# Search for Latent Periodicity in Amino Acid Sequences with Insertions and Deletions 

Valentina Pugacheva ${ }^{1}$, Alexander Korotkov ${ }^{2}$ and Eugene Korotkov ${ }^{1,2}$<br>${ }^{1}$ Institute of Bioengineering, Research Center of Biotechnology of the Russian Academy of Sciences, Leninsky Ave. 33 bld. 2, 119071, Moscow, Russia<br>${ }^{2}$ National Research Nuclear University "MEPhI", Kashurskoe shosse, 31. Moscow 115409, Russia

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#### Abstract

The aim of this study was to show that amino acid sequences have a latent periodicity with insertions and deletions of amino acids in unknown positions of the analyzed sequence. Genetic algorithm, dynamic programming, and random weight matrices were used to develop the new mathematical algorithm for latent periodicity search. The method makes the direct optimization of the position-weight matrix for multiple sequence alignment without using pairwise alignments. The developed algorithm was applied to analyze the amino acid sequences of a small number of proteins. This study showed the presence of latent periodicity with insertions and deletions in the amino acid sequences of such proteins, for which the presence of latent periodicity was not previously known. The origin of latent periodicity with insertions and deletions is discussed.


## 1 INTRODUCTION

The development and application of mathematical methods in the study of symbolic sequences is of particular importance to achieve great success in the sequencing of various genomes. It also increases the accumulation of information about the complete genomes of many species (Ekblom and Wolf, 2014). If mathematical methods are not applied, a big part of the known nucleic and amino acid sequences will be stored away in computer data banks, without significant usage. This is especially true for eukaryotic genomes. The task of developing new mathematical methods entails finding new mathematical laws to explain sequence organization and the relationship of these laws with the biological functions of various parts of the genome (Almirantis et al., 2014). These studies show the relationship between certain mathematical regularities observed in sequences with their biological properties.

Latent periodicity is one of the structural regularities of sequences and is widely represented in amino and DNA sequences (Korotkov et al., 2003a, 2003b). A periodicity is considered as latent if the similarity between any two periods is not statistically significant or if it belongs to the twilight
zone (Durbin et al., 1998). Perfect periodicity can become latent periodicity if it accumulates over 1.0 mutations per amino acid in the studied sequence (Suvorova et al., 2014). The distinctive property of latent periodicity is that it cannot be detected by pairwise comparisons of amino acid sequences (Turutina et al., 2006). However, latent periodicity can be found if we apply a mathematical method to directly detect the multiple alignment of amino acid sequences without constructing pairwise alignments. The periods of a sequence with latent periodicity are sequences for multiple alignment and the multiple alignment can be statistically significant. The goal of this study was to find multiple alignments of amino acid sequences (periods) in the absence of statistically important pairwise alignments.

There is a significant gap in the mathematical approaches presently used to search for latent periodicities in symbolic and numeric sequences. Spectral approaches enable the discovery of enough "fuzzy" periodicity in protein sequences without insertion(s) or deletion(s) of amino acids. Fourier transform, wavelet transform, information decomposition and some other methods can be attributed to a number of spectral methods (Tiwari et al., 1997; Lobzin and Chechetkin, 2000; Kravatskaya et al., 2011; Korotkov et al., 2003a; de

Sousa Vieira, 1999; Meng et al., 2013; Suvorova et al., 2014; Sosa et al., 2013; Kumar et al., 2006). However, these approaches have a significant limitation, such as the fact that they do not allow the detection of periodicity with insertions and deletions.

On the other hand, methods based on dynamic programming can accurately find insertions and deletions (Pellegrini, 2015). However, methods based on dynamic programming cannot detect latent periodicity, in a situation where the statistical significance of similarity between any two periodic sequences is small (Korotkov et al., 2003; Turutina et al., 2006). This is due to the fact that the periodicity of amino acid sequences (with the number of periods greater than or equal to 4 ) was detected by pairwise alignment between periods. In the absence of statistically significant pairwise alignments, these approaches are incapable of finding latent periodicity. First of all, it concerns algorithms and programs such as REP (Andrade et al., 2000), Internal Repeat Finder (Marcotte et al., 1999), Prospero (Mott, 1999), RADAR (Heger \& Holm, 2000), REPRO (Heringa \& Argos, 1993) TRUST (Szklarczyk \& Heringa, 2004) and PTRStalker (Pellegrini et al., 2012). It is also difficult to detect latent periodicity by the programs XSTREAM (Newman \& Cooper, 2007) and TREKS (Jorda \& Kajava, 2009) because the similarity between different periods is very low in the case of latent periodicity. This leads to lack of seeds and identical short strings. The Markov models and neural networks are inefficient for finding latent periodicity, since there are no training samples. The following programs were used in previous studies HHrep (Söding et al., 2006), HHRepID (Biegert \& Söding, 2008) and the approaches developed in the works of Palidwor et al. (2009) and Rubinson \& Eichman (2012).

Therefore, in this study a mathematical method was proposed that considers this gap and finds the latent periodicity of any symbolic sequence in the presence of insertions and deletions (in unknown positions of the analyzed sequence) and in the absence of a known position-weight matrix.

Any periodicity of the sequence $S$ with length $N$ can be characterized by either the frequency matrix (Korotkov et al., 2003b) or the position-weight matrix $M$ (Shelenkov et al., 2006) calculated from frequency matrix. Amino acids are the signs of the rows of this matrix while period positions serve as the signs of the columns. The element of this matrix $m(i, j)$ indicates the weight which has the amino acid $i$ in position $j$ of the period. The positions of the
period changed from 1 to $n$. The sequence $S_{1}$ of length $N$, which is an artificial periodic sequence $1,2, \ldots, n$, was introduced. Here, the numbers were treated as symbols and columns in the matrix $M$ were consistent with them. For a period equal to $n$, the sequence $S$ corresponds to a certain frequency matrix and weight matrix $M(20, n)$. The problem was formulated as follows. We have a sequence $S$ with length $N$. It is necessary to find such optimal weighting matrix $M_{0}$, where the local alignment of sequences $S_{1}$ and $S$ have the greatest statistical significance. Under the statistical significance, the probability $P$ is that $F_{r}>F_{\max }$, where $F_{\max }$ is the maximum weight of a local alignment of sequences $S_{r}$ and $S_{1}$, using the some optimal matrix $M_{0}$. Here, $F_{r}$ is the maximum weight of a local alignment of randomly mixed sequences $S_{r}$ and $S_{1}$ using the some optimal matrix $M_{r}$. We search a matrix $M_{0}$, which have the lowest probability $P$. It is always possible to set the threshold level of the probability $P_{0}$ and if the probability $P\left(F_{r}>F_{\max }\right)$ is less than $P_{0}$, then a local alignment of sequences $S$ and $S_{1}$ is found, using the some optimum matrix $M_{0}$ and this alignment can be considered as statistically significant.

It is possible to use the local alignment algorithm, for alignment of the amino acid sequence $S$ and an artificial periodic sequence $S_{1}$, relative to the known weight matrix (Smith and Waterman, 1981). It is necessary to find the optimal weight matrix $M_{0}$. The objective of this study was to develop a mathematical approach for finding the matrix $M_{0}$, as well as a method for assessing the probability $P$. To find the optimal weight matrix, a genetic algorithm was used, as well as a local alignment algorithm. The Monte Carlo method was used to estimate the probability $P$.

A mathematical method was developed in this paper to find more than 3 tandem repeats in amino acid sequences. The method was used for direct optimization of the position-weight matrix for multiple sequence alignment without using pairwise alignments. This means that for each $n$, a matrix $M_{0}$ is found, the probability $P$ is estimated and we build the alignment of the sequences $S$ and $S_{1}$ using $M_{0}$ matrix. It is not the goal of this study to analyze all the known amino acid sequences, since the developed method requires very large computer resources. The developed algorithm was applied to search for latent periodicity with insertions and deletions in the amino acid sequences of a small number of proteins This study showed the presence of latent periodicity with insertions and deletions in the amino acid sequences of proteins, for which the presence of latent periodicity was not previously
known.

## 2 MATHEMATICAL METHODS AND ALGORITHMA

A genetic algorithm was used to search for the optimal weight matrix $M_{0}$ for period $n$. A genetic algorithm is a heuristic search algorithm for solving optimization problems and is a form of direct random search (Mitchell, 1998). It is often used to optimize the functions of several variables. The general view of the algorithm is as shown in Figure 1. Usually, the problem is formalized, so that a solution could be found as a vector, where each element can be a bit, a number, or some other object. This vector is considered as an "organism." Usually, a set of initial organisms are randomly created (Gondro and Kinghorn, 2007). Each of these organisms was measured using an objective function, which is regarded as a "fitness function." As a result, every organism is associated a certain fitness value, which determines how well the organism solves the problem. Organisms are selected from this set of organisms (it can be called "generation") for application of the "genetic operators" ("crossing" and "mutation", taking into account the value of "fitness"). The new organisms were gotten as a result of the application of these operators. The value of fitness was also calculated for new organisms, and then selection of the best organisms to the next generation was done. This set of actions was repeated iteratively, and thereby simulating the "evolutionary process". This process was allowed to continue for several life cycles (generations), before executing the stop criterion of the algorithm. Such a criterion can be either finding the global or suboptimal solutions or exhaustion of the number of generations released for evolution. In this study, the organisms are the weighting matrix of the periodicity. This set was called $Q_{n}$ or population. Each matrix has 20 rows and $n$ columns. Matrix elements $m(i, j)$ are some numbers that show the weight amino acids $i$ to column number $j$. A larger weight of the element $m(i, j)$ corresponds to a high probability of the presence of the amino acid $i$ at position $j$ of the period. As the assessment of fitness (objective function) for the organism (weight matrix $M$ ), the maximum value of the similarity function $F_{\max }$ was considered for the local alignment (Altschul et al., 1990). A local alignment was built between the sequences $S_{1}$ and $S$, using a weight matrix $M$ to calculate the objective function. The
calculation of $F_{\text {max }}$ was conducted for each organism (weight matrix $M$ ). The process was repeated after applying genetic operators to the organisms. The process was stopped after a stable population was achieved, that is, increase in the values of $F_{\max }$ was stopped. As a result, the matrix $M_{0}$ was defined for the period length $n$ with the greatest $F_{\max }$. The alignment of sequences $S_{1}$ and $S$ was well built using the matrix $M_{0}$. The algorithm discussed is as shown in Figure 1. The algorithm was repeated for $n$ from 2 to 100 .


Figure 1: The main stages of the genetic algorithm used in the study.

### 2.1 Initialization

The first step of the algorithm is to provide a zero generation of organisms (weight matrix in our case) for the local alignment. A random population of organisms was selected as zero generation. The zero generation of organisms must be maximally diverse
in order to more quickly achieve a stable population and find matrix $M_{0}$, maximum of $F_{\max }$. Organisms (matrix of the size $20 \times n$ ) can be viewed as points in space with a size of $20 \times n$. It is possible to achieve the maximum diversity of organisms, if the points are selected in space $20 \times n$ spaced at a distance $D>D_{0}$. The coordinates of these points are the initial matrices (organisms). The distance between both matrices (organisms), the Euclidean distance between the two points in space $20 \mathrm{x} n$, was taken.

$$
\begin{equation*}
D=\sqrt{\sum_{i=1}^{20} \sum_{j=1}^{n}\left(m_{1}(i, j)-m_{2}(i, j)\right)^{2,0}} \tag{1}
\end{equation*}
$$

where $m_{1}(i, j)$ and $m_{2}(i, j)$ are elements of the two matrices ( $M_{1}$ and $M_{2}$ ) compared. The population size should be large enough. Organisms having a high fitness function are distributed too quickly in populations with a small size. The population becomes homogeneous and the probability of continuation of the evolution becomes very small. This means that the algorithm can find the local rather than the global maximum of $F_{\text {max, }}$, in the case of a small population size. At the same time, descendants produced in large populations are likely to be more varied, although an increase in $F_{\max }$ is much slower. A population size equal to $10^{4}$ was used for all the results presented here. These $10^{4}$ weight matrices were chosen so as to cover the space $20 \times n$, as fully as possible. Each matrix $M$ (organism) was created by comparing the sequence $S_{1}$ with a random sequence of length $N$. The random sequence $S r_{i}$ was obtained by mixing the original sequence $S$, with $i$ varied from 1 to $10^{4}$. The frequency matrix $V(20, n)$ was completed as follows. To elements of the matrix $v\left(s r(k), s_{1}(k)\right)$, a value of 1 was added for all $k$ from 1 to $N$, where $s r(k)$ is an element of the sequence $S r_{i}$. Then, based on the matrix $V$, the weighting matrix $M(20, n)$ was calculated as:

$$
\begin{equation*}
m(i, j)=\frac{v(i, j)-N p(i, j)}{\sqrt{N p(i, j)(1-p(i, j))}} \tag{2}
\end{equation*}
$$

where the partial sums for lines are $x(i)=\sum_{j} v(i, j)$ and for columns are $y(j)=\sum_{i} v(i, j), N=\sum_{i, j} v(i, j)$ and probabilities $p(i, j)=x(i) y(j) / N^{2}$. If this matrix is the first in the population, then it is automatically included in the initial population. If this matrix is not the first matrix, it is compared with all the matrices (organisms) already included in the population and the distance from each matrix was calculated using Formula 1. If the distances are
greater than the $D_{0}$, then the matrix is included in the initial population. Otherwise, this matrix is rejected and a new matrix is created. The level of $D_{0}$ was chosen so that the initial population will have from $10^{4}$ to $1.05 \times 10^{4}$ from $5 \times 10^{5}$ random matrices. Let us call the population of organisms (matrices) as $Q_{n}$.

### 2.2 Calculation of Fitness and Statistical Significance of the Organism

After the birth of a new organism (creating a new matrix $M$ ), the first step is to assess the fitness of the organism. This is the determination of $F_{\max }$ of the local alignment (Smith and Waterman, 1981) for sequences $S_{1}$ and $S$, using a weighting matrix $M$. The higher the value, $F_{\max }$ corresponds to a better alignment and to a lower probability $P\left(F_{r}>F_{\max }\right)$. The fitness of the organism (chapter 2) is higher for larger values of $F_{\text {max }}$. In more detail, the construction of a local alignment is discussed subsequently in paragraph 2.6. After completion of the genetic algorithm, an argument of the normal distribution for the organism $M_{\max }$ (which have the highest $F_{\max }$ ) was calculated using the formula:

$$
\begin{equation*}
Z_{m k}=\frac{F_{\max }-M[\vec{F}]}{\sqrt{D\left[\vec{F}_{\max }\right]}} \tag{3}
\end{equation*}
$$

where $\vec{F}_{\text {max }}=F_{\text {max }}^{1}, F_{\text {max }}^{2}, \ldots, F_{\text {max }}^{N}$ are the maximum weights of the local alignments between random sequence $S r_{i}$ and the sequence $S_{1}$, is determined using the best weight matrix $M_{\max }$.

Calculation of the vector $F_{\max }$ was performed using random sequences $S r_{i}$ derived from the amino acid sequence $S$ by random mixing. In total, 200 random sequences were created ( $N_{R}=200$ ). The necessity of using values $Z_{m k}$ instead of $F_{\max }$ at the end of the calculation, is due to the fact that the direct calculation of the probability $P\left(F_{r}>F_{\max }\right)$ is difficult, because of the very large amount of computations. Furthermore, while reducing the probability $P\left(F_{r}>F_{\max }\right)$, the amount of computations grew very quickly and for a good periodicity in the sequence $S$, the calculations could not be performed within a reasonable time. Therefore, it is convenient to use $Z_{m k}$ as a measure of statistical significance of $F_{\max }$ for the matrix $M_{\max }$. A similar calculation was performed for all the investigated period of length $n$ and the dependence $Z_{m k}(n)$, was obtained.

### 2.3 Completion of the Genetic Algorithm

Proofs that the genetic algorithm necessarily reach the global optimum, even for an infinite number of iterations, are currently non-existent (Mitchell, 1998). This necessitated the decision to stop the algorithm adopted by the heuristic criteria. Therefore, a decision was reached to use a combination of the two most common genetic algorithm stopping criteria (Banzhaf et al., 1998). The evolutionary process was continued as long as the best organism (matrix with the highest $F_{\max }$ ) will not be repeated for several generations, or will limit the number of iterations reached $\left(10^{4}\right)$. In this paper, the resulting solution is considered as the found global optimum. Figure 2 shows an example of the growth of $F_{\max }$ for the best organism in the population.


Figure 2: The graph of growth of the fitness $F_{\max }$ for the best individual in the population in the process of evolution.

### 2.4 Choice of Parents in the Genetic Algorithm

The choice of parents was made using a combination of approaches: the elite and fitness proportionate selection, also known as roulette-wheel selection (Bäck, 1996). To do this, firstly, all organisms were sorted on the degree of $F_{\max }$ increase and then, 20\% of organisms with the highest $F_{\max }$, were selected. Thereafter, two parents were selected among them with a probability that depends on the $F_{\max }$. If $F_{\max }^{i}$ is the fitness of the organism $i$ in the population, then the probability of the organism selection is as follows: $P_{i}=F_{\max }^{i} / \sum_{k=1}^{K} F_{\max }^{i}$ where $K$ is the size of the population. It is more likely that more adapted organisms will be selected as parents, if this approach is used. However, for the less fit
individuals, there is still a chance of being selected for reproduction and survival during evolution. This is an advantage over the purely elite strategy, despite the impracticality, an organism (weighting matrix) can contain successful portions (successful matrix elements). Then, these properties of the organism can be taken up by evolution and can contribute to the global maximum.

### 2.5 Reproduction of Organisms

The recombination operator was used immediately after the selection of parents for the creation of descendants. The essence of recombination is that created descendants should inherit genetic information from both parents. Then, the mutation operator was applied for each descendant.

### 2.5.1 Recombination of Organisms and the Creation of Descendants

A combination of the two-point crossover and differential crossing was used to create descendants. In this case, the organisms (matrix) were considered as a linear vector. This means that the matrix rows were built one behind the other in a line. These vectors were then closed in a ring formed by a compound at the ends of these vectors. Then, the random selection of two points on the ring was performed and the segment from one ring was used to replace the segment of the other ring (Fogel, 1998; Fogel, 2010). Two-point crossover showed an improvement over the single point crossover. Further addition of crossover points impairs the activity of the genetic algorithm as the increased destruction of organisms and evolutionary process slows down (Spears and De Jong, 1991; Sywerda, 1989).

Afterwards, the intermediate recombination was used. The values of "genes" of the organism (weight matrix elements) other than the value of the parental "genes", occur at an intermediate recombination. This leads to the emergence of new organisms with fitness that could be better than that of the parents. Such recombination operator in the literature is sometimes called differential crossing. If $x$ and $y$ are two organisms in a population (two weight matrix with elements $x(i, j)$ и $y(i, j)$ ), then the descendant is calculated by the formula (Radcliffe, 1991): $\quad z_{i j}=x_{i j} \pm \alpha\left(x_{i j}-y_{i j}\right) \quad$ where $i=1,2, \ldots, 20$, $j=1,2, \ldots, n$ and $\alpha \in[0,1]$ are random values with a uniform distribution. Here, the matrix of weights (organism) was considered as a vector. To create
descendants after the two-point crossover, two parents were involved. Then, two descendants (w and $v$ ) were formed using Formulae 4 and 5:

$$
\begin{gather*}
w(i, j)=\alpha x(i, j)+(1-\alpha) y(i, j)  \tag{4}\\
v(i, j)=(1-\alpha) x(i, j)+\alpha y(i, j) \tag{5}
\end{gather*}
$$

### 2.5.2 Creation of Mutations

By one of two methods, mutations were introduced in the descendants $W$ and $V$. The initial method of introducing mutations (probability for each method was 0.5 ) was randomly chosen. The first method replaced the randomly selected element of the weight matrix on a random number that is uniformly distributed in the range from -1 to 1 . The probability for a replacement $p_{1}$ is equal to 0.01 . All elements of all descendants exposed a random change of values. Changes were made to the whole matrix (all its values) on some small value, in the second method of making mutations. The intensity of the whole matrix mutation was determined by the probability $p_{2}$, which was randomly selected from the range of 0.001 to 0.03 . Each descendant element $w(i, j)$ of the matrix $W$ was replaced with a new element, calculated according to the formula: $v(i, j)=w(i, j) \pm p_{2} w(i, j)$ where $i=1,2, \ldots, 20$ and $j=$ $1,2, \ldots, n$. After making mutational changes, the fitness of descendants ( $W$ and $V$ ) was evaluated, that is, $F_{\max }$ was calculated for them. The descendant with a maximum value of $F_{\max }$ was added to the population $Q_{n}$. Concurrently, the worst organism with the smallest value of $F_{\max }$ was removed from the population $Q_{n}$. This method of replacing organisms in the population maintains the population size.

### 2.6 Construction of the Alignment and Choice of Weight for Deletion

### 2.6.1 Alignment of Amino Acid Sequence using the Random Matrices

A local alignment of sequences $S_{1}$ and $S$ was conducted using the weight matrices (organisms) and affine function penalty for insertions and deletions, to search $F_{\max }$ and the matrix $M_{0}$ (Durbin et al., 1998). To construct the alignment, the matrices for similarity functions $F, F_{1}$ and $F_{2}$ were filled for each matrix $M$ from the population (set $Q_{n}$ ). Matrix $M$ changed and turned into a matrix $M^{\prime}$.

$$
\begin{align*}
& F(i, j)=\max \left\{\begin{array}{l}
0 \\
F(i-1, j-1)+m^{\prime}\left(s_{1}(i), s(j)\right) \\
F_{1}(i-1, j-1)-d \\
F_{2}(i-1, j-1)-d
\end{array}\right\} \\
& F_{1}(i, j)=\max \left\{\begin{array}{l}
F(i-1, j)-d \\
F_{1}(i-1, j)-e
\end{array}\right\}  \tag{6}\\
& F_{2}(i, j)=\max \left\{\begin{array}{l}
F(i, j-1)-d \\
F_{2}(i, j-1)-e
\end{array}\right\}
\end{align*}
$$

where $s_{1}(i)$ and $s(i)$ are letters from the sequences $S_{1}$ and $S, d$ is the price for opening insertion or deletion in the sequences $S_{1}$ and $S, e$ is the price for the continued insertion or deletion in the sequences $S_{1}$ and $S$. Here, $i$ and $j$ changed from 1 to $N$. The matrices $F, F_{1}$ and $F_{2}$ have a dimension equal to $N \times N$, where $N$ is the length of sequences $S_{1}$ and $S$. $F_{\max }$ was selected as the maximum element of the matrix $F$. The coordinates of this element are $i_{m}$ and $j_{m}$.

Simultaneously, by calculating the matrixes $F, F_{1}$ and $F_{2}$ inverse transition matrix $F^{\prime}$ (same dimensions as the matrix $F$ ) were also filled. Each element of the matrix $F^{\prime}(i, j)$ contains the number of the matrix (1 for $F, 2$ for $F_{1}$ and 3 for $F_{2}$ ) and the number of element of the matrix $F$ or $F_{1}$ or $F_{2}$, which has a maximum value in Formula 6. Using the inverse transition matrix $F^{\prime}$, the alignment of the sequences $S_{1}$ and $S$ was built. The path in the matrix $F^{\prime}$ from the point $\left(i_{m}, j_{m}\right)$ to the point $\left(i_{0}, j_{0}\right)$, corresponds to the created alignment. At the first instance, the point $\left(i_{0}, j_{0}\right) F^{\prime}$ is equal to zero and serves as the beginning of the alignment. The matrix $M$ (organisms) from the set $Q_{n}$ (population) was used to create the alignment of sequences $S_{1}$ and $S$. For every matrix $M$ from the set $Q_{n}$, the values $R$ and $K_{d}$ were calculated before carrying out the alignment as:

$$
\begin{gather*}
R^{2}=\sum_{i=1}^{20} \sum_{j=1}^{n} m(i, j)^{2}  \tag{7}\\
K_{d}=\sum_{i=1}^{20} \sum_{j=1}^{n} m(i, j) f(i) t(j) \tag{8}
\end{gather*}
$$

where $f(i)=b(i) / N, b(i)$ is the number of amino acids of type $i$ in the sequence $S, t(j)=1 / n, N$ is the total number of amino acids in the sequence $S$. For calculation of the alignment, a changed matrix $M^{\prime}$ has to satisfy two conditions. The first condition is that $R$ for the matrix $M^{\prime}$ with the same period length $n$ would be identical and equal to $5(20 n)^{1 / 2}$. The dependence $R \sim n^{1 / 2}$ allows a similar distribution for $F_{\max }$ to be obtained, for a study of the different
random sequences $S r_{i}$. These random sequences were obtained by mixing the original sequence $S$.

The second condition is that the distribution functions for $F_{\text {max }}$ for each matrix from the set $Q_{n}$ should be close to each other. Such a distribution function can be determined for each matrix from the set $Q_{n}$, if this matrix is used to calculate the alignments of the sequence $S$ with each random sequence from the set $S r_{i}$. $K_{d}$ was selected for each matrix from the set $Q_{n}$ which would provide maximum identity $\bar{l}$ (see below this paragraph).

The above two conditions enabled the replacement of the matrix $M$ by the matrix that satisfies Equations 7 and 8. Equation 7 is the equation of the sphere in space $20 \times n$ and Equation 8 is an equation of the plane. If the matrix satisfies these conditions, then it lies on the circle $C$ formed by the intersection of the sphere (Equation 7) by the plane (Equation 8). Matrix $M$ was considered as a point in space $20 \times n$ and from this point, the nearest point was taken which lies on the circle $C$. The coordinates of this point are the desired matrix $M^{\prime}$. It is possible to use Equations 7 and 8 and to calculate the matrix $M^{\prime}$. Actually, it means that if we have the constant $R, K_{\mathrm{d}}$, matrix $M$ and calculate $f(i)$ for the sequence $S$, then the matrix $M^{\prime}$ (if there is the circle $C$ ) can be clearly defined. Matrix $M^{\prime}$ is used in Equation 6.

The next task was to choose the constant $K_{d}$ for each matrix from the set $Q_{n}$, which would provide the maximum identity of the distribution function of the $F_{\max }$. The average length of a random alignment
$\bar{l}$ for each matrix from the $Q n$ set, as the average for difference $\left(j_{m}-j_{0}\right)$ along with the calculation of the distribution function of the $F_{\max }$. Here, $j_{m}$ is the coordinate of $F_{\max }$ in sequence $S, j_{0}$ is the coordinate where $F=0.0$ in the calculation of the alignment (coordinate of the beginning of the alignment in the sequence $S$ ). The average length of the random alignment chosen is equal to $N / 5$. This value provides the best determination of the alignment boundaries with respect to the actual boundaries for the model sequences of length $N$. As model sequences, random sequences were selected for the insertion of a local alignment with periodicity for which $\mathrm{Z}>10.0$ (Korotkov et al,. 2003a) and length is from $N / 10$ to $N / 2$.

The constant $K_{d}$ was selected iteratively. $K_{d}$ provides $\bar{l}$ to be approximately $N / 5$ and obviously lies in the range from $K_{1}=0$ to $K_{2}=-20$. Then, the middle of this interval was taken. If $\bar{l}$ was more than $N / 5$, then $K_{1}=\left(K_{1}+K_{2}\right) / 2$ is calculated and if $\bar{l}$ is less than $N / 5, K_{2}=\left(K_{1}+K_{2}\right) / 2$ is calculated and the
process was repeated. Upon reaching the value $\bar{l}$ $=N / 5 \pm 20$, selection of the constant $K_{d}$ stopped.

Random sequences were created by the following algorithms. A number sequence was generated using a random number generator of the same length as the amino acid sequence. Thereafter, the sequence of random numbers was arranged in ascending order and the permutations made were memorized. These changes were applied to the amino acid sequence. Random amino acid sequences of good quality were created by this algorithm.

### 2.6.2 Weights of the Deletions and Other Constants

The constant $d$ for each period $n$ was determined separately. The constant $e$ was selected as $0.25 d$. A total of 100 test sequences were analyzed which were created for the period $n$ as follows. Artificial sequences were created with length equal to 1000 amino acids and contained a period $n$. The statistical significance of this periodicity $Z(n)$ defined by the information decomposition method is equal to 7.0 (Korotkov et al., 2003a). Insertions or deletions were introduced into the sequence randomly for every 50 amino acids. A constant $d$ was chosen which provides the greatest value $Z_{m k}$ by using Formula 3 . This value was applied for alignments using weighting matrices from the set $Q_{n}$.

### 2.7 Selection of the Threshold $Z_{0}$

Initially, $Z_{0}$ was estimated as the threshold for $Z_{m k}(n)$ to cut the influence of statistical noise. The method of this study was used to analyze 300 amino acid sequences. Therefore, the estimation of $Z_{0}$ for 300 random amino acid sequences was done. The sequence had a length equal to 600 amino acids and a period equal to 19 amino acids with 1.5 random changes per amino acid. To create the mutation, random positions were chosen in the sequence. Then, we changed the amino acid in a selected position that was randomly chosen (with probability which is equal for all amino acids). This was done 900 times for each sequence. From 4 to 15 inserts having the length, one amino acid was added in each sequence at random locations. This set was called $Q_{19}$. The ability of the developed approach to detect periodicity in a multitude $Q_{19}$ was tested. The results showed that periodicity can be detected in $93 \%$ of cases. We believe it is possible to achieve $100 \%$ result, but the number of iterations should be increased to approximately $10^{5}$ (see paragraph 2.3).

Then, these 300 sequences were analyzed and
$Z_{m k}(19)$ was calculated for each of them. Next, these 300 sequences were shuffled and a random sequence was obtained. Then, the random sequences were analyzed and a set of values $Z_{m k}(19)$ were obtained. Then, $Z_{0}$ equal to 10.0 was chosen since $N_{\text {random }}(10.0) / N_{\text {real }}(10.0)<5 \%$. It means that the number of errors of the first kind is less than 0.05 . Therefore, $N_{\text {random }}(10.0)$ shows a number of $Z_{m k}^{R}$ (19) with values equal to or more than $10.0 ; N_{\text {real }}(10.0)$ indicates the number of $Z_{m k}(19)$ equal to or greater than 10.0. The level of 10.0 was chosen for all $n$. The computational complexity of the algorithm is the reason why only 300 amino acid sequences were analyzed. An analysis of 300 sequences required about 6 months of calculations on a computer cluster with 10 AMD FX-8350 processors. Therefore, the task of analyzing the entire Swiss-prot database was not done, because it would require a lot of computer resources. The intention of the authors was to show that periodicity exists in amino acid sequences with many substitutions as well as where there are amino acid insertions and deletions. This periodicity can be detected by the approach developed in this study, despite being combined with other methods. The 300 amino acid sequences are enough to solve this problem.

## 3 EXAMPLES OF AMINO ACID SEQUENCES

In total, 300 amino acid sequences randomly selected from the Swiss-prot data bank (Boeckmann et al., 2003) were studied. In the process of selection, any sequence having already known amino acid repeats or repetitive domains (Kajava, 2012) were excluded from the set. As a result, 71 sequences were detected by our algorithm (any $\mathrm{Z}(n)>10.0)$ of having regions with the periodicity of various lengths. Lengths of regions with periodicity are more than 40 amino acids and number of periods is more than 3 . Three typical examples of sequences having insertions and deletions were considered and were found to have latent periodicity.

Figure 4 shows a second example of the spectrum $Z(n)$ for the sequence Q1D823 (Yang et al., 2004), which contains the adventurous-gliding motility protein. The region from 35 to 1373 amino acids contains periodicity with length equal to 7 amino acids, which can be revealed with deletions and insertions only. The $Z(7)$ of this region has a maximum value for all period lengths and is equal to 15.6. This region contains 4 extended coiled coil
regions. Alignment containing 20 deletions and insertions of different lengths, that is, the average length between the insertions and deletions is about 67 amino acids. Periodicity equal to 7 amino acids is typical for the coiled coil regions. This periodicity has the form HPPHCPC, where the positions of the period is referred to as abcdefg. Here, H represents hydrophobic residues, $C$ represents typically charged residues, and $P$ represents polar (and therefore, hydrophilic) residues. The positions of the heptad repeat are commonly denoted by the lowercase letters $a$ through $g$. These motifs are the basis for most coiled coils, particularly leucine zippers, which have predominantly leucine in the $d$ position of the heptad repeat. The periodicity observed in sequence Q1D823, is different from the periodicity specific for the coiled coil. It can be assumed that there are different heptad repeats, capable of forming a coiled coil. It is also likely that such a difference is due to insertions or deletions of amino acids. The findings of the present work indicate that the resulting matrix probably can be used to locate regions with long coiled coils.


Figure 3: Spectrum $Z(n)$ for the sequence O 42918 .


Figure 4: Spectrum $Z(n)$ for the sequence Q1D823.

Figure 5 shows the $Z(n)$ for the amino acid sequence P48681 (Dahlstrand et al., 1992) in a region from 182 to 1248 amino acids. The period which is equal to 11 amino acids is clearly visible. The periods of 22 and 33 amino acids are induced by the main period which equals 11 amino acids. The sequence with periodicity includes some coil regions and tail. The periodicity was discovered only in the presence of 24 amino acid insertions or deletions of various lengths. In the absence of insertions and deletions, this periodicity is not detectable.

We analyzed 71 amino sequences by the programs REP (Andrade et al., 2000), Internal Repeat Finder (Marcotte et al., 1999), Prospero (Mott, 1999), RADAR (Heger \& Holm, 2000), REPRO (Heringa \& Argos 1993), TRUST (Szklarczyk \& Heringa 2004) and PTRStalker (Pellegrini et al., 2012). These programs found periodicity in these sequences, if Z is more than 18.2. If Z lies in the interval from 18.2 to 15.5 , these programs found only $\sim 34 \%$ of our results. Also, if $10.0<Z<15.5$, then these methods found nothing. Totally, these methods found 6 regions with latent periodicity from 71 which was found in this work. As is written above (see paragraph 1), it is the consequence of using pairwise alignments between periods for the detection of latent periodicity (number of periods is more than 3 ).


Figure 5: Spectrum $Z(n)$ for the sequence P 48681 .
The question arises about the role of the observed periodicity in the structure and functions of proteins. Two assumptions were put forward about the functional role of the detected periodicity. Firstly, the periodicity found could be some property which provides a certain secondary structure (Jernigan and Bordenstein, 2015). This assumption has been expressed for the amino acid repeats, which were found earlier (Jorda et al., 2010; Kajava, 2012). In this study, there are periods of length 6 and 7 amino
acids which may participate in the formation of $\alpha$ helixes. Secondly, the periodicity found may reflect a certain spatial repeatability of protein parts belonging to 3D structures. For known repeats, this can be observed for the Zn -finger domains (Lee et al., 1989), Ig-domains (Sawaya et al., 2008) and the human matrix metalloproteinase (Elkins et al., 2002). In the work of Kajava (2012), "the structural classification of the repetitive proteins based on the length of their repeats" provides additional information.

The origin of multiple tandem repeats in proteins can be associated with the processes of multiple tandem duplications in DNA (De Grassi and Ciccarelli, 2009). It may come to the formation of new proteins (Björklund et al., 2006). Further evolution and accumulation of mutations (amino acid substitutions, deletions and insertions) could lead to the creation of latent periodicity with many amino acid substitutions, insertions and deletions. Periodicity was detected in the present work.

In the future, the computation time for this algorithm can be reduced and all known amino acid sequences accumulated in the Swiss-prot database will be analyzed again. Increase in performance is possible due to the use of other methods instead of a genetic algorithm for optimization of the weight matrix $M$ or application for calculations using large computing clusters.

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