An Optimum-Rounding 5/3 IWT based on 2-Level Decomposition for Lossless/Lossy Image Compression

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Keywords: 5/3 IWT, 2-Level Decomposition, Rounding Effect, Image Compression.

Abstract: Lifting structures and rounding operations are main tools to construct integer wavelet transforms (IWT) that are well applied in lossless/lossy compression. However, the rounding operation generates its non-linear noise that makes its performance worse. In this report, we propose a new optimum-rounding 5/3 IWT based on 2-level decomposition for lossless/lossy image compression. Our proposed 5/3 IWT is designed to reduce rounding operation as much as possible. Filter characteristics of our proposed 5/3 IWT are the same as the conventional 2-level 2D 5/3 IWT excluded rounding effect. Coding performances of the proposed 5/3 IWT are better than those of conventional 5/3 IWT in lossy performance, because of reduction of rounding effects. Especially, its performance in near lossless compression is much better than the conventional one. However, they have almost the same lossless performance. Simulation results confirm effectiveness of our proposed 5/3 IWT.

1 INTRODUCTION

Many researchers have been paying attention to the standardization of new image compression system JPEG 2000 (Christopoulos, C., 2000). The Integer Wavelet Transform (IWT) (Calderbank, A.R., 1998) is one of the famous lossless algorithms because the IWT-based coding system can provide not only lossy coding but also lossless coding thanks to lifting structures (LS) (Daubechies, I., 1998) and rounding operations. However, the error generated from rounding operation causes PSNR degradation in lossy coding (Reichel, J., 2001) when quantization is applied. The conventional IWT is a one-dimensional (1D) filter bank (FB) (Vaidyanathan, P.P., 1993) constructed from double LS. To perform 2D FB for image application, the 1D LWT is applied twice in horizontal and vertical dimension, successively. Namely, it is a separable 2D IWT.

Recently, many researchers proposed a nonseparable 2D IWT. The number of rounding operations of those IWT is less than that of conventional 2D IWT, whereas filter characteristics of a nonseparable 2D IWT (Chokchaitam, S., 2002) are the same as those of conventional 2D IWT when error generated by the rounding operation is negligible. Therefore, coding performance of nonseparable 2D IWT is better than that of the conventional 2D IWT in lossy coding, especially at high bit rate when quantization errors are relatively small compared to the rounding errors. However, if nonseparable 2D IWT is applied for multi-stage, the rounding operations are not optimized.

In this report, a new optimum-rounding 5/3 IWT based on 2-level decomposition is proposed for lossless/lossy compression. The proposed optimum-rounding 5/3 IWT is mainly reduced rounding operations based on two methods: 1) Reducing rounding from nonseparable 2D IWT and 2) Reducing rounding from redundancy of 2-level decomposition. In simulation results, lossy coding performances of the proposed 5/3 IWT confirm its effectiveness comparing to the conventional IWT. Coding performance of our new proposed 2D 5/3 IWT is better than those of both the conventional 2-level 2D IWT and the existing 2-level nonseparable 2D IWT. However, their performance in lossless coding are almost the same results.

This report is organized as follows. In section 2, we review signal processing of the conventional 2-level 2D 5/3 IWT based on applying the conventional 1D 5/3 IWT in horizontal and vertical direction independently twice. Then, we review a signal processing of the conventional 2-level nonseparable 2D 5/3 IWT for image compression in

section 3. In section 4, we propose the 2-level optimum-rounding 5/3 IWT. Simulation results confirm effectiveness of our proposed IWT in both lossless coding and lossy coding in section 5. Finally, we summarize our proposed IWT in section 6.

2 THE CONVENTIONAL 2D 5/3 IWT

2.1 The Conventional 2D 5/3 IWT for 1-Level Decoposition

The conventional 2D 5/3 IWT is reconstructed by applying the conventional 1D 5/3 IWT in horizontal and vertical direction independently as illustrated in figure 1. Input signal (X) is decomposed into 4 subbands (Y_{LL}, Y_{LH}, Y_{HL}, Y_{HH}). For example, Y_{LH} indicates horizontally low-passed and vertically high-passed subband. The z_1 and z_2 denotes horizontal and vertical dimension, respectively. The Q_{LL}, Q_{LH}, Q_{HL}, Q_{HH} denote quantization in subband LL, LH, HL, and HH, respectively. The LS denotes lifting structure. The ® and "↓2" denote the rounding operation and the down-sampler by two. As shown in Fig. 1, six rounding operations are required to perform the conventional 2D 5/3 IWT where parameter $P_1(z)$ and $P_2(z)$ are the following

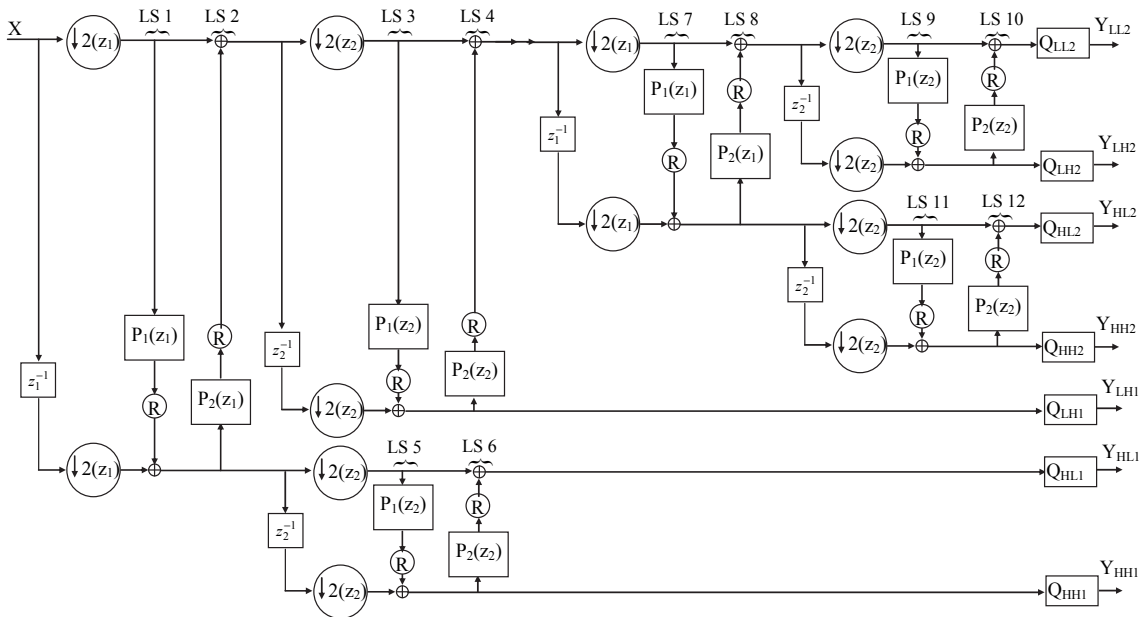$$P_1(z) = -\frac{1 + Z^{-1}}{2} \qquad (1)$$

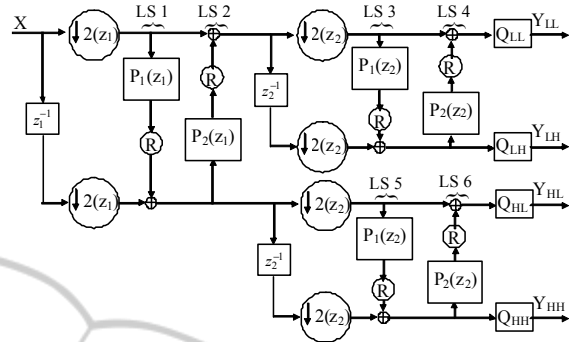$$P_2(z) = -\frac{1 + Z^1}{4} \qquad (2)$$

Figure 1: Analysis part of the conventional 2D 5/3 IWT.

2.2 The Conventional 2D 5/3 IWT for 2-Level Decomposition

The conventional 2-level 2D 5/3 IWT is reconstructed by applying the conventional 2D 5/3 IWT in previous section to decompose subband Y_{LL} again. Therefore, subband Y_{LL} is decomposed into 4 subbands (Y_{LL2}, Y_{LH2}, Y_{HL2}, Y_{HH2}). As shown in figure 2, twelve rounding operations are required to perform the conventional 2-level 2D 5/3 IWT.

Figure 2: Analysis part of the conventional 2D 5/3 IWT for 2-level decomposition.

3 THE CONVENTIONAL NON-SEPARABLE 2D 5/3 IWT

3.1 The Conventional Nonseparable 2D 5/3 IWT for 1-Level Decomposition

The signal processing of the conventional nonseparable 2D 5/3 IWT is designed to reduce rounding operation for 1 level. As shown in figure 3, analysis part of the conventional nonseparable 2D 5/3 IWT requires only four rounding operations; whereas, its filter characteristics are same as those of conventional 2D 5/3 IWT. Because of advantages of the nonseparable 2D FB, parameters in different LS of conventional 2D 5/3 IWT can be combined. For example, parameters of LS 1 and LS 5 in figure 1 are combine into those of LS 1' in figure 3. Therefore, the number of rounding operations required to perform the conventional nonseparable 2D 5/3 IWT is reduced to four rounding operations.

3.2 The Conventional Nonseparable 2D 5/3 IWT for 2-Level Decomposition

Similar to the conventional 5/3 IWT, the conventional 2-level nonseparable 2D 5/3 IWT is reconstructed by applying the conventional nonseparable 2D 5/3 IWT to decompose subband

Y_{LL} into 4 subbands (Y_{LL2}, Y_{LH2}, Y_{HL2}, Y_{HH2}). As shown in figure 4, eight rounding operations are required to perform the conventional 2-level 2D 5/3 IWT.
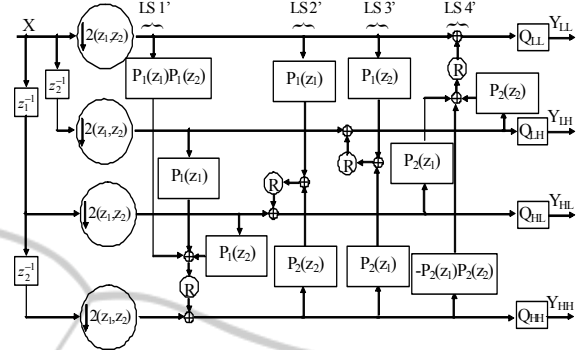

Figure 3: Analysis part of the conventional nonseparable 2D 5/3 IWT.

4 THE OPTIMUM-ROUNDING 5/3 IWT FOR 2-LEVEL DECOMPOSITION

4.1 Rounding-optimization Concept

In this section, we illustrate concept how to optimize number of rounding operations. First, we simplify
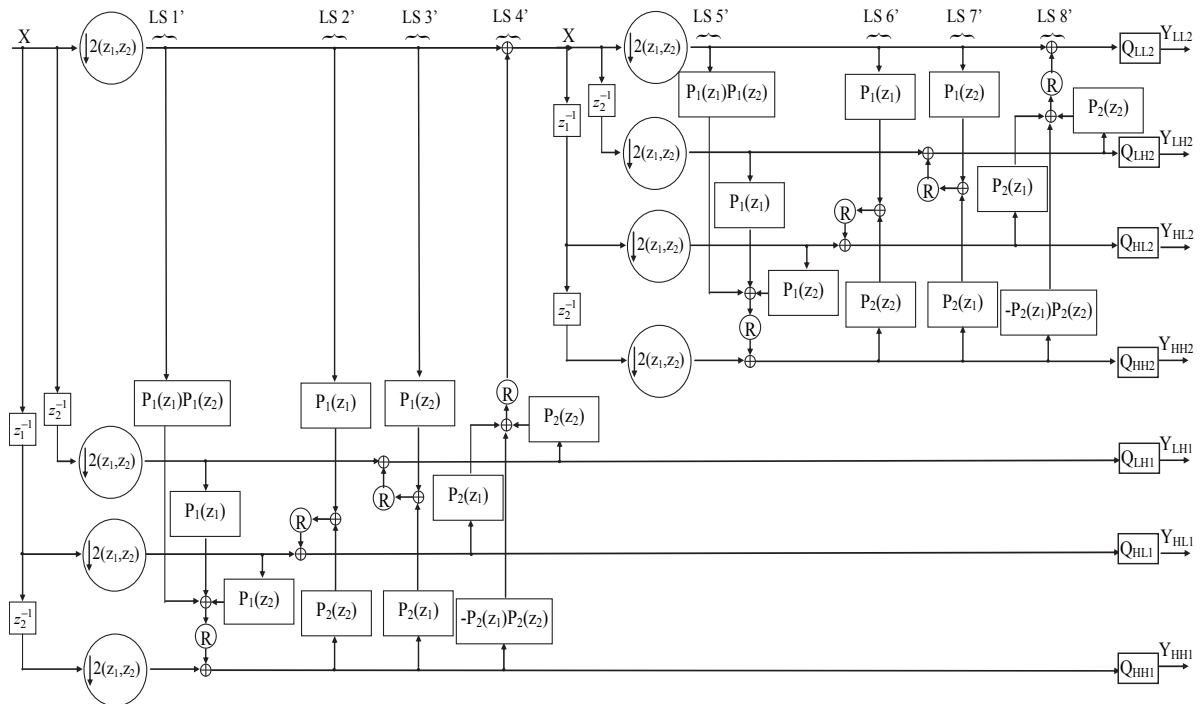

Figure 4: Analysis part of the conventional nonseparable 2D 5/3 IWT for 2-level decomposition.

the interested part before optimizing as shown in figure 5(a). Then, the lifting structure 1 in figure 5(a) is expanded into the lifting structure 1' and 1'' in figure 5(b). Finally, the lifting structure 1 in figure 5(a) can be combined with the lifting structure 2 and 3 in figure 5(a) to the lifting structure 2' and 3' in figure 5(c), respectively.
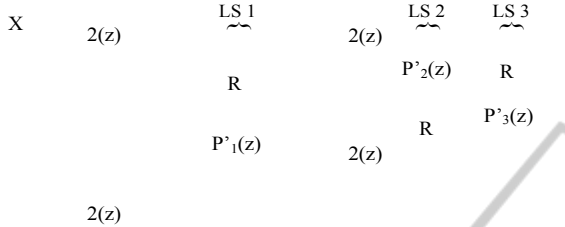

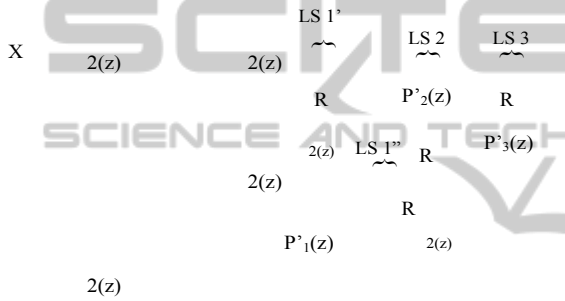Figure 5(a): The interested part before optimizing.

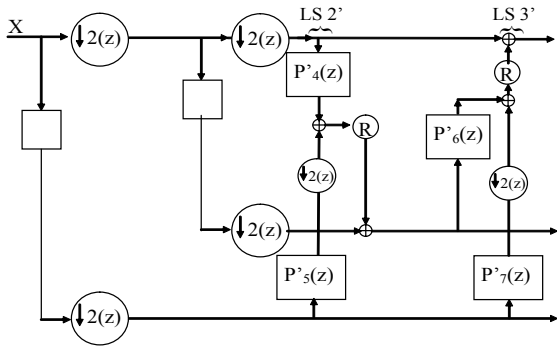Figure 5(b): The expanded part of LS1.

Figure 5(c): The Optimization result.

The coefficients in figure 5(c) are the following

$$P'_4(z) = Z^{-1}P'_1(z) + P'_1(z)P'_2(z) \quad (3)$$

$$P'_5(z) = P'_2(z) \quad (4)$$

$$P'_6(z) = P'_3(z) \quad (5)$$

$$P'_7(z) = P'_1(z)(1 + P'_2(z)P'_3(z)) + z^{-1}P'_3(z) \quad (6)$$

4.2 The Optimum-rounding 5/3 IWT for 2-Level Decomposition

The proposed optimum-rounding 5/3 IWT for 2-level decomposition subbands Y_{LL} into 4 subbands (Y_{LL2}, Y_{LH2}, Y_{HL2}, Y_{HH2}), similar to the other. However, the proposed optimum-rounding 5/3 IWT is designed to reduce rounding operation in LS4' (in figure 4) by expanding all coefficients of LS4' into LS5'', LS6'', LS7'' and LS8'' (in figure 6) Therefore, the proposed 5/3 IWT requires only seven rounding operations. The expanded coefficients in LS5'' are following:

$$P_3(z_1, z_2) = \frac{(1 + z_2)}{4} LS_5(z_1, z_2) \quad (7)$$

$$P_4(z_1, z_2) = \frac{(1 + Z_1)}{4} LS_5(z_1, z_2) \quad (8)$$

$$P_5(z_1, z_2) = -\frac{(1 + z_1)(1 + z_2)}{16} LS_5(z_1, z_2) \quad (9)$$

where

$$LS_5(z_1, z_2) = \frac{(1 + Z_1^{-1})^2(1 + Z_2^{-1})^2}{4} \quad (10)$$

The expanded coeffiecients in LS6'' are following:

$$P_6(z_1, z_2) = \frac{(1 + Z_2)}{4} LS_6(z_1, z_2) \quad (11)$$

$$P_7(z_1, z_2) = \frac{(1 + Z_1)}{4} LS_6(z_1, z_2) \quad (12)$$

$$P_8(z_1, z_2) = -\frac{(1 + Z_1)(1 + Z_2)}{16} LS_6(z_1, z_2) \quad (13)$$

where

$$LS_6(z_1, z_2) = \frac{(Z_2 + 4 + Z_2^{-1})Z_2^{-1}}{4} - \frac{(1 + Z_2^{-2})}{2} \quad (14)$$

The expanded coeffiecients in LS7'' are following:

$$P_9(z_1, z_2) = \frac{(1 + Z_2)}{4} LS_7(z_1, z_2) \quad (15)$$

$$P_{10}(z_1, z_2) = \frac{(1 + Z_1)}{4} LS_7(z_1, z_2) \quad (16)$$

$$\begin{aligned} P_{11}(z_1, z_2) \\ = -\frac{(1 + Z_1)(1 + Z_2)}{16} LS_7(z_1, z_2) \end{aligned} \quad (17)$$

where

$$LS_7(z_1, z_2) = \frac{(Z_1 + 4 + Z_1^{-1})Z_2^{-1}}{4} - \frac{(1 + Z_2^{-2})}{2} \quad (18)$$

The expanded coefficients in LS8'' are following:

$$P_{12}(z_1, z_2) = \frac{(1 + Z_2)}{4} \quad (19)$$

$$P_{13}(z_1, z_2) = \frac{(1 + Z_1)}{4} \quad (20)$$

$$P_{14}(z_1, z_2) = -\frac{(1 + z_1)(1 + z_2)}{16} \quad (21)$$

5 SIMULATION RESULTS

In this section, we apply some 8-bit gray-level images as input signals to illustrate effectiveness of our proposed method comparing those of the conventional 5/3 IWT. Lossless-coding performance is tested in section 5.1. Effectiveness of the proposed method is illustrated as the results from lossy-coding performance in section 5.2.

5.1 Lossless-coding Performance

Table 1 illustrate lossless coding performance of all IWT in term of the entropy rate calculated by

$$H = -\sum_s P_s \log_2 P_s \qquad (22)$$

where P_s indicates probability of a symbol "s". The "Conv.", "Nonsep." and "Prop." indicate "the conventional separable 2D 5/3 IWT for 2-level decomposition", "the conventional nonseparable 2D 5/3 IWT for 2-level decomposition" and "the proposed optimum-rounding 5/3 2D IWT for 2-level decomposition", respectively. The best entropy rate of each category is highlighted. Lossless performance of the proposed IWT almost the same as those of all 5/3 IWT. The optimized rounding operation doesn't directly affect entropy rate of output.

Table 1: Entropy rate in lossless coding of all 5/3 IWT.

Image Name	Conv.	Nonsep.	Prop.
Couple	4.43	**4.41**	4.42
Aerial	**5.80**	5.81	5.82
Girl	**4.72**	**4.72**	4.74
Chest	6.28	**6.24**	6.26
Mobile	5.10	**5.09**	5.10
Barbara	**5.19**	**5.19**	5.20
Flower	**5.42**	**5.42**	5.45
Lena	**5.06**	**5.06**	5.08

5.2 Lossy-coding Performance

Table 2-3 illustrate lossy coding performance of both methods in term of PSNR (Peak Signal to Noise Ratio) defined as

$$PSNR = 10\log_{10}\left(\frac{255^2}{\sigma_E^2}\right) \ [dB] \qquad (23)$$

where σ_E^2 denotes variance of error signal between original signal and decoded signal. The "∞" denotes infinity because of no error. The best PSNR of each category is highlighted. From the results in table 2-3, lossy coding performances of the proposed IWT are the best in many images at 5 bpp. because number of rounding operation is less. However, lossy coding performances of the proposed IWT and those of nonseparable 2D 5/3 IWT are best at 4 bpp. Therefore, our proposed method is the best for lossy coding especially in high bit rate. Figure 7-9
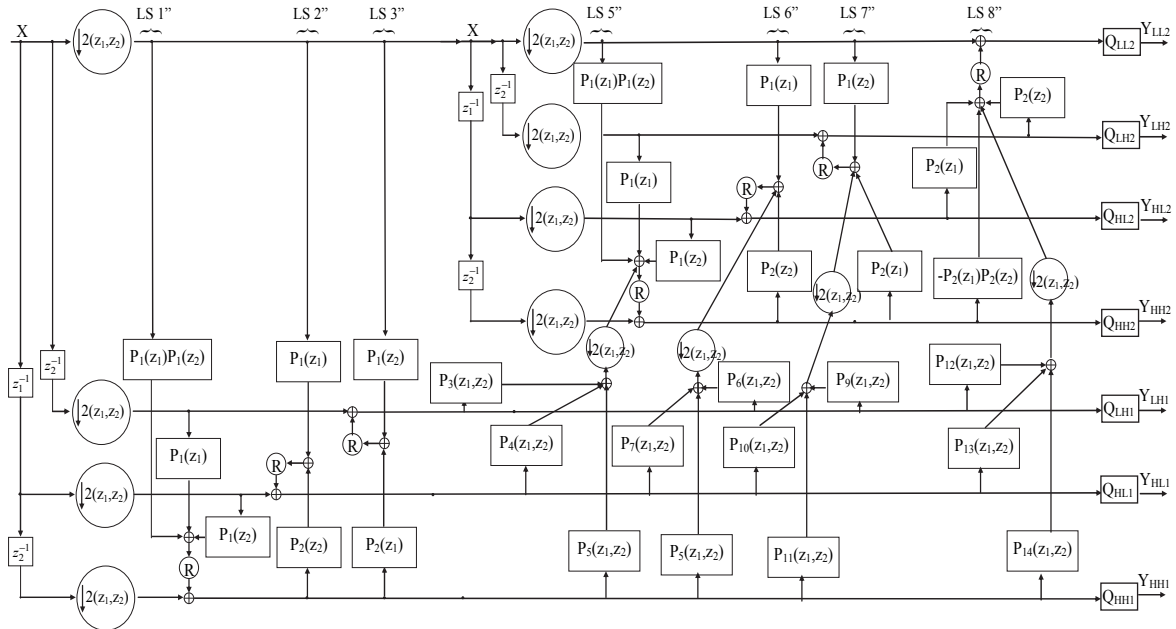

Figure 6: Analysis part of the proposed optimum-rounding 5/3 IWT for 2-level decomposition.

illustrate a part of image "Barbara" based on an original image, a nonseparable 2D 5/3 IWT at 5 bpp. and our proposed optimum-rounding 5/3 IWT at 5 bpp., respectively.

Table 2: PSNR of decoded image at 5 bpp.

Image Name	Conv.	Nonsep.	Prop.
Couple	∞	∞	∞
Aerial	46.1	47.3	**47.6**
Girl	∞	∞	∞
Chest	44.4	**45.9**	**45.9**
Mobile	47.1	47.7	**47.8**
Barbara	47.9	48.3	**48.9**
Flower	46.5	47.6	**47.9**
Lena	47.9	**48.2**	**48.2**

Table 3: PSNR of decoded image at 4 bpp.

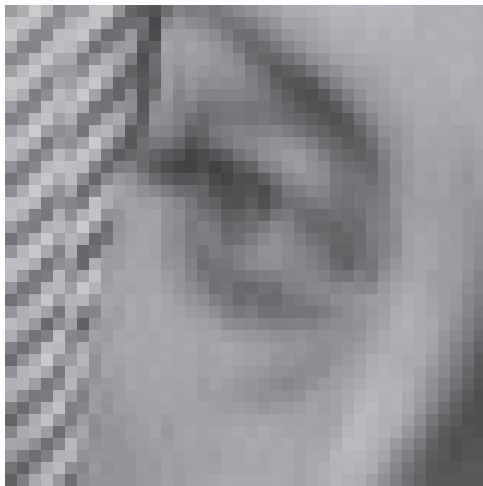| Image Name	Conv.	Nonsep.	Prop.
Couple	46.8	48.1	**48.2**
Aerial	43.0	**44.0**	43.9
Girl	46.3	**47.9**	**47.9**
Chest	41.5	**42.2**	**42.2**
Mobile	45.8	**47.6**	**47.6**
Barbara	46.0	**47.5**	**47.5**
Flower	44.6	**45.8**	45.7
Lena	46.1	**47.7**	**47.7**

Figure 7: A part of original image "Barbara".

6 CONCLUSIONS

In this report, we proposed an optimum-rounding 5/3 IWT for 2-level decomposition. The lossy-coding performance of our proposed IWT is better than those of the conventional IWT because the

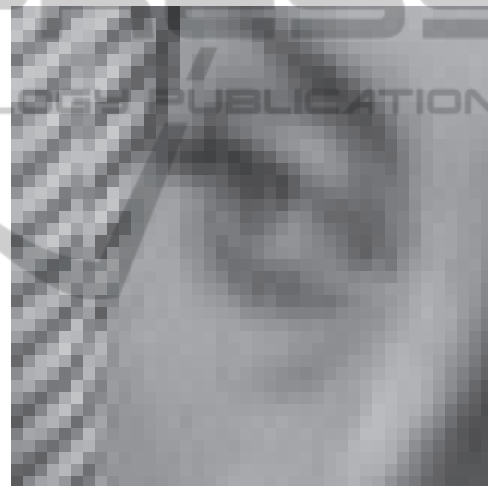Figure 8: A part of decoded image "Barbara" based on nonseparable 2D 5/3 IWT at 5 bpp.

Figure 9: A part of decoded image "Barbara" based on our proposed optimum-rounding 5/3 IWT at 5 bpp.

proposed IWT has less number of rounding operations; whereas, filter characteristics of all methods are exactly same if rounding effects are neglected. Simulation results confirm effectiveness of our proposed method in lossy coding especially at high bit rate.

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