Algorithms for Computing Inequality Joins

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Abstract: Although the problem of joins has been known ever since the concept of relational databases was introduced, much of the research in that area has addressed the question of equijoins. In this paper, we look at the problem of inequality joins, which compares attributes using operators other than equality. We develop an algorithm for computing inequality joins on two relations with comparisons on two pairs of attributes and then extend the work to queries involving more than two comparisons. Our work also derives a lower bound for inequality joins on two relations and show that the two-comparisons algorithm is optimal.

1 INTRODUCTION

Database joins is an old problem (Codd, 1970, Graefe, 1993, Mishra, 1992) with much of the research on the topic addressing the problem of equijoins. Inequality joins (again an old problem, (DeWitt, 1991, Klug, 1988)), where attributes are compared for inequality are less frequently used than equijoins in traditional database systems, but find utility in temporal databases and other applications such as database cleaning (Wang, 2013, Cao, 2012, Enderle, 2004, Khayyat, 2015), and sophisticated problems even in conventional applications.

For example, consider a company that sells a large number of products. By one estimate, a well-known online retailer offers close to half a billion different products in the United States alone. Among the various factors that play a role in the profits of a company is the amount of storage space required by a product. Often, the size of a specific product is not directly proportional to the profit on that item. For example, jewelry items are often small, but they typically generate more profit per unit volume than some bulkier items such as plastic chairs.

Assume that products that vary widely in their features are implemented as separate relations. (Jewelry and chairs might be two such products.) As an example, suppose C and D are two separate relations representing two categories of products. Among the numerous attributes, assume that the following are identically named in the two relations: *key*, the primary key of the relation; *vol*, the amount of space occupied by one unit of the product for storage; *profit*, the average profit for one unit of the product; and *unitsSold*, the number of units of the product sold per year.

An example of relation C is given in Table 1. Table 2 is an instance of D.

The company wants to minimize the cost of storage while maximizing profits. In connection with this, suppose it is considering increasing the inventory of some products in C relative to the inventory of some products in D. For this, it may wish to check for all pairs of tuples c in C and d in D whether $d.vol$ is more than $c.vol$ and $c.profit$ is more than $d.profit$; if the condition is true, and if space is an issue, the company *might* decide to stock a larger quantity of c at the expense of storage for d. It is thus worthwhile to execute the following query (call it STORAGE).

```
select c.key, d.key from C c, D d where
d.vol > c.vol and c.profit > d.profit;
```

Executing the query on the sample relations yields the following pairs of keys: (*c1, d2*), (*c1, d5*), (*c1, d7*), (*c2, d1*), (*c2, d2*), (*c2, d6*), (*c2, d7*), (*c3, d1*), (*c3, d2*), (*c3, d3*), (*c3, d4*), (*c3, d5*), (*c3, d6*), (*c3, d7*), (*c4, d2*), (*c5, d2*), (*c7, d2*).

The company may want to do more analysis: it might decide that comparing the *unitsSold* field should also be a factor in making a decision. It may then decide to execute a query such as the following.

```
select c.key, d.key from C c, D d where
d.vol > c.vol and c.profit > d.profit
and c.unitsSold > d.unitsSold
```

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The result of this query is (*c1, d7*), (*c2, d7*), (*c3, d1*), (*c3, d3*), (*c3, d4*), (*c3, d7*).

Table 1: Relation C.

key	vol	profit	unitsSold
c1	35	45	15
c2	15	35	10
c3	5	55	30
c4	35	12	10
c5	18	15	15
c6	90	55	80
c7	17	11	2

Table 2: Relation D.

key	vol	profit	unitsSold
d1	20	30	20
d2	50	10	35
d3	15	12	10
d4	16	52	12
d5	40	35	40
d6	20	20	30
d7	40	30	5
d8	2	57	15

Despite such applications, the problem of inequality joins remains an insufficiently researched area. A recent paper (Khayyat, 2017) makes quite a compelling case for more efficient inequality join algorithms. This inefficiency is not surprising because surprising because commercial database systems solve the problem using a nested loop that examines every pair of tuples.

Assume that *S* and *T* are relations with both relations containing fields named *k*, *A,* and *B*. The inequality join query, which we denote by *Q1*, involving these two fields *A* and *B* is the following, where θ_1 and θ_2 are relational operators (<, >, <=, and >=).

```
select s.k, t.k from S s, T t where
s.A  θ1  t.A AND s.B  θ2  t.B
```

In this work, we present an algorithm to solve inequality join queries. Our paper makes the following contributions: 1) It first develops an algorithm to solve the inequality join problem involving two comparisons. 2) It provides an extension to the algorithm in (1) to an arbitrary number of comparisons. 3) It derives a lower bound for the problem of inequality joins of the form given in *Q1*. The result applies to algorithms that employ comparisons to determine which tuple pairs belong to the result. 4) It shows that the algorithm in (1) above is optimal.

The rest of the paper is organized as follows. In the next section, we illustrate our algorithm with an example. For exposition purposes, we describe a good part of the algorithm using query STORAGE. In Sect. 3, we formally describe the algorithm and extend it to multiple comparisons. In Sect. 4, we derive a lower bound for the problem for two comparisons. An examination of related work is done in Sect. 5. Section 6 concludes the paper.

2 ALGORITHM CONCEPTS

In this section, we introduce the algorithm using the query STORAGE.

2.1 Page Setup

We rename the smaller relation *S* and the larger relation *T*. If the two relations are equal in size, we may name either one of them *S* and the other *T*. The attributes are named *k* (for key), *A* (the attribute used in the first comparison), and *B* (the attribute in the second comparison). The `where` clause is then changed so that all inequality comparisons are of the form s.X <rel_op> t.X.

For the rest of this section, we illustrate the algorithm using the query STORAGE. *C* is the smaller relation, so it is renamed *S*, and *D* becomes *T*. The attributes *key*, *vol*, and *profit* are renamed *k, A*, and *B*, respectively. For uniformity, we refer to the key values of *S* as *s1, s2*, etc. instead of *c1, c2*, and so on. Similarly, the key values of *T* are referred to as *t1, t2*, etc. The query becomes

```
select s.k, t.k from S s, T t where
s.A < t.A and s.B > t.B
```

We will use the letter *s* to denote an arbitrary *S* tuple and *t* to denote an arbitrary *T* tuple.

2.2 Regions

We now introduce the concept of **regions**. Clearly, the query result depends on the relative order of the *S* and *T* tuples based on attribute *A* (as well as *B*), that is, which *S* tuples have an *A* attribute value less than the *A* attribute value of the *T* tuples. Consider sorting the union of *S* and *T* on attribute *A* in the ascending order. We get the sequence *t8, s3, t3, s2, t4, s7, s5, t1, t6, s1, s4, t5, t7, t2, s6.*

Tuple *s3* has an *A* attribute value less than the *A* attribute values of all tuples other than *t8*, so it could be paired with all *T* tuples except *t8 as far as attribute*

A is concerned. It is this idea that we formalize into the notion of regions.

Since no *S* tuple has an *A* value less than *t8.A* and no *T* tuple has an *A* value greater than *s6.A*, tuples *t8* and *s6* cannot be part of the query result and are ignored. We can divide tuples other than *t8* and *s6* into sub-sequences with a non-empty sequence of *S* tuples followed by a non-empty sequence of *T* tuples. The sub-sequences are $(s3, t3)$, $(s2, t4)$, $(s7, s5, t1, t6)$, and $(s1, s4, t5, t7, t2)$.

Each of these four sub-sequences is called a **region** and are numbered 0 through 3. We call the numbers **region numbers**. Every tuple also has a region number, which is that of the region the tuple is in. For example, *s3* and *t3* have the *A* region number of 0. Similarly, *s2* and *t4* have the region number 1, *s7*, *s5*, *t1*, and *t6* have the region number 2, and *s1*, *s4*, *t5*, *t7*, and *t2* have the region number 3.

For attribute *B*, the comparison is $s.B > t.B$, so we sort the union of *S* and *T* in the descending order on *B*. We get 5 sub-sequences and the tuples are assigned region numbers 0 through 4: $(s3, t4)$, $(s1, t5)$, $(s2, t7, t1, t6)$, $(s5, t3)$, and $(s4, s7, t2)$. (Tuples *t8* and *s6* are omitted and in the case of a tie, we place the *T* tuple before the *S* tuple because the comparison is a less than.)

Every *S* and *T* tuple thus has two region numbers, one for each of the two sorted sequences. We call them *A* region number and *B* region number. Note that *s6* and *t8* do not get region numbers and are no longer considered in further stages of the algorithm.

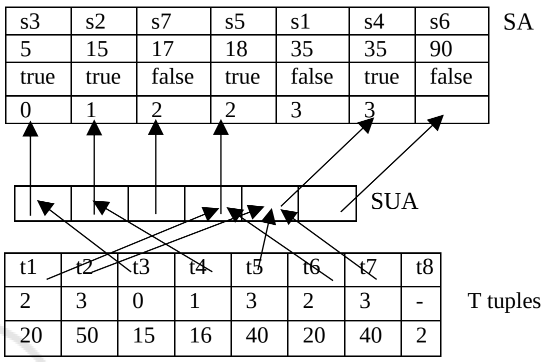
Although the concepts of regions and region numbers were described using the two sorted sequences of $S \cup T$, for the sake of efficiency, we compute the region numbers using a different approach, which we describe next.

2.2.1 Computing the Region Numbers

To compute the *A* region numbers, we first sort *S* on attribute *A* to get the table *SA* as shown in Fig. 1. Each cell contains, or will contain, the key, the *A* attribute value, a flag, and the region number. We create a second list *SUA* with one entry for each unique value of attribute *A*. The entries in *SUA* point to entries in *SA* as shown in the figure.

For each *T* tuple, we determine its *insertion point* in *SUA*. For example, *t4.A* is 16, so its insertion point is between *s2* and *s7* (because we have $s2.A < t4.A < s7.A$); we set the flag in *SA* to *true* to note that there is a *T* tuple *t* with *t.A* between *s2.A* and *s7.A*. Tuple *t4* stores a reference to *s2*. Similarly, we find insertion points for all *T* tuples. Note that *s7*'s flag is *false* because there is no *T* tuple *t* such that $s7.A < t.A < s5.A$. Also, *t8* does not have an insertion point because of its key value. There is no *T* tuple *t* with $t.A > s6.A$. Both *s6* and *t8* are therefore ignored.

We next compute the *A* region number of the *S* tuples. The region number is incremented whenever we encounter an *S* tuple s with the flag set to *true*. The *A* region number of a *T* tuple is the *A* region number of the *S* tuple it points to. For example, *t4* points to *s2*, so *t4*'s *A* region number is the same as *s2*'s *A* region number.



s3	s2	s7	s5	s1	s4	s6
5	15	17	18	35	35	90
true	true	false	true	false	true	false
0	1	2	2	3	3	

t1	t2	t3	t4	t5	t6	t7	t8
2	3	0	1	3	2	3	-
20	50	15	16	40	20	40	2

Figure 1: Region Number Computation.

For attribute B, since the relational operator is $>$ for the second comparison, we sort *S* in the descending order of the *B* attribute values. The *A* and *B* region numbers are given in the Fig. 2.

2.2.2 Computing the Query Result

If an attribute has exactly one region, that attribute need not be considered in computing the query result, so we assume the more general and non-trivial case where both attributes have at least two regions.

Let $s.r_A$ denote the region number of tuple *s* of relation *S*. Similar notations apply for the *B* region numbers and *T* tuples. A pair (*s, t*) is in the query result if and only if $s.r_A \leq t.r_A$ and $s.r_B \leq t.r_B$.

To compute the result, we need to access the tuples efficiently. We can access the *S* tuples in the region order using *SA* and *SB*, but the *T* tuples are a problem because they are not sorted. So we employ two separate collections called **region ordered tables** to access the *T* tuples in the region order, but not necessarily in the sorted order of *A* or *B*.

	s1	s2	s3	s4	s5	s7
A	3	1	0	3	2	2
B	1	2	0	4	3	4

	t1	t2	t3	t4	t5	t6	t7
A	2	3	0	1	3	2	3
B	2	4	3	0	1	2	2

Figure 2: A and B region numbers.

The table is implemented as a balanced tree, with one leaf node per region. The leaves are stored in a linked list, and each leaf points to a linked list of T tuples that have the region number of that leaf node.

The first step in computing the query result is to create an A region ordered table for the T tuples. This table shown in Fig. 3. The leaves hold region numbers and are shown as circles. We also create an empty region ordered table. This will contain the T tuples and will be ordered on the B region numbers.

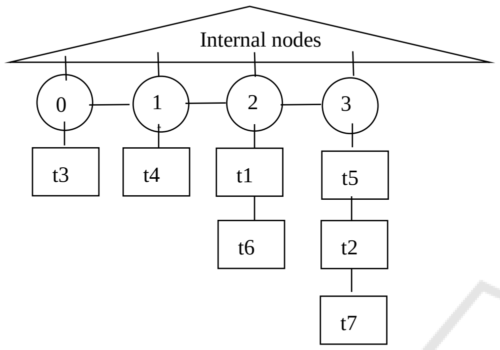


Figure 3: Data Structure Holding All the T tuples.

We then enter a loop starting with the highest numbered A region and work our way down to 0. For each region number i, we add to the B region ordered table those T tuples with $t.r_A = i$.

The highest A region is 3. We pick from the A region ordered table all the T tuples with A region number of 3. These are $t5$, $t2$, and $t7$. They are stored in the B region ordered table. The result is shown in Fig. 4. The S tuples with A region equal to 3 are $s1$ and $s4$. Since $s1.r_B = 1$ and $t5$, $t7$, and $t2$ all have $r_B \geq 1$, they are all paired with $s1$. Since $s4.r_B = 4$, $s4$ is paired with only $t2$ because $t2.r_B = 4$.

We proceed to A region 2. T tuples $t1$ and $t6$ have 2 for their A region number and are added to the table. The S tuples with $s.r_A = 2$ are $s5$ and $s7$. $s5.r_B = 3$, so $s5$ is paired with $t2$. $s7.r_B = 4$, so $s7$ is also paired with $t2$.

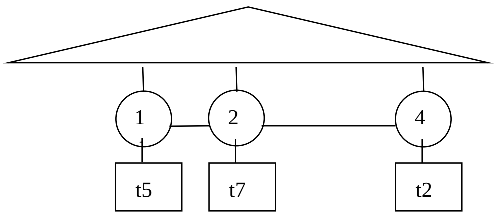


Figure 4: Table After Adding T Tuples with $t.r_A = 3$.

Moving down to A region 1, we add $t4$ to the table. We pair $s2$ ($s2.r_A = 1$) with $t1$, $t6$, $t7$, and $t2$ because they all have a B region number that is greater than or equal to $s2.r_B$. Finally, for A region 0, we add $t3$. Since $s3.r_B = 0$, $s3$ is paired with every tuple in the table.

3 FORMAL DESCRIPTIONS

In this section, we state the region computation and the final query result computation algorithms more formally and describe our extension to multiple comparisons.

3.1 The Two-Comparisons Algorithm

The algorithm for computing the A region numbers is shown in Fig. 5. The B region numbers are computed similarly. The complexities for the steps are indicated as we progress through the steps.

Lines 1-4 sort S on attribute A in the ascending or descending order (complexity: $O(m \log m)$) and lines 5-6 initialize the flag (complexity: $O(m)$). Steps 9-14 create the table SUA (complexity: $O(m)$).

Lines 15-41 compute the insertion points for the T tuples. The loop 23-37 is the standard binary search algorithm augmented to determine the insertion point. With the loop starting in line 19, the complexity of lines 15-41 is $O(n \log m)$.

```
1)  if rel_op is < or rel_op is <=
2)    SA = S sorted ascending on S.A
3)  else
4)    SA = S sorted descending on S.A
5)  for s in SA
6)    s.fA = false
7)  SUA = new array of indices to SA
8)  uniques = 0
9)  for 0 <= i < m - 1
10)   if SA[i].A != SA[i + 1].A
11)     SUA[uniques].index = i
12)     uniques = uniques + 1
13) SUA[uniques] = SA[m - 1]
14) uniques = uniques + 1
15) if rel_op is <  or  rel_op is >
16)   offset = -1
17) else
18)   offset = 0
19) for t in T
20)   low = 0
21)   high = uniques - 1
22)   found = false
23)   while low <= high and not found
24)     mid = (low + high)/2
25)     if SA[SUA[mid]].A == t.A
26)       found = true
27)     else if rel_op is <  or <=
28)       if SA[SUA[mid]].A < value
29)         low = mid + 1
```

Figure 5: Region Computation.

```
30)         else
31)           high = mid - 1
32)       else
33)         if SA[SUA[mid]].A > value
33)           low = mid + 1
34)         else
35)           high = mid - 1
36)       if found
37)          ins_pt = mid - offset
36)       else
37)          ins_pt = high
37)    if ins_pt == -1
38)      remove t from T
39)    else
40)      SA[SUA[ins_pt]].fA = true
41)      t.spos = SUA[ins_pt]
42) r = 0
43) for s in sorted order SA
44)    s.rA = r
45)    if s.fA
46)      r++
47) for t in T
48)    t.rA = t.spos.rA
```

Figure 5: Region Computation (Cont.).

The region numbers are computed in lines 42-48. The complexity is $O(n)$.The final result is computed by the algorithm in Fig. 6. Since there are at most m regions, TA can be created in time $O(n \log m)$.The iterator can be constructed in $O(\log m)$ time. The complexity of lines 5-14 is $O(n \log m + e)$, where e is the number of tuples in the result. Clearly, we have

Theorem 1. The algorithm complexity is $O(n \log m + e)$.

Theorem 2. The algorithm computes the inequality join correctly.
Proof. Omitted due to lack of space.

```
1 $T_A$: region ordered table
2 $T_B$: region ordered table
3 for each t in T
4    $T_A$.add(r, t)
5 for r from $N_A$ down to 0
6   itA = $T_A$.iterator(r)
7   while itA.hasNext()
8     t = itA.next()
9     $T_B$.add(t.$r_B$, t)
10  for each s with s.$r_A$ == r
11    itB = $T_B$.iterator(s.$r_A$)
12    while itB.hasNext()
13      t = itB.next()
14      output the pair (s, t)
```

Figure 6: Computing the Final Result.

3.2 Multiple Comparisons Algorithm

We start off as in the two-comparisons algorithm by renaming the relations and fields in the obvious manner. Suppose there are p comparisons. Let us denote the attributes involved in these comparisons by $F_1, F_2, \dots, F_p$. The algorithm maintains p copies of the S table, $S_1, S_2, \dots, S_p$, with S_i sorted on field $F_i, 1 \le i \le p$. It computes the region numbers for the p regions, one for each field $F_i, 1 \le i \le p$. It constructs p region ordered tables each storing all the tuples of the T table. Let us denote these by $TI_1, TI_2, \dots, TI_p$. TI_i is ordered on region $i, 1 \le i \le p$. It constructs p region ordered tables, each initially empty. Let us denote these by $TO_1, TO_2, \dots, TO_p$. TO_i is ordered on region $i, 1 \le i \le p$.

The algorithm in Fig. 5 in its entirety and lines 1-4 of the algorithm in Fig. 6 are employed to implement all of the above functionalities. The difference between this extension and the two-comparisons algorithm is in the way the final query result is computed. The extension employs a greedy approach. For this, we make the observation that in the two-comparisons algorithm, we perform far fewer comparisons at the beginning when we deal with the higher region numbers. With a uniform distribution of the tuples over all regions, we would expect the number of T tuples in region F_i to be n/N_{F_i} and the number of S tuples to be m/N_{F_i}, where N_{F_i} is the number of regions for field F_i. Therefore, the number of pairs of S and T tuples we need to consider *for the highest region number* for F_i is at most $(mn)/(N_{F_i} N_{F_i})$.

```
tuplesLeft = m, the number of S tuples
while tuplesLeft > 0
    select A, a field (see discussion)
    r = largest-numbered unprocessed
            region number in A;
    execute processField(A, r)
```

Figure 7: Handling Multiple Fields.

```
processField(A, r):
$TI_A$: region ordered table for field X
$TO_A$: region ordered table for field X
for each t in $TI_A$
  $TO_A$.add(r, t)
B = any field $\neq$ A with $N_A > 1$
itA = $TI_A$.iterator(r)
while itA.hasNext()
   t = itA.next()
   $TO_B$.add(t.$r_B$, t)
for each s with s.$r_A$ == r and
   s.processed == false
   itB = $T_B$.iterator(s.$r_B$)
   while itB.hasNext()
     t = itB.next()
     for every field g $\neq$ A or B
        if $s.r_g > t.r_g$
          exit the outer for loop
      output (s, t);
   s.processed = true
   tuplesLeft--
```

Figure 8: Computing the Final Result.

The above observation is exploited in the algorithm shown in Fig. 7. We select a field A for which we judge the number of comparisons to be the smallest. For each field F_i, assume the following: the largest unprocessed region number is r_i; the number of S tuples in region r_i is US_i; the number of tuples in the region ordered table TO_i is V_i; and the number of T tuples in region r_i is UT_i. Then we select field A for which the expression $\frac{US_i}{V_i+UT_i}$ is the smallest.

The tuple pairs are computed by the algorithm in Fig. 8. After we retrieve a T tuple t from the region ordered table TO_A that matches an S tuple s for region number based on B, we check if $t.r_g \geq s.r_g$ for all other fields g as well. As the region numbers decrease, more and more T tuples enter the region ordered tables, but we also flag more S tuples at higher numbered regions when we encounter fewer T tuples. This way, the total number of pairs to be considered does not increase sharply.

4 A LOWER BOUND

In this section, we derive a lower bound for the number of operations needed to enumerate all pairs of tuples that satisfy an inequality join query of the form given in *Q1* (Sect. 1). We consider only algorithms that employ comparisons to determine the query result. We will assume that the number of tuples in S and T are m and n, respectively, and that $m \leq n$. For simplicity, the two relations S and T are assumed to have identically named attributes.

Any deterministic algorithm to compute the query result needs to determine the tuple pairs in the result and enumerate them. If there are e tuple pairs in the result, any algorithm must take $O(e)$ steps to enumerate them.

Consider two relations S and T such that for any pair of tuples o_1 and o_2 in $S \cup T$, either $o_1\theta_1 o_2 \ and \ o_1\theta_2 o_2$ is true or $o_2\theta_1 o_1 \ and \ o_2\theta_2 o_1$ is true; that is, all pairs of tuples are totally ordered on the pair of comparisons. Clearly, this is a weaker problem than the one we addressed in Sec. 2 and 3. With a total order, to determine the query result, we need to determine which of the possible total orders of the S and T tuples is present in the input *with one restriction*: the relative order of the S tuples themselves or the relative order of the T tuples themselves do not play a role in the output.

4.1 Number of Input Sequences

The tuples of S could be distributed into k *non-empty* sets, where $1 \leq k \leq m$; these subsets are denoted by $S_1, \ldots, S_k$. The number of ways in which a set of m elements can be divided into k non-empty subsets is given by Stirling number of the second kind (Graham, 1988) and is denoted by $S(m,k)$. The n tuples of T could be distributed in-between (and/or before and after) these sets in four different ways.

1. Distribute the n tuples between the pairs of S tuples. There are $k-1$ such locations. The m tuples of S are distributed in $S(m,k)$ ways. Although their relative order within a set should not be counted, the relative order of each subset S_i with respect to the subsets of T that occupy the $k-1$slots is important and should be counted. Thus, there are $S(m,k)k!$ possible sequences of the m tuples. The T tuples can be distributed in $S(n,k-1)$ ways into the $k-1$ slots and the permutation of these $k-1$ subsets must also be counted. This gives us $S(n,k-1)(k-1)!$ sequences. Thus, the total number of sequences is $S(m,k)k!\,S(n,k-1)(k-1)!$.

2. In addition to using up all $k-1$ locations between the S tuples, we could attach an additional non-empty set of T tuples before S_1, (which is the first subset of S). This gives rise to k non-empty sets of T tuples. The number of sequences is $S(m,k)k!\,S(n,k)k!$.

3. This is similar to item 2 above. We could have k non-empty sets of T tuples by putting a non-empty set of T tuples after S_k. The number of sequences is again $S(m,k)k!\,S(n,k)k!$.

4. This case applies only if there are enough T tuples to go around. That happens if $n > m$ or $k < m$. We form $k+1$ subsets by putting a non-empty set of T tuples before S_1 and after S_k. The number of sequences is $S(m,k)k!\,S(n,k+1)(k+1)!$.

Therefore, unless $k=m=n$, the number of sequences with k non-empty subsets of S tuples is obtained by adding up the four terms given above to get the following expression.

$$\begin{aligned} &S(m,k)k!\,S(n,k-1)(k-1)! \\ &\quad + 2S(m,k)k!\,S(n,k)k! \\ &\quad + S(m,k)k!\,S(n,k+1)(k+1)! \end{aligned}$$

If $k=m=n$, the number of sequences is

$$S(m,k)k!\,S(n,k-1)(k-1)! \\ +\ 2S(m,k)k!\,S(n,k)k$$

To get a lower bound we can simply use the fact that

$$\sum_{k=1}^{m} S(m,k)k!\,S(n,k)k! \geq \sum_{k=1}^{m} S(m,k)k!\,S(n,k)$$

But $S(m,k) \geq \binom{m}{k}$ because we can divide the m tuples into k subsets by having $m-k$ tuples in one subset and putting the remaining k tuples into $k-1$ non-empty subsets, giving at least $\binom{m}{k}$ distributions. The number of input sequences is thus no less than $\sum_{k=1}^{m}\binom{m}{k}k!\,S(n,k) = m^n$(Griffith, 2010).

The algorithm must pick the correct input sequence from the m^n possible inputs. We can easily employ an oracle argument as in the case of the derivation for the lower bound for sorting and see that in the worst case, a comparison may reduce the number of possible sequences by half. Therefore, the number of comparisons needed is at least $O(n \log m)$. In addition, if the result contains e tuple pairs, the algorithm must spend $O(e)$ time to enumerate them. This gives us the following result.

Theorem 3. Let $\mathcal{A}$ be any inequality join query processing algorithm that uses comparisons to determine the tuple pairs in the result and enumerate them. The minimum number of steps for $\mathcal{A}$ to complete its execution when processing a query on two relations S and T with cardinalities m and n respectively, where $m \leq n$, is $O(n \log m + e)$, where e is the number of tuple pairs in the result.

From Theorem 2 and Theorem 3, we have

Theorem 4. The two-comparisons algorithm is optimal.

5 RELATED WORK

Classes of joins other than equijoins that have received less attention but have their own applications include inequality joins (Klug, 1988, Chandra, 1977, DeWitt, 1991) and similarity joins (Silva, 2012). The MapReduce framework has been used to compute joins. A work using this framework to compute inequality joins is by Okcan (Okcan, 2011). There are other important implementations of equijoins and similarity joins using the MapReduce framework including works by Blanas et al, (Blanas, 2010) Silva and Reed (Silva, 2012), Vernica, et al (Vernica, 2010), and Afrati and Ullman (Afrati, 2010).

A significant work by Khayyat et al (Khayyat, 2017) essentially addresses the same problem we took up here. Their resort to sorting both S and T, whereas we sort only the smaller relation. A difference between the works is that their algorithm takes $O(mn + e)$ time, while our approach takes $O(n \log m + e)$ time. Our algorithm is optimal for a pair of relations on two pairs of fields.

Inequality joins have found applications in areas such as XML query processing to perform containment joins (Wang 2003). A containment join between a set of ancestor nodes (denoted as *A*) and a set of descendant nodes (denoted as *D*) is to find all pairs of $(a,d), a \in A, d \in D$, such that a is an ancestor of d. A solution to inequality joins can be applied to help process these queries.

Inequality joins can be applied to address questions in temporal databases (Cao, 2012, Enderle, 2004) and have a role in database cleaning (Chaudhuri, 2006, Khayyat, 2015).

6 CONCLUSIONS

In this paper, we looked at the problem of inequality joins, an important class of joins that has received less attention than equijoins. We derived a lower bound for the problem of inequality joins of two relations and came up with an optimal algorithm that solved the problem for two comparisons. We showed how to extend the approach to more than two comparisons.

We plan to investigate how the multiple comparisons algorithm could be parallelized. The approach seems to support a high degree of concurrency because it processes tuples by region.

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