t}^{(i)}\right|>1, \tag{44}$$

roots, where

$$\overline{B}{}^{\bullet}{}_t^{(i)}\left(z^{-1}\right)=\hat{b}_{2t}^{(i)}+\hat{b}_{1t}^{(i)}z^{-1}, \quad F^{\bullet}{}_t^{(i)}\left(z^{-1}\right)=-\hat{a}_{1t}^{(i)}+z^{-1}, \tag{45}$$

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5 EXPERIMENTAL ANALYSIS

The realization of digital self-tuning control is performed by employing the industrial Beckhoff BK9000 programmable logic controller (PLC). The PLC controller is configured and controlled by TwinCat software.

The experimental analysis has been performed for 3 cases: digital self-tuning PID control by algorithms (4), (9), (18) and digital self-tuning PID control with unoptimal closed-loop parameters and optimal sampling period and digital predictor-based self-tuning control with constraints.

The predefined conditions of experimental analysis of self-tuning PID control are as follows: the initial uncertainty intervals of closed-loop parameters $\omega\in[0.02\text{rad/s}, 1.0\text{rad/s}]$, $\zeta\in[0.02, 2.0]$, and sampling period $T_0\in[0.04\text{s}, 0.2\text{s}]$ have been selected with $\Delta T_0=0.06\text{s}$. The optimization (30) is started with an initial damping factor $\zeta^{(0)}=1.0$. The control signal boundaries $u_{\min}^{(i)}=3.5\text{V}$, $u_{\max}^{(i)}=10\text{V}$, with the change rate $\delta_t^{(i)}=10\text{V}$ of the control law (31)-(32) for the ith tube is used.

The same step-shape reference signal for both tubes has been applied with repeatable values of 75 and 40. The observation time of each signal is 1000 second, but only the last 800 values are used in criterion calculations, in order to eliminate the influence of initial adaptation process.

The searches of sampling period and closed-loop parameters by golden section algorithm are depicted in Figure 5 and Figure 6.

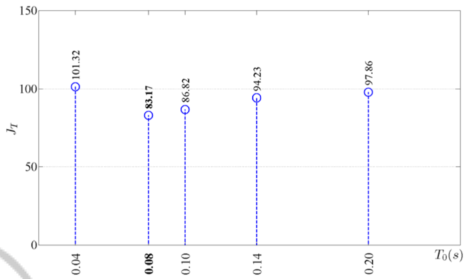

Figure 5: Search of sampling period (20) by golden section algorithm.

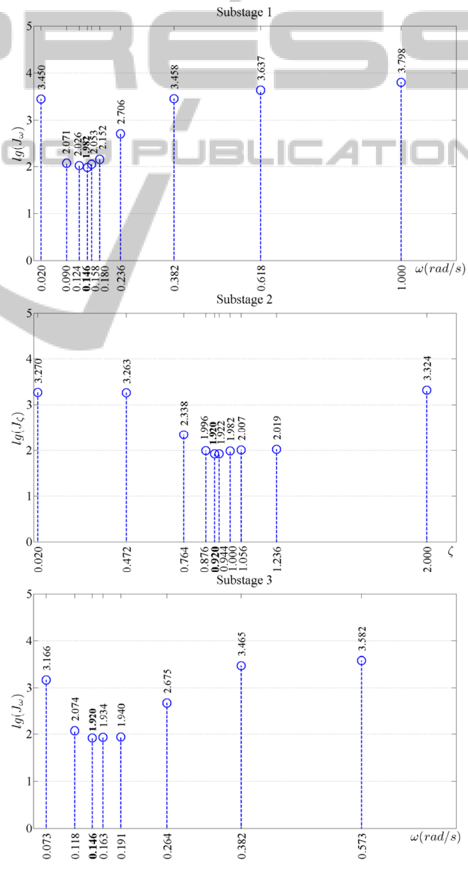

Figure 6: Subcomponent search of closed-loop parameters (28) by golden section algorithm with optimal sampling period $T_0^*=0.08\text{s}$.

Figure 5 demonstrates that the optimal sampling period obtained by golden section algorithm is with $T_0^*=0.08\text{s}$. and Figure 6 depicts the search of closed-loop parameters with that value of sampling period.

Notice that criterion, with a fixed sampling period to its optimal value, is optimized within 3 steps of subcomponent search procedure by golden section algorithm thus is necessary to find only 28 criterion values.

The optimization results by golden section algorithm have shown that optimal closed-loop parameters and optimal sampling period are $\omega^* = 0.146\text{rad/s}$, $\zeta^* = 0.92$, $T_0^* = 0.08\text{s}$. with minimal criterion value $Q^* = 83.17$, when $\rho = 9$ ($Q^* = 81.90$, when $\rho = 0$).

The control performance of digital self-tuning PID control of pressure process with optimal closed-loop parameters and optimal sampling period is demonstrated in Figure 7. The on-line identification of model parameters of this case is illustrated in Figure 8.

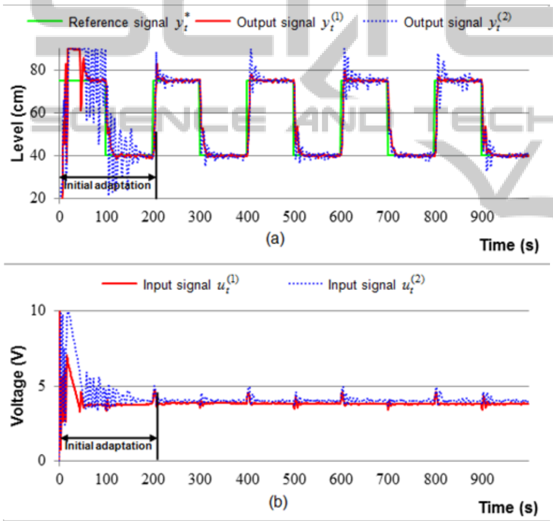

Figure 7: Control performance of self-tuning PID control of pressure process with optimal closed-loop parameters and optimal sampling period - $\omega^* = 0.146\text{rad/s}$, $\zeta^* = 0.92$, $T_0^* = 0.08\text{s}$., (a) – reference and output signals, (b) – control signals.

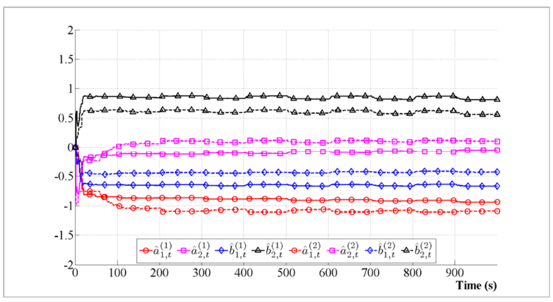

Figure 8: On-line identification of model parameters – self-tuning PID control with optimization.

If we select slightly shifted values from optimal closed-loop parameters ($\omega = 0.174\text{rad/s}$, $\zeta = 1.0$) with sampling period remaining unchanged, the control performance by self-tuning PID control (Figure 9) is degraded - the steady state error is significantly increased and the variance of control signals change are also raised. The on-line identification of model parameters of this case is illustrated in Figure 10.

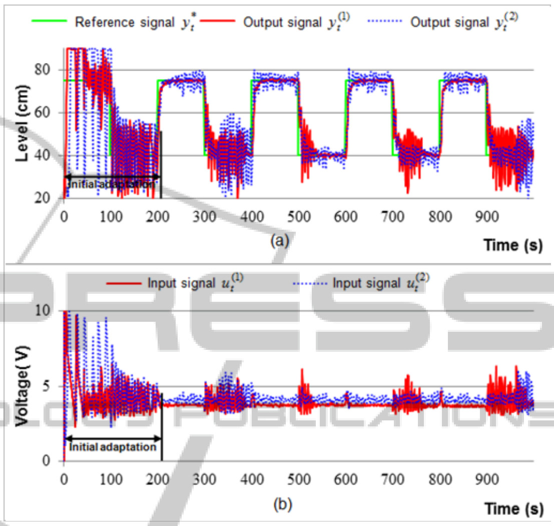

Figure 9: Control performance of self-tuning PID control of pressure process with closed-loop parameters and sampling period - $\omega = 0.174\text{rad/s}$, $\zeta = 1.0$, $T_0^* = 0.08\text{s}$., (a) – reference and output signals, (b) – control signals.

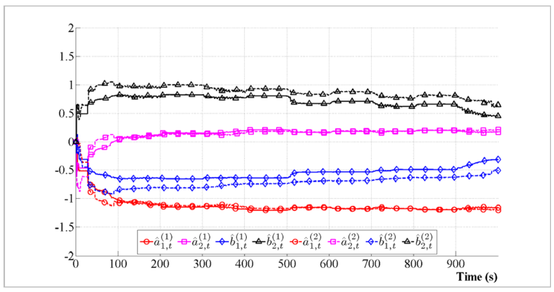

Figure 10: On-line identification of model parameters – self-tuning PID control.

The sampling period (T_0) is commonly selected from equation (Åström and Wittenmark, 1997)

$$0.1 \le \omega T_0 \le 0.6\,, \tag{46}$$

but optimization results show that with optimal closed-loop parameters and sampling period is outside of the interval (46). Figure 11 demonstrates that with unoptimal sampling period, from the interval (46), the control quality of pressure process is significantly decreased.

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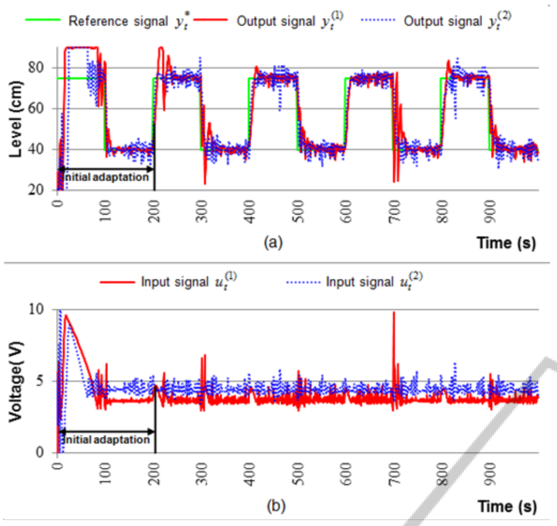

Figure 11: Control performance of self-tuning PID control of pressure process with closed-loop parameters and sampling period - $\omega = 0.118$rad/s, $\zeta = 0.662$, $T_0 = 1.0$s., (a) – reference and output signals, (b) – control signals.

The control performance of pressure process by predictor-based self-tuning controllers with constraints is depicted in Figure 12. It is seen from the graph that control quality is poor: the output signals of both tubes not settle in certain time, oscillating with high amplitudes.

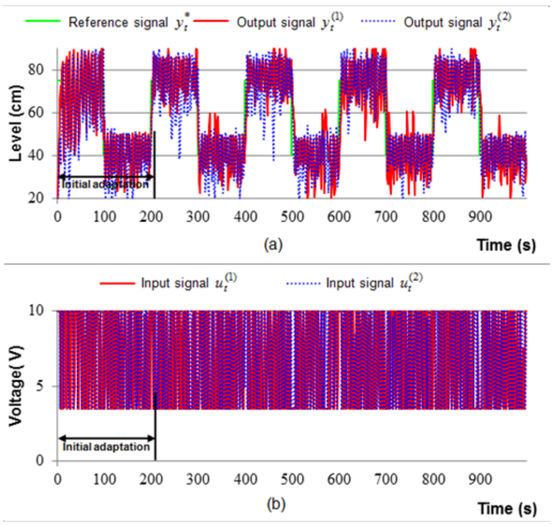

Figure 12: Control performance of predictor-based self-tuning control with constraints for pressure process with $T_0 = 0.1$s., $u_{\min}^{(i)} = 3.5$V, $u_{\max}^{(i)} = 10$V, $\delta_t^{(i)} = 10$V : $Q = 2675.9$, when $\rho = 9$ ($Q = 186.02$, when $\rho = 0$), (a) – reference and output signals, (b) – control signals.

The on-line identification of model parameters of this case is illustrated in Figure 13.

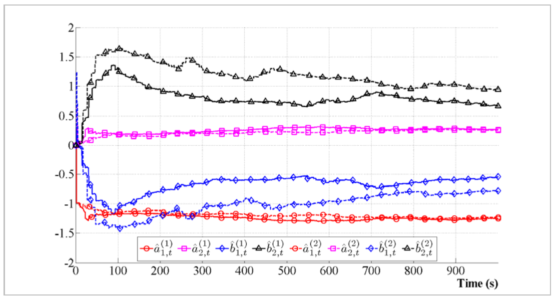

Figure 13: On-line identification of model parameters – predictor-based self-tuning control with constraints.

Considering only the variance of control errors, i.e. not taking into account the influences of control signals from criterion (19), the optimized self-tuning PID control (Figure 7) has up to 2 times lower as compared to predictor-based self-tuning control with constraints (Figure 12).

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6 CONCLUSIONS

The design method of digital self-tuning PID control with optimization of closed-loop parameters and sampling period for pressure process has been proposed.

The multidimensional optimization problem of closed-loop parameters and sampling period by subcomponent search method may be divided into optimization problems of one-variable functions.

The predictor-based self-tuning control with constraints for both - minimum-phase and nonminimum-phase - process models is proposed.

Experimental analysis has demonstrated that the control quality of pressure process by digital self-tuning PID control with closed-loop parameters and sampling period optimization is significantly better as compared to the predictor-based self-tuning control with constraints.

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