Localization Method According to Collect Data from an Acoustic Wireless Sensor Network
Example of Homarus Gammarus in Natural Area

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Abstract: The platform STELLA MARE (Sustainable TEchnologies for LittoraL Aquaculture and MArine REsearch) has for objective to bring responses in the management of the sea in relation with the professional fishing. In this paper, we introduce an experiment on the monitoring of Homarus Gammarus. Using a passive tracking methodology using an acoustic wireless sensor network (AWSN), we try to follow some individuals, to define movement. The main objective is to build a lobster path according to the collected signal according a classical localization method to test our tracking method. Using a smoothing method and some resolution algorithms we are able to deduct a behaviour of tagged lobsters in an experimental area. This paper describes our methodology to estimate lobsters position according to data collected by an AWSN. This work represent a first step before the building a lobster model to simulate its behaviour in Corsican Mediterranean conditions.

1 INTRODUCTION

The global changes which affect the Earth have more important consequences in closed spaces. It's the case of the Mediterranean Sea, and especially in the island area. To relay in this subject, the University of Corsica decides to create in 2010 the platform STELLA MARE (Sustainable TEchnologies for LittoraL Aquaculture and MArine Research). This research center has been certified by the CNRS, and the platform became the unit UMS n°3514 Stella Mare University of Corsica/CNRS. This unit is specialized in Marine and Littoral Ecological Engineering. In this way, we have selected, in interaction with the professional fishing, species both studied in laboratories and in situ.

After a first experiment of Maja squinado (Gualtieri, 2013) based only on an activity tracking, we propose a second study of the Homarus Gammarus. In Corsica, the population of Homarus Gammarus is threatened and decreases strongly with an impact also on the fishers' activity. Stella Mare wants to provide sustainable model for the lobsters fishers in Corsican area. Indeed the distribution of the target species can be affected by different environmental conditions. However it is important to collect data on the European lobster in Corsican area because it doesn't exist this kind of stud. These works represent the first step before the building of lobster model to simulate the behaviour of this species. The European lobster, Homarus Gammarus, has a broad geographical distribution. In its northern range, it occurs from the Lofoten Islands in Northern Norway to south-eastern Sweden and Denmark, but is absent in the Baltic Sea probably due to lowered salinity and temperature extremes.

Its distribution southwards extends along the mainland European coast around Britain and Ireland, to a southern limit of about 30° north latitude on the Atlantic coast of Morocco. The species also extends, though less abundantly, throughout the coastal and islands areas of the Mediterranean Sea and has been reported from the westernmost end of the Black Sea in the Straits of Bosporus (Prodöhl, 2006).

According to our objective to build and simulate a lobster model, we decided to track several Homarus Gammarus using a VEMCO (VEMCO Ltd, Nova Scotia) wireless sensor system to collect

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behaviour data. Acoustic telemetry systems are an increasingly common way to examine the movement and behaviour of marine organisms. However, there has been little published on the methodological and analytical work associated with this technology (How, 2012).

Figure 1: Geographical distribution of Homarus Gammarus (Prodöhl, 2006).

In this paper, we try to develop a simple way to localize lobsters according to collected data from an AWSN. In the first part of this paper, we present several works on the lobster Homarus Gammarus tracking. In the second part, we describe the material and the used method to monitor the different lobsters. We introduce our method to reconstitute a position according to data collected by data transceivers. The data collected are analysed in the third part and the first results are shown. In part four, we suggest the perspectives of this work and expected results.

2 ACOUSTIC TELEMETRY AND LOBSTER MONITORING

The acoustic telemetry is an important research area. Indeed we can find an important volume of articles of this problematic. In (Kilfoyle, 2000), the authors explain that the acoustic telemetry channel is bandlimited and reverberant which poses many obstacles to reliable, high-speed digital communications. In our study, we focus on lobster monitoring using acoustic telemetry.

The north of the Europe is the main localization of the most important studies on the Homarus Gammarus. In south-eastern of Norway, the Norvegian University of life science leads some research on the lobster's activities (Wiig, 2012). 50 males were tagged with acoustic sensors (VEMCO V13). Tags were programmed to transmit signals (69 kHz) at 110 - 250 seconds random interval, equipped also by a pressure sensitive sensor for the depth. To follow lobster movement, 44 acoustic receivers were deployed. The lobster's beacons were able to transmit a special signals for GPS data reconstitution. This kind of research aims to identify patterns of lobster movements through the use of a sophisticated acoustic telemetry (AT) array which will continuously map the movement of tagged individuals, to within metres, within a large area over several months. Improvements in AT now allow us to tag and track large numbers of wild lobster in situ with minimal amount of disturbance, permitting studies which were previously impossible using traditional techniques such as catch data, thus improving quantification of movements, habitat utilisation and zonation in a way that was previously impossible (Skerritt, 2013). In (Moland, 2011), the Florida wildlife research institute leads some experiments with tagged a total of 10 lobsters in May 2003, five males (mean ± SD: 95.8 ± 17.0 mm carapace length, CL) and five females (80.6 ± 14.3 mm CL). The tags (VEMCO, V16 coded tags) were 16 mm in diameter, 58 mm long and produced a coded signal in a randomised interval between 60 and 180 s. One year before deployment of receivers, they conducted range tests of tags at Washerwoman shoals. They defined that some rules to define the position of lobsters according to the detections/transmissions. This method allows them to predict the lobster positon.

The precision of location estimates was determined by the hourly centroids of a tagged lobster that had entered a trap, which fell within a 30 m radius. The distance and velocity were estimated using the Haversine formula. We can see that the VEMCO low-cost solutions can represent a good alternative for a quick deployment for an experimental study (Heupel, 2006). It was precised by the authors that the definition of the type of data required will dictate the type of acoustic telemetry the project requires and how best to deploy the selected technology. The detection range is also defined by the deployment of the receivers. In (Simpfendorfer, 2008) the authors evaluated the performance of receivers and explained that several factors can affect the performance.

- A large number of transmitters within the range of receivers,

- A noisy environment,
- A behaviour of the tagged animals,
- The deployment method of the receivers,
- The heterogeneity of the environment relative to the transmission of acoustic signal (example: estuarine area).

According to these different researches, methodology and limits in the positions evaluation, we defined an experimental area and we have equipped 7 lobsters with VEMCO beacons. We present the materials and method in the part III.

3 MATERIALS AND METHOD

3.1 Tracking Lobsters

For the tracking, we use VEMCO materials. During this experiment, we have tagged 7 Homarus Gammarus (male and female) with V13TP VEMCO transmitters. The receivers are VEMCO VR2W illustrated by the Figure 2 and 3. The research area is near of Bastia city in Corsica. We can find a battle wreck at a depth of 44 m. This area represents a great zone for the lobster activities. We deployed 6 VEMCO receivers forming a grid around the wreck showed by the Figure 4. This research area presents two interests:

- Signal analysis: the behaviour of the transmitters – receivers of an AWSN in metallic environment.
- Biological: the behaviour of Lobster in special environment (wreck).

Figure 2: Hommarus Gammarus tagged with V13TP VEMCO.

Figure 3: VR2W acoustic receiver.

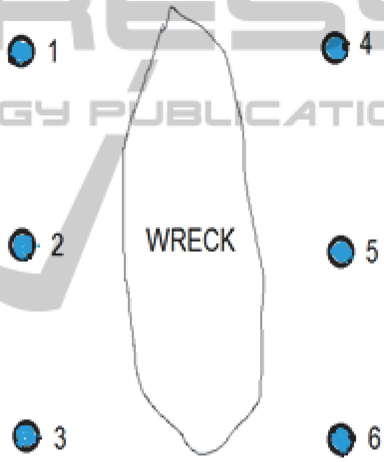

Figure 4: Experimental area.

3.2 Noise Problem on Signal Reception

The collected data from the n=6 receivers (hydrophone) (called Insuma 1 to Insuma 6) define the presence of the different lobsters. The collected data allow us to know if a lobster VEMCO signal was received by the receivers. Each lobster can send alternately two kinds of signals (ID Code): temperature and pressure.

For the data treatment we group the two ID for each lobster. In the deployment of two kinds of beacons are deployed. The difference is on the sampling frequency.

- Transmitters 8788, 8790, 8792, 8794 et 8796 with a sampling frequency every 60 seconds
- Transmitters 8798, 8800, 8002, 8804, 8806, 8810, 8812, 8814 and 8816 with a sampling frequency every 90 seconds.

Several problems can disturb the signal reception:

- If two beacons transmit at the same time a signal, a collision can be created with information lost. It is a reason of irregularity of the transmission.
- A signal can be lost if a disturbance exists between the lobster and the receiver.
- A lobster can be invisible because there is an obstacle between the V13TP and the VR2W.

According to these different researches, methodology and limits in the positions evaluation, we defined an experimental area and we have equipped 7 lobsters with VEMCO transmitters.

3.3 Method to Deduct Position According Data Collected

In this part, we present the used method to deduct lobster position according data collected. We use classical methods, described more precisely in the 3.3.2. Indeed, the objective of this work was to have a simple method to have lobster position independent of VEMCO systems and manufactures, based only on the received signals integrating a possible development way towards a real time analysis.

The first analyze of the data collected allows us to have a presence or not of a signal (of a lobster) on the different receivers as showed on our example in the Figure 5 during a day.

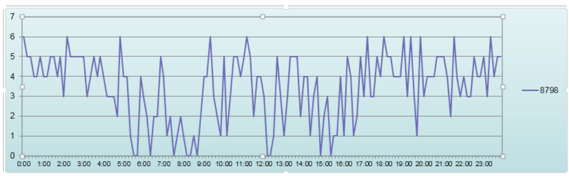

Figure 5: Data collected by the 6 deployed receivers for a day and a tagged lobster 8798 (y = receivers VR2W (1 to 6) and x= Time of arrival of a message on a receivers).

We see that the possible noise of the environment and the possible loss of signals could be explained by the experimental metallic area of the wreck. We can estimate that it is a first result, clearly different of our first tracking activity (Gualtieri, 2013). The reception of message is clearly dependant of the distance between a lobster and a receiver. To remove the noise we use a smoothing method according to Savitzky-Golay algorithm. According to the smoothed data we present a method to deduct the localization of the lobster and a global behavior.

3.3.1 Smoothing Method

On the Figure 5 we can observe the data collected for tagged Lobster 8798.

In each point x of the Figure 5, we treat the curve as a polynomial P(*t*) in order d with d <2L+1. L is the window of values.

$$\bar{\chi} = P(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots + a_d t^d$$

In the point x-L,…, x, x+L the polynomial must coincide with the values of the function to smooth. If we center the resolution in 0, i.e. by setting x = 0, a set of equations is obtained to resolve –L, ...1 , 0 , 1, ...L. The α_0 value corresponds to the smoothed value x α_1 and its derivative.

$$\begin{pmatrix} \chi_{k-L} \\ .. \\ \chi_k \\ .. \\ \chi_{k+L} \end{pmatrix} = \begin{bmatrix} 1 & -L^1 & .. & (-L)^d \\ .. & .. & .. & .. \\ 1 & 0 & .. & 0 \\ .. & .. & .. & .. \\ 1 & L^1 & .. & L^d \end{bmatrix} * \begin{pmatrix} a_0 \\ a_1 \\ .. \\ a_d \end{pmatrix}$$

The Figure 6 shows the smoothed curved. The effectiveness of a smoothing increases with the number of points considered, the dispersion values is reduced. After smoothing the error on the points decrease as much as the convolution involves a large number of points.

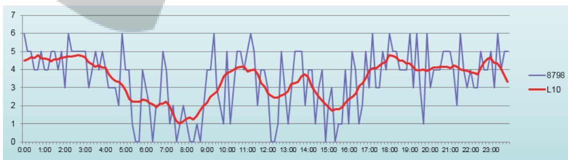

Figure 6: Smoothed curve according to Figure 5.

If we do not wish to have access to different derivatives, it is not necessary to choose a high degree for the polynomial. For a polynomial of degree 3 and a window of size L = 10 are obtained.

3.3.2 Localization Method

To determine the position of individuals in relation to the stations, we need to receive functions R. It is assumed that the reception is not oriented, and it depends only on the distance D of the individual from the station. In this case, it is inversely proportional to the distance, which we estimate based on a distance -related exponential D.

$$R(x,y) = R_{max} - R^d(x,y)$$

With

$$R^d(x,y) = \frac{N_0 R_{max}}{(N_0 + (R_{max} - N_0)e^{-rD})}$$

R is the reception rate; R_{max} is the maximum value of R (100). N_0 is a starting constant to receive curve (the higher this value is small and the curve will remain longer in the vicinity of R_{max} for small values of D. r is a damping coefficient. On the Figure T we can distinguish the impact of r on the R receives functions. According this parameter r, we can describe the derived functions.

$$\begin{pmatrix} \frac{\partial R(x,y)}{\partial x} \\ \frac{\partial R(x,y)}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{rxR^d(x,y)}{D}\left(\frac{R^d(x,y)}{R_{max}} - 1\right) \\ \frac{ryR^d(x,y)}{D}\left(\frac{R^d(x,y)}{R_{max}} - 1\right) \end{pmatrix}$$

To find the position of an individual at time t, we will minimize the quadratic error in measuring the difference between the R function (x, y) and the measures taken at the time t.

$$\delta^2(x,y,t) = \sum_{i=1}^{n}\left[\frac{\bar{\chi}_i(t) - R_i(x,y,t)}{\omega_i}\right]^2$$

δ(x, y, t) is the error to be minimized. χ_i correspond to the smoothed values for each station. ω_i the weights of each measure The idea is from a point $p_j = (x_j, y_j)$ to calculate a new point $p_{j+1} = (x_{j+1}, y_{j+1})$ which decreases the value δ. At each iteration, we calculate η a disturbance that will evolve the squared error to a minimum.

The gradient descent algorithm allows to find the minimum. In this case the disturbance occurs in the direction of the steepest gradient provided by $R_i(x_j, y_j, t)$. The disturbance η_{dg} that minimizes δ^2 is found when de gradient of the squared error is null.

$$\frac{\partial \delta^2(p)}{\partial p} = -2J^tW[\bar{\chi}_i - R_i(p)] = 0$$

$$\eta_{dg} = \alpha J^tW[\bar{\chi}_i - R_i(p_j)]$$

Where W is the weighting matrix with $W_{ii} = 1/\omega_i^2$, and J is the Jacobian matrix.

This algorithm is simple to implement, but the starting point should be close to the global minimum. With a not too large step the method can quickly become chaotic, whereas with a not too small step this method can be very long and converge to a local minimum. The η_{dg} perturbation is proportional to previous described value, where the positive scalar α determines the length of the step in the steepest-descent direction. It is possible to change this value by increasing or decreasing it when the error decreases or increases.

The Gauss-Newton method is a well-known iterative technique used regularly for solving the nonlinear least squares problem (NLSP) (Press, 1992). This method is more stable than that of the gradient, it allows to converge in most cases. We approximate the δ(x, y, t) by a quadratic function in the vicinity of the solution, which avoids using the Hessian of R_i. That is to limit the development R_i to the first derivative.

$$[R_i(p_{j+1})] \approx [R_i(p_j)] + J\eta_{gn}$$

If we inject the value of R in p_j to calculate the value of the square error p_{j+1} gives an estimate of the derivative of $\delta(x_{j+1}, y_{j+1}, t)$.

$$\frac{\partial \delta^2(p_{j+1})}{\partial \eta_{gn}} \approx -2[\bar{\chi}_i - R_i(p_j)]^tWJ + 2\eta_{gn}{}^tJ^tWJ$$

And when squared error is null we can obtain the local disturbance.

$$\eta_{gn} = [J^tWJ]^{-1}J^tW[\bar{\chi}_i - R_i(p_j)]$$

The Levenberg-Marquardt (LM) algorithm is an iterative technique that locates the minimum of a function that is expressed as the sum of squares of nonlinear functions (Levenberg, 1944), (Lourakis, 2005), (Marquardt, 1963). It has become a standard technique for nonlinear least-squares problems and can be thought of as a combination of steepest descent and the Gauss-Newton method.

$$\eta_{lm} = [J^tJ + \lambda diag(J^tJ)]^{-1}J^tW[\bar{\chi}_i - R_i(p_j)]$$

The disturbance corresponds is given by the equation. If λ tends to 0 we approach the Gauss-Newton method and so λ is more important we approach the gradient descent algorithm. At the start, we will fix a λ value that will decrease as the square error increases. This value will increase as soon as the square error decreases, to be closer to the gradient method.

The following algorithm is used to compute the optimal solution of the squared error between collected data and the receive functions R. The Levenberg Marquardt algorithm (Nocedal, 1999), (Kelley, 1999) requires an initial value for p_0 to be estimated at t time. We chose p_0 equal to p_{min} evaluate at the previous time (the last position of the lobster). When the lobster is hidden, this position is not available, so we use the latest known position.

This algorithm computes the disturbance and evaluates the error at the new parameter vector. If the error has increased as a previous result, λ is increase by a factor of 10, and the step is rejected. If the error has decreased as a result of the update we accept the step and decrease λ by a factor of 10. We stop the steps when the algorithm met the desired convergence criteria or has exceeded the limit of function evaluations or iterations.

Algorithm.

for each *time t*

{ initialization of p_0 *;*

choose λ_0 $=\max_i[diag(J^tJ)]_{i,i}$ *and continue=true ;*

while *(continue and iteration<limit)*

{ Compute the new disturbance η_{gn}

Evaluated the new squared error $\delta^2(p+\eta_{lm})$

if $\|\delta^2(p)-\delta^2(p+\eta_{lm})\| < \varepsilon\|p\|$ *continue=false;*

else

{ evaluate an acceptability factor

$$\rho(\eta_{lm}) = \frac{\|\delta^2(p)\| - \|\delta^2(p+\eta_{lm})\|}{2\eta_{lm}(\lambda\eta_{lm} + J^t[\bar{\chi} - R_i(p)])}$$

if *(*$\rho(\eta_{lm}) < 0$ *)* $p = p + \eta_{lm}$ *; decrease* λ *;*

else *increase* λ *;*

}

iteration = iteration +1;} }

ε and limit are user specified.These methods were implemented in C# and the treated data are presented on the following figures.

4 RESULTS

In this part we present the activity of each tagged lobster after position calculation according to the previous methods.

On the presented figures we can see only a static activity. It is important to precise that we have an activity monitoring step by step. On the different figures we can observe the blue points which represent the experimental area as showed on the Figure 4. On the table 1 we introduce the monitoring period for each lobster that corresponds at the usable collected data are.

We can observe on the different figures that we are able to distinguish an activity for each observed lobster. The signal loss can appear on different figures (Figure 7 and Figure 14).

Table 1: List of figures.

Lobster	Period (0h00 to 0h00)	Figure
8794	6 June 2014 to 27 June 2014	Figure 7.
8798	17 September 2014 to 5th January 2015	Figure 8.
8806	17 September 2014 to 12 October 2014	Figure 9.
8812	6th June to 12 August 2014	Figure 10.
8812 (bis)	14 August to 1st October 2014	Figure 11.
8814	6th June to 12 August 2014	Figure 12.
8816	6th June to 12 August 2014	Figure 13.
8816 (bis)	14 August to 1st October 2014	Figure 14.

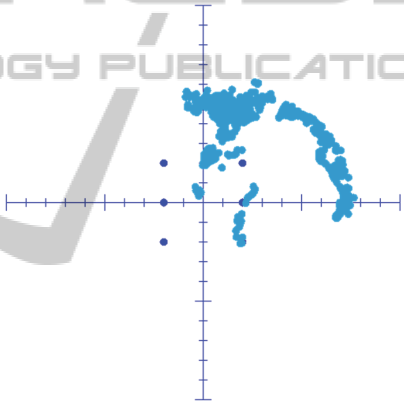

Figure 7: Activity of tagged lobster 8794.

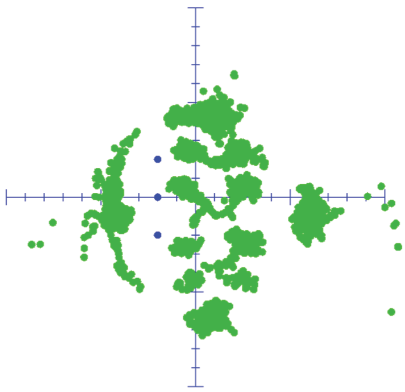

Figure 8: Activity of tagged lobster 8798.

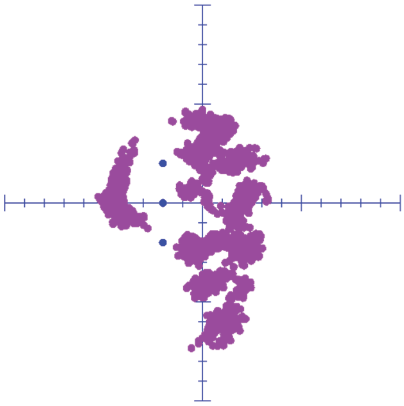

Figure 9: Activity of tagged lobster 8806.

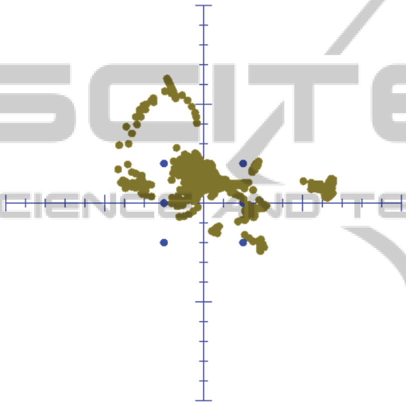

Figure 10: Activity of tagged lobster 8812.

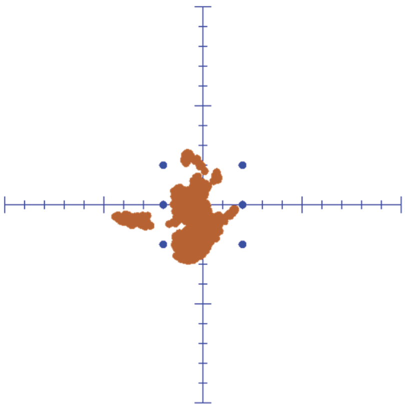

Figure 11: Activity of tagged lobster 8812 (bis).

These figures show that the used method allow us to deduct a behaviour during a period according collected data using a subaquatic wireless sensor network.

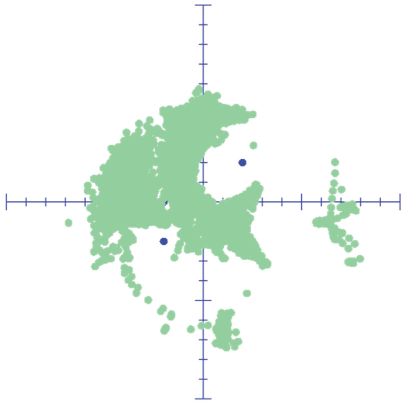

Figure 12: Activity of tagged lobster 8814.

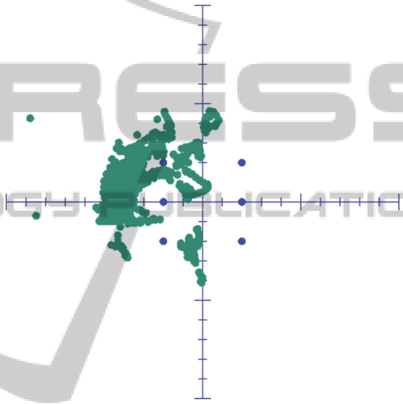

Figure 13: Activity of tagged lobster 8816.

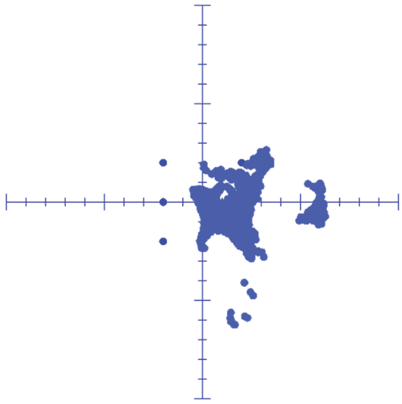

Figure 14: Activity of tagged lobster 8816 (bis).

It is important to precise the collected data is dependant by the capacity of the receivers to receive the signals. Indeed the activity of the lobster is clearly is diurnal and during the day the lobster is in its habitat, in a metallic area with an important impact on the signal transmission. The figures show only the behaviour during the night corresponding at the hunt behaviours of the lobsters. During, the lobster leaves its habitat and allows a new transmission of the Vemco units. The wreck is rich in foods and we can observe a stillness of the lobsters. When the lobster leave the wreck we can observe a loss of signals because the research area is limited by 6 receivers and the distance between transmitters – receiver is too important.

5 CONCLUSIONS

These results show that the used method allow us to deduct a behaviour during a period according collected data using a subaquatic wireless sensor network. The monitoring of Homarus Gammarus in the Mediterranean Sea can play an important role on the economic development of sustainable fisheries activity. In this context, this paper want to provide a simple way to predict the behaviour of this species according collected data by an acoustic wireless sensor network.

This paper presented a part of the state of art on the monitoring of Hommarus Gammarus using a passive tracking with an acoustic wireless sensor network from VEMCO technology. According to the data collected we tried to define the position and the global behaviour of 7 tagged lobsters. We can see that we are able to build a global behaviour of the lobsters from collected signal. This behaviour building is deducted from classical localization method. This method is a hybridization of two classical methods. Indeed, according to a given value λ we use either gradient descent or Gauss Newton method.

The goal of this study was not to develop a new method of localization but to build a simple way to deduct from collected data the relative position of the tags and to extract a behavior. In this experiment, the results are generally positive but not sufficient. Indeed we are able to have a position but we are not sure of the precision. The impact of the damping coefficient r must be measured and reported in our estimation of the position. We must a measurement campaign during a year to observe the complete behavior of a lobster.

The impact of receiver’s positions must be better appreciate. Indeed we made the choice to deploy the VR2W in the fund of the sea however it seems to be more precise to deploy under the sea surface to improve reception quality. The choice of the research area (wreck with metallic body) has an impact on the signal.

However these first results allow us to build a first behavior model of individual lobster according to the collected and interpreted data form the sensor network in the Mediterranean Sea. Indeed the diurnal activity, the stillness of the lobster on a rich foods area are the first elements of a corsican lobster behavior to find some solutions in the repopulation of this species.

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REFERENCES

Gualtieri, J. S., Aiello, A., Antoine-Santoni, T., Poggi, B., & De Gentili, E. (2013). Active Tracking of Maja Squinado in the Mediterranean Sea with Wireless Acoustic Sensors: Method, Results and Prospectives. Sensors, 13(11), 15682-15691.

Prodöhl, P. A., Jørstad, K. E., Triantafyllidis, A., Katsares, V., & Triantaphyllidis, C. (2006). European lobster-Homarus gammarus. Genetic Impact of Aquaculture Activities on Native Populations. Final Scientific Report, 91-98.

How, J. R., & de Lestang, S. (2012). Acoustic tracking: issues affecting design, analysis and interpretation of data from movement studies. Marine and Freshwater Research, 63(4), 312-324.

Kilfoyle, D. B., & Baggeroer, A. B. (2000). The state of the art in underwater acoustic telemetry. Oceanic Engineering, IEEE Journal of, 25(1), 4-27.

Wiig, R., Jorgen, (2012), Acoustic monitoring of lobster (Homarus Gammarus) behaviour survival during fisheries, Master Thesis.

Skerritt, D.J., Fitzsimmons, C., Hardy, M.H., Polunin, N.V.C. (2013) Mapping European lobster (Homarus gammarus) movement and habitat-use via acoustic telemetry – Implications for management. Progress report to the Marine Management Organisation. Oct 2013.

Moland, E., Olsen, E. M., Andvord, K., Knutsen, J. A., & Stenseth, N. C. (2011). Home range of European lobster (Homarus gammarus) in a marine reserve: implications for future reserve design. Canadian Journal of Fisheries and Aquatic Sciences, 68(7), 1197-1210.

Heupel, M. R., Semmens, J. M., & Hobday, A. J. (2006). Automated acoustic tracking of aquatic animals: scales, design and deployment of listening station arrays. Marine and Freshwater Research, 57(1), 1-13.

Simpfendorfer, C. A., Heupel, M. R., & Collins, A. B. (2008). Variation in the performance of acoustic receivers and its implication for positioning algorithms in a riverine setting. Canadian Journal of Fisheries and Aquatic Sciences, 65(3), 482-492.

Lourakis, M. I. (2005). A brief description of the Levenberg-Marquardt algorithm implemented by levmar. Foundation of Research and Technology, 4, 1-6.

Press W, Teukolsky B.F.S, & Vetterling W. (1992) Numerical Recipes: The Art of Scientific Computing. second ed. Cambridge: Cambridge University Press; 1992.

Levenberg, K. (1944) A Method for the Solution of Certain Non-linear Problems in Least Squares. Quarterly of Applied Mathematics, 2(2):164–168, Jul. 1944.

Marquardt D. (1963) An algorithm for least squares estimation of nonlinear parameters. J Soc Ind Appl Math 1963;11:431–441.

Nocedal J. & Wright, S. (1999) Numerical Optimization. Springer, New York, 1999.

Kelley C. (1999) Iterative Methods for Optimization. SIAM Publications, Philadelphia, 1999.