A Novel Reliable Leader-Following Consensus for Continuous-Time Multi-Agent Systems Under Nonhomogeneous and Asynchronous Markov Network Topology

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Abstract: This paper tackles the challenge of achieving robust leader-following consensus in multi-agent systems facing actuator faults and asynchronous network-dependent controllers. Specifically, it establishes sufficient conditions for ensuring reliable consensus, including: i) integration of actuator faults and network topology asynchronism into the control synthesis for each follower, ii) solutions to convex problems arising from the multiplication between time-varying transition rates and conditional probabilities, and iii) development of an innovative relaxation technique that reformulates $\mathcal{H}_\infty$ stabilization conditions into linear matrix inequalities.

1 INTRODUCTION

In recent years, multi-agent systems (MASs) have drawn attention in various applications such as unmanned vehicles, rendezvous, distributed sensor networks, formation control, flocking control, and swarming control. The most interesting issue in the control problem of MASs is consensus control, which empowers a team of agents to achieve a unified agreement. Consensus control is typically classified into two categories: leader-following consensus and leaderless consensus. The main advantage of the leader-following consensus control is that it enhances communication efficiency, conserves energy, and reduces control costs. In leader-following consensus, the leader functions independently of other agents, while the remaining agents track the leader's state trajectories. However, the network topology of MASs in practical network environments can be influenced and continuously fluctuating over time due to various factors such as connectivity disturbances, bandwidth limitations, and random packet dropout. To alleviate the concerns raised by the drawback of fixed network topology of MASs, the concept of time-varying network topology has been recently focused. Especially, Markov process has been emerged as an effective modeling approach for representing the random abrupt variation of variables in controlled systems. Therefore, one of the effective way to capture the sudden random shifts in network topology in MASs is embedding the framework of Markov process in the considered MASs. In particular, to determine the network topology in MASs under Markov network topology, it is essential to obtain the values of transition rates. However, measuring the exact value of transition rates is challenging due to high equipment costs, complexities in sensor interface, and limited sensor precision. Therefore, the concept of a nonhomogeneous Markov process is ideal for representing network topology in leader-following consensus of MASs, where the transition rates are either partially known or entirely unknown.

Concurrently, the phenomenon of actuator faults can occur in many control systems due to various reasons, including the inherent physical limitations of actuators, electrical or mechanical failures, and environmental factors. In this context, reliable control has become an inevitable trend in the state-of-the-art control engineering, aiming to ensure that control systems operate satisfactorily even in the presence of actuator faults. At the same time, packet dropout and time delay can frequently happen during the analysis of network topology modes and the transmission of that information to each follower's controller. Thus, this makes it challenging for the controller to identify

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control topology that stays in synchronization with the dynamic network topology. Based on these observations, it is comprehensively necessary to investigate the impact of asynchronous control and actuator faults on the leader-following consensus of MASs under nonhomogeneous Markov network topology. The concept of Markov switching network topologies for MASs has been firstly applied in (Li et al., 2015); however, the network topologies were assumed to be completely fixed and constant. Later, (Sakthivel et al., 2018) studied the leader-following exponential consensus of MASs by investigating a set of Lyapunov function candidates containing integral terms. However, the network topology was also assumed to be characterized by a constant Markov network topology. Most recently, (Yang et al., 2023) addressed the leader-following consensus control problem for discrete-time MASs with actuator faults. To handle the uncertain semi-Markov network topology, the transition rates in (Yang et al., 2023) were assumed to be bounded and confined to a polytope. Based on the above discussion, the leader-following consensus for MASs, considering actuator faults and asynchronous control topology, should be given greater focus, especially when analyzing the transitions in the nonhomogeneous Markov network topology.

This paper tackles the challenge of achieving reliable asynchronous leader-following consensus for MASs under a nonhomogeneous Markov network topology with $\mathcal{H}_\infty$ performance. The main contribution of this paper can be summarized as follows:

- Unlike (Li et al., 2015; Sakthivel et al., 2018; Wang et al., 2019), the network topology is characterized by a nonhomogeneous Markov process, which includes a more general and practical formulation of transition rates. Since obtaining transition rates that provide valuable information about the network topology in real-time is challenging, considering partially known or entirely unknown transition rates can result in a less conservative stabilization condition.
- As mentioned above, the controller at each follower must detect the network topology to determine important network-dependent matrices, such as the Laplacian matrices and the leader adjacency matrices, and use them to construct the control signals. However, it is practically difficult to detect the exact network topology and fully utilize these matrices at each follower's control side. As a result, asynchronous reliable controllers with actuator faults have been designed using an asynchronous control-topology-dependent Lyapunov function and a linear matrix inequalities (LMIs) approach.
- To manage the parameterized linear matrix inequalities (PLMIs) influenced by both time-varying transition rates and conditional probabilities, the relaxation technique is necessary to convert PLMIs-based stabilization conditions in solvable ones. Compared to (Yang et al., 2023), the continuous-time nonhomogeneous Markov network topology we are considering is more complex due to the presence of both negative and positive transition rates, whereas only positive rates are considered in the discrete-time case in (Yang et al., 2023). The differing signs of transition rates are crucial in relaxation techniques, as they make the relaxation technique more intricate. Furthermore, unlike (Nguyen and Kim, 2019; Nguyen and Kim, 2021), the relaxation technique is based on an augmented matrix formed from both the partly known transition rates and the entirely unknown transition rates, allowing the inclusion of a broader scope of transition rates in the relaxation technique.

Notations. The term $(*)$ is utilized to express symmetrical terms in symmetric matrices; $\mathbf{E}\{\cdot\}$ represents for the mathematical expectation; $\mathbf{He}\{Q\} = Q + Q^T$ where Q^T is the transpose matrix of Q; $\otimes$ denotes the Kronecker product. Moreover, for scalar or vector a_i, $\mathbf{col}(a_1, a_2, \cdots, a_n) = [a_1^T \ a_2^T \ \cdots \ a_n^T]^T$ and $\mathbf{diag}(\cdot)$ is a diagonal matrix. Correspondingly, some useful augmented matrices are utilized as follows: For $\mathbf{A} = \{a_1, a_2, \cdots, a_n\}$, $[Q_{a_i}]_{a_i \in \mathbf{A}} = \mathbf{col}(Q_{a_1}, Q_{a_2}, \cdots, Q_{a_n})$ and $[Q_{a_i}]^{\mathbf{d}}_{a_i \in \mathbf{A}} = \mathbf{diag}(Q_{a_1}, Q_{a_2}, \cdots, Q_{a_n})$, where Q_{a_i} denotes real sub-matrices with appropriate dimensions.

2 PROBLEM STATEMENT

2.1 Markov Network Topology

Multi-agent systems under the network topology are established as time-varying directed graph $\mathbf{G}_{\phi_t} = (\mathbf{S}, \mathbf{E}_{\phi_t}, \mathbf{A}_{\phi_t})$, where $\mathbf{S} = \{n_i\}$ denotes the nodes set or agents set where $i \in \mathbf{N} = \{1, 2, \ldots, N\}$; $\phi_t \in \mathbf{N}_\phi = \{1, 2, \cdots, n_\phi\}$ denotes the network topology mode; $\mathbf{E}_{\phi_t} \subseteq \{(n_j, n_i) \mid n_i, n_j \in \mathbf{S}, j \neq i\}$ denotes the edge set with the ordered pairs (n_j, n_i) meaning the information flow from node n_j to node n_i; and $\mathbf{A}_{\phi_t} = [a_{ij,\phi_t}]_{i,j \in \mathbf{N}}$ denotes the adjacency matrix with $a_{ij,\phi_t} = 0$ if and only if the pair $(n_j, n_i) \notin \mathbf{E}_{\phi_t}$ and $a_{ij,\phi_t} > 0$, otherwise. The leader depends on an extended directed graph $\mathbf{G}^0_{\phi_t} = (\mathbf{S}^0, \mathbf{E}^0_{\phi_t})$ with $\mathbf{S}^0 = \mathbf{S} \bigcup \{n_0\}$ and $\mathbf{E}^0_{\phi_t} \subseteq \{(n_0, n_i) \mid n_i \in \mathbf{S}\}$. In addition, the network-topology-dependent Laplacian matrix of $\mathbf{G}_{\phi_t}$ is constructed by $L_{\phi_t} = \left[l_{ij,\phi_t}\right]_{i,j\in\mathbf{N}} = D_{\phi_t} - A_{\phi_t}$, where $D_{\phi_t} = \left[d_{i,\phi_t}\right]^{\mathbf{d}}_{i\in\mathbf{N}}$ and $d_{i,\phi_t} = \sum_{j\in\mathbf{N}} a_{ij,\phi_t}$. Furthermore, the network-topology-dependent leader adjacency matrix of $\mathbf{G}^0_{\phi_t}$ is established by $M_{\phi_t} = \left[m_{i,\phi_t}\right]^{\mathbf{d}}_{i\in\mathbf{N}}$, where $m_{i,\phi_t} > 0$ if and only if the leader n_0 transmits information to the follower n_i and $m_{i,\phi_t} = 0$, otherwise.

Notably, the network topology is defined by a continuous-time nonhomogeneous Markov process $\{\phi_t, t \geq 0\}$ subject to the following time-varying transition probabilities:

$$\mathbf{Pr}\left(\phi_{t+\theta} = h \middle| \phi_t = g\right) = \begin{cases} \pi_{gh}(t)\theta + o(\theta) & \text{if } h \neq g \\ 1 + \pi_{gg}(t)\theta + o(\theta) & \text{if } h = g \end{cases}$$

where $\theta > 0$, $\lim_{\theta\to 0}(o(\theta)/\theta) = 0$; and $\pi_{gh}(t)$ stands for the transition rates (TRs) from topology mode g at time t to topology mode h at time $t+\theta$, satisfying

$$\pi_{gh}(t) \geq 0 \text{ and } \pi_{gg}(t) = -\sum_{h\in\mathbf{N}_\phi\setminus\{g\}} \pi_{gh}(t). \tag{1}$$

Moreover, based on the precision and availability of $\pi_{gh}(t)$, the network topology mode set can be composed of three subsets such that $\mathbf{N}_\phi = \mathbf{N}^\star_\phi + \widetilde{\mathbf{N}}_\phi + \mathbf{N}^\times_\phi$

$$\begin{aligned} \mathbf{N}^\star_\phi &= \left\{h \mid \pi_{gh} \text{ is known and constant }\right\} \\ \widetilde{\mathbf{N}}_\phi &= \left\{h \mid \pi_{gh}(t) \text{ is time-varying and bounded by } \pi_{gh}(t) \in [\underline{\pi}_{gh}, \overline{\pi}_{gh}]\right\} \\ \mathbf{N}^\times_\phi &= \left\{h \mid \pi_{gh}(t) \text{ is entirely unknown }\right\}. \end{aligned} \tag{2}$$

From the characteristics of each topology set (2), one can conclude the two main formulations related to the transition rates $\pi_{gh}(t)$:

- $\pi_{gh}(t) \in [\underline{\pi}_{gh}, \overline{\pi}_{gh}]$ for $h \in \widetilde{\mathbf{N}}_\phi$ (3)
- $0 = \pi_{gg} + \sum_{h\in\mathbf{N}^\star_\phi\setminus\{g\}} \pi_{gh} + \sum_{h\in\widetilde{\mathbf{N}}_\phi\setminus\{g\}} \pi_{gh}(t) + \sum_{h\in\mathbf{N}^\times_\phi\setminus\{g\}} \pi_{gh}(t)$, if $g \in \mathbf{N}^\star_\phi$. (4)

2.2 Multi-Agent Systems

Let us consider the following continuous-time dynamics of the ith follower and the leader with nonhomogeneous Markov network topology:

$$\begin{cases} \dot{x}^i_t = Ax^i_t + B\Gamma_t u^i_t + Df(x^i_t) + Ew^i_t \\ \dot{x}^0_t = Ax^0_t + Df(x^0_t) \\ z^i_t = C\left(x^i_t - x^0_t\right) \end{cases} \tag{5}$$

where $x^i_t \in \mathbf{R}^{n_x}$, $u^i_t \in \mathbf{R}^{n_u}$, $w^i_t \in \mathbf{R}^{n_w}$, $f(x^i_t) \in \mathbf{R}^{n_f}$, $x^0_t \in \mathbf{R}^{n_x}$, $f(x^0_t) \in \mathbf{R}^{n_f}$, and $z^i_t \in \mathbf{R}^{n_z}$ denote the state of the ith follower, the control input of the ith follower, the external disturbance of the ith follower satisfying $w^i_t \in \mathbf{L}_2[0,\infty)$, the nonlinear function of the ith follower, the state of the leader, the nonlinear function of the leader, and the performance output of the ith follower, respectively. Furthermore, $\Gamma_t = \left[\Gamma^\ell_t\right]^{\mathbf{d}}_{\ell\in\{1,2,\ldots,n_u\}}$ is employed to represent the actuator fault model, where the ℓth element Γ^ℓ_t satisfies $0 < \underline{\Gamma}^\ell \leq \Gamma^\ell_t \leq \overline{\Gamma}^\ell \leq 1$. Thus, Γ_t can be represented by $\Gamma_t = R + \Delta_t S$, where $R = \left[(\underline{\Gamma}_\ell + \overline{\Gamma}_\ell)/2\right]^{\mathbf{d}}_{\ell\in\{1,2,\ldots,n_u\}}$, $S = \left[(\overline{\Gamma}_\ell - \underline{\Gamma}_\ell)/2\right]^{\mathbf{d}}_{\ell\in\{1,2,\ldots,n_u\}}$, $\Delta_t = \left[\Delta^\ell_t\right]^{\mathbf{d}}_{\ell\in\{1,2,\ldots,n_u\}}$ with $|\Delta^\ell_t| \leq 1$.

Continuously, let us consider the following control protocol with asynchronous network topology:

$$u^i_t = F_{\rho_t}\left(\tilde{x}^i_t + m_{i,\phi_t}\left(x^i_t - x^0_t\right)\right) \tag{6}$$

where $\tilde{x}^i_t = \sum_{j\in\mathbf{N}} l_{ij,\phi_t}\left(x^j_t - x^0_t\right)$. In (6), $\rho_t \in \mathbf{N}_\rho = \{1,2,\cdots,n_\rho\}$ represents the control network topology, which is asynchronous to the multi-agent system's network topology described in (5). Here, $\tilde{x}^i_t$ denotes the synthesized signal, and F_{ρ_t} refers to the asynchronous control gain that will be designed later. Especially, to demonstrate the relationship between the nonhomogeneous Markov network topology and asynchronous control network topology, we construct the condition probability as follows:

$$\mathbf{Pr}\left(\rho_t = p \mid \phi_t = g\right) = \varpi_{gp} \tag{7}$$

where

$$\varpi_{gp} \in [0,1] \text{ and } \sum_{p\in\mathbf{N}_\rho} \varpi_{gp} = 1. \tag{8}$$

In the next step, the error between the leader and the ith follower are introduced by

$$\bar{x}^i_t = x^i_t - x^0_t \text{ and } \bar{f}^i_t = f(x^i_t) - f(x^0_t).$$

Based on the brief representation of $O_g = O(\phi_t = g)$ and $O_p = O(\rho_t = p)$, the dynamics of the ith error system can be formulated by:

$$\dot{\bar{x}}^i_t = A\bar{x}^i_t + B\Gamma_t F_p\left(\sum_{j\in\mathbf{N}} l_{ij,g}\bar{x}^j_t + m_{i,g}\bar{x}^i_t\right) + D\bar{f}^i_t + Ew^i_t. \tag{9}$$

Subsequently, let us introduce some following augmented vectors: $\bar{x}_t = \left[\bar{x}^i_t\right]_{i\in\mathbf{N}}$, $\bar{f}_t = \left[\bar{f}^i_t\right]_{i\in\mathbf{N}}$, $w_t = \left[w^i_t\right]_{i\in\mathbf{N}}$, and $z_t = \left[z^i_t\right]_{i\in\mathbf{N}}$. The resultant closed-loop system is given by:

$$\begin{cases} \dot{\bar{x}}_t = \left((I_N \otimes A) + (L_g + M_g) \otimes B\Gamma_t F_p\right)\bar{x}_t + (I_N \otimes D)\bar{f}_t + (I_N \otimes E)w_t \\ z_t = (I_N \otimes C)\bar{x}_t. \end{cases} \tag{10}$$

The following assumption and mathematical lemmas serve benefit platform for our future derivation:

Assumption 1. *(Valentine, 1945)* Given a constant $\varepsilon > 0$, the nonlinear function $f(x_t^0)$ and $f(x_t^i)$ satisfy the following Lipschitz condition:

$$||f(x_t^i) - f(x_t^0)|| \leq \varepsilon ||x_t^i - x_t^0||. \tag{11}$$

Lemma 2.1. *(Xie and de Souza, 1990)* For any matrices $\mathcal{S} \in \mathbf{R}^{n\times m}$, $\mathcal{T} \in \mathbf{R}^{n\times m}$, and $0 < \mathcal{P} = \mathcal{P}^T \in \mathbf{R}^{n\times n}$, the following inequality holds:

$$\mathbf{He}\{\mathcal{S}^T\mathcal{T}\} \leq \mathcal{S}^T\mathcal{P}\mathcal{S} + \mathcal{T}^T\mathcal{P}^{-1}\mathcal{T}. \tag{12}$$

Lemma 2.2. *(Takaba, 1998)* Let Ξ, $\mathcal{R}_1$, Δ_t, and $\mathcal{R}_2$ be real matrices with proper dimensions such that $\Delta_t^T\Delta_t \leq I$ and there exits a positive scalar α. Then, the following condition holds $0 > \mathcal{Z} + \mathbf{He}\{\mathcal{R}_1\Delta_t\mathcal{R}_2\}$, if the following condition is ensured:

$$0 > \begin{bmatrix} \mathcal{Z} + \alpha\mathcal{R}_1\mathcal{R}_1^T & (*) \\ \mathcal{R}_2 & -\alpha I \end{bmatrix}.$$

The target of this paper is to design a reliable asynchronous controller such that the closed-loop system (10) of the continuous-time multi-agent system (5) is stochastically stable with $\mathcal{H}_\infty$ performance level.

3 MAIN RESULTS

Let us choose the Lyapunov function candidate dependent on the asynchronous control network topology mode $\rho_t = p$:

$$V(t,\rho_t) = \bar{x}_t^T\big(I_N \otimes P_p\big)\bar{x}_t \tag{13}$$

where $0 < P_p = P_p^T \in \mathbf{R}^{n_x\times n_x}$ and $I_N \otimes P_p = \mathbf{diag}(\underbrace{P_p, P_p, \cdots, P_p}_{N})$. Then, the weak infinitesimal operator of $V(t,\rho_t)$ is provided by

$$\begin{aligned}
&\nabla V(t,\rho_t) \\
&= \lim_{\theta\to 0}\frac{1}{\theta}\mathbf{E}\Big\{V(t+\theta,\rho_{t+\theta} = q|\phi_t = g) \\
&\qquad - V(t,\rho_t = p|\phi_t = g)\Big\} \\
&= \bar{x}_t^T\mathbf{He}\Big\{(I_N\otimes\widetilde{\mathbf{P}}_g)\big((I_N\otimes A)+(L_g+M_g)\otimes B\Gamma_tF_p\big)\Big\}\bar{x}_t \\
&\quad + \mathbf{He}\Big\{\bar{x}_t^T(I_N\otimes\widetilde{\mathbf{P}}_gD)\bar{f}_t\Big\} + \bar{x}_t^T\big(I_N\otimes\mathbf{P}_g^\times\big)\bar{x}_t \\
&\quad + \mathbf{He}\Big\{\bar{x}_t^T(I_N\otimes\widetilde{\mathbf{P}}_gE)w_t\Big\}
\end{aligned} \tag{14}$$

where

$$\widetilde{\mathbf{P}}_g = \sum_{p\in\mathbf{N}_\rho}\varpi_{gp}P_p,\ \mathbf{P}_g^\times = \sum_{h\in\mathbf{N}_\phi}\sum_{q\in\mathbf{N}_\rho}\pi_{gh}(t)\varpi_{hq}P_q.$$

In parallel, it follows from Assumption 1 that $||\bar{f}_t^i|| \leq \varepsilon||\bar{x}_t^i||$, which leads to

$$\bar{f}_t^T\bar{f}_t \leq \bar{x}_t^T(I_N\otimes\varepsilon^2 I_{n_x})\bar{x}_t. \tag{15}$$

Then (15) and Lemma 2.1 allow

$$\begin{aligned}
&\mathbf{He}\Big\{\bar{x}_t^T(I_N\otimes\widetilde{\mathbf{P}}_gD)\bar{f}_t\Big\} \\
&\leq \bar{x}_t^T\Big(I_N\otimes\widetilde{\mathbf{P}}_gDD^T\widetilde{\mathbf{P}}_g\Big)\bar{x}_t + \bar{f}_t^T\bar{f}_t \\
&\leq \bar{x}_t^T\Big(I_N\otimes\widetilde{\mathbf{P}}_gDD^T\widetilde{\mathbf{P}}_g\Big)\bar{x}_t + \bar{x}_t^T(I_N\otimes\varepsilon^2 I_{n_x})\bar{x}_t \\
&= \bar{x}_t^T\Big(I_N\otimes\sum_{p\in\mathbf{N}_\rho}\varpi_{gp}P_pDD^TP_p\Big)\bar{x}_t + \bar{x}_t^T(I_N\otimes\varepsilon^2 I_{n_x})\bar{x}_t.
\end{aligned}$$

Next, let us define $\eta_t^T = \big[\bar{x}_t^T \mid w_t^T\big]$, the $\mathcal{H}_\infty$ leader-following consensus condition of (10) is constructed by

$$\begin{aligned}
&||z_t||^2 - \gamma^2||w_t||^2 + \nabla V(t,\rho_t) \\
&\leq \eta_t^T\Xi\eta_t < 0
\end{aligned} \tag{16}$$

where

$$\begin{aligned}
\Xi &= \left[\begin{array}{c|c} \Xi_{11} & (*) \\ \hline (I_N\otimes E^T\widetilde{\mathbf{P}}_g) & -\gamma^2 I_{N\cdot n_w}\end{array}\right] \\
\Xi_{11} &= \mathbf{He}\Big\{I_N\otimes\widetilde{\mathbf{P}}_gA\Big\} + \big(I_N\otimes\mathbf{P}_g^\times\big) + \big(I_N\otimes\varepsilon^2 I_{n_x}\big) \\
&\quad + \mathbf{He}\Big\{(I_N\otimes\widetilde{\mathbf{P}}_g)\big((L_g+M_g)\otimes B\Gamma_tF_p\big)\Big\} \\
&\quad + (I_N\otimes C^TC) \\
&\quad + \Big(I_N\otimes\sum_{p\in\mathbf{N}_\rho}\varpi_{gp}P_pDD^TP_p\Big).
\end{aligned}$$

To deal with the non-convexity of time-varying transition rates $\pi_{gh}(t)$ in parameterized linear matrix inequalities (PLMIs), the following theorem provides the relaxed reliable $\mathcal{H}_\infty$ control synthesis conditions, formulated in terms of linear matrix inequalities (LMIs). 5

Theorem 3.1. For given scalars ε and β, there exists $0 < \bar{P}_p = \bar{P}_p^T \in \mathbf{R}^{n_x\times n_x}$, $Q_{pq} = Q_{pq}^T \in \mathbf{R}^{n_x\times n_x}$, $\bar{F}_p \in \mathbf{R}^{n_u\times n_x}$, $T_{gh} = T_{gh}^T \in \mathbf{R}^{n_x\times n_x}$, $X_g \in \mathbf{R}^{n_x\times n_x}$, scalars $0 < \gamma$, and $0 < \alpha_g$, such that the following conditions hold: for $g \in \mathbf{N}_\phi$ and $p \in \mathbf{R}_\rho$

$$0 > \left[\begin{array}{c|c|c} \Omega_{(1)} + \mathbb{T}_{(1)} + \mathbb{X}_{(1)} & (*) & (*) \\ \hline \Omega_{(2)} + \mathbb{T}_{(2)} + \mathbb{X}_{(2)} & \mathbb{T}_{(3)} & 0 \\ \hline \Omega_{(3)} + \mathbb{X}_{(3)} & 0 & 0 \end{array}\right] \tag{17}$$

$$0 \leq \begin{bmatrix} Q_{pq} & (*) \\ \bar{P}_p & \bar{P}_q \end{bmatrix},\ \forall q \in \mathbf{N}_\rho \tag{18}$$

where

$$\Omega_{(1)} = \left[\begin{array}{c|ccc|c} \Omega^{\star}_{11} & (*) & (*) & (*) & (*) \\ \hline (I_N \otimes E^T) & -\gamma^2 I_{N\cdot n_w} & 0 & 0 & 0 \\ (I_N \otimes \varepsilon \bar{P}_p) & 0 & -I_{N\cdot n_x} & 0 & 0 \\ (I_N \otimes C\bar{P}_p) & 0 & 0 & -I_{N\cdot n_z} & 0 \\ \hline (I_N \otimes S\bar{F}_p) & 0 & 0 & 0 & -\alpha_g I_{N\cdot n_u} \end{array}\right]$$

$$Q_q(\beta) = \sum_{q\in \mathbf{N}_\rho} \varpi_{hq} Q_{pq} + \sum_{q\in \mathbf{N}_\rho} \varpi_{gq} \beta^2 \bar{P}_q - 2\beta \bar{P}_p$$

$$\begin{aligned}\Omega^{\star}_{11} = & I_N \otimes \left(\mathbf{He}\{A\bar{P}_p\} + DD^T\right) \\ & + \mathbf{He}\{(L_g+M_g)\otimes BR\bar{F}_p\} \\ & + \alpha_g \left((L_g+M_g)\otimes B\right)\left((L_g+M_g)\otimes B\right)^T \\ & + \left(I_N \otimes \sum_{h\in \mathbf{N}^{\star}_g\setminus\{g\}} \pi_{gh} Q_q(\beta)\right)\end{aligned}$$

$$\Omega_{(2)} = \left[\left(I_N \otimes \frac{1}{2} Q_q(\beta)\right)\chi\right]_{h\in \tilde{\mathbf{N}}_g\setminus\{g\}}$$

$$\Omega_{(3)} = \left[\left(I_N \otimes \frac{1}{2} Q_q(\beta)\right)\chi\right]_{h\in \mathbf{N}^{\times}_g\setminus\{g\}}$$

$$\mathbb{T}_{(1)} = \chi^T \left(I_N \otimes \sum_{h\in \tilde{\mathbf{N}}_g\setminus\{g\}} \bar{\pi}_{gh}\underline{\pi}_{gh} T_{gh}\right)\chi$$

$$\mathbb{T}_{(2)} = \left[\left(I_N \otimes \frac{1}{2}(-\underline{\pi}_{gh} - \bar{\pi}_{gh}) T_{gh}\right)\chi\right]_{h\in \tilde{\mathbf{N}}_g\setminus\{g\}}$$

$$\mathbb{T}_{(3)} = [(I_N\otimes T_{gh})]^{\mathbf{d}}_{h\in \tilde{\mathbf{N}}_g\setminus\{g\}}$$

$$\mathbb{X}_{(1)} = \begin{cases} \chi^T \left(I_N \otimes \mathbf{He}\left\{\sum_{h\in \mathbf{N}^{\star}_g} \pi_{gh} X_g\right\}\right)\chi, \text{ if } g\in \mathbf{N}^{\star}_g \\ 0, \text{ otherwise}\end{cases}$$

$$\mathbb{X}_{(2)} = \begin{cases} [(I_N\otimes X_g)\chi]_{h\in \tilde{\mathbf{N}}_g\setminus\{g\}}, \text{ if } g\in \mathbf{N}^{\star}_g \\ 0, \text{ otherwise}\end{cases}$$

$$\mathbb{X}_{(3)} = \begin{cases} [(I_N\otimes X_g)\chi]_{h\in \mathbf{N}^{\times}_g\setminus\{g\}}, \text{ if } g\in \mathbf{N}^{\star}_g \\ 0, \text{ otherwise}\end{cases}$$

$$\chi = \left[\begin{array}{c|ccc|c} I_{N\cdot n_x} & 0 & 0 & 0 & 0 \end{array}\right] \in \mathbf{R}^{N\cdot n_x \times N\cdot n_\chi}$$

$$n_\chi = 2n_x + n_w + n_z + n_u.$$

Proof: From the property of (8) that $\sum_{p\in \mathbf{N}_\rho} \varpi_{gp} = 1$, it can be referred (16) to the following control-topology-dependent $\mathcal{H}_\infty$ leader-following consensus:

$$0 > \Xi = \sum_{p\in \mathbf{N}_\rho} \varpi_{gp}\Xi_p \tag{19}$$

where

$$\Xi_p = \left[\begin{array}{c|c} \begin{array}{l}\mathbf{He}\{I_N\otimes P_pA\} + \left(I_N \otimes \mathbf{P}^{\times}_g\right) \\ +\mathbf{He}\{(L_g+M_g)\otimes P_pB\Gamma_t F_p\} \\ +\left(I_N\otimes P_pDD^TP_p\right) \\ +\left(I_N\otimes \varepsilon^2 I_{n_x}\right) + (I_N\otimes C^TC)\end{array} & (*) \\ \hline (I_N\otimes E^TP_p) & -\gamma^2 I_{N\cdot n_w}\end{array}\right].$$

In addition, from the property of (8) that $\varpi_{gp}\in[0,1]$, it can be referred (19) to the following control-topology-dependent $\mathcal{H}_\infty$ leader-following consensus:

$$0 > \Xi_p \tag{20}$$

where its Schur complement can be represented by

$$\Xi_p = \left[\begin{array}{c|ccc} \Xi_{11,p} & (*) & (*) & (*) \\ \hline (I_N\otimes E^TP_p) & -\gamma^2 I_{N\cdot n_w} & 0 & 0 \\ (I_N\otimes \varepsilon I_{n_x}) & 0 & -I_{N\cdot n_x} & 0 \\ (I_N\otimes C) & 0 & 0 & -I_{N\cdot n_z}\end{array}\right]$$

$$\begin{aligned}\Xi_{11,p} = & I_N\otimes\left(\mathbf{He}\{P_pA\} + P_pDD^TP_p\right) + \left(I_N\otimes \mathbf{P}^{\times}_g\right) \\ & + \mathbf{He}\left\{(L_g+M_g)\otimes P_pB\Gamma_tF_p\right\}.\end{aligned}$$

Next, let us perform the congruence transformation on (20) by $\mathbf{diag}(I_N\otimes \bar{P}_p, I_{N\cdot n_w}, I_{N\cdot n_x}, I_{N\cdot n_z})$ and its transpose, which leads to

$$0 > \left[\begin{array}{c|ccc} \bar{\Xi}_{11,p} & (*) & (*) & (*) \\ \hline (I_N\otimes E^T) & -\gamma^2 I_{N\cdot n_w} & 0 & 0 \\ (I_N\otimes \varepsilon\bar{P}_p) & 0 & -I_{N\cdot n_x} & 0 \\ (I_N\otimes C\bar{P}_p) & 0 & 0 & -I_{N\cdot n_z}\end{array}\right] \tag{21}$$

where

$$\begin{aligned}\bar{\Xi}_{11,p} = & I_N\otimes\left(\mathbf{He}\{A\bar{P}_p\} + DD^T\right) + \left(I_N\otimes \bar{P}_p\mathbf{P}^{\times}_g\bar{P}_p\right) \\ & + \mathbf{He}\left\{(L_g+M_g)\otimes B\Gamma_t\bar{F}_p\right\}\end{aligned}$$

$$\bar{P}_p = P_p^{-1},\ \bar{F}_p = F_p\bar{P}_p.$$

Especially, to deal with the non-convexity emerged from the multiplication time-varying transition rates and topology-dependent conditional probability, based on (1), the following equality is definitely necessary

$$\begin{aligned}\bar{P}_p\mathbf{P}^{\times}_g\bar{P}_p = & \sum_{h\in \mathbf{N}_\phi\setminus\{g\}} \pi_{gh}(t)\left(\sum_{q\in \mathbf{N}_\rho} \varpi_{hq}\bar{P}_pP_q\bar{P}_p\right. \\ & \left. - \sum_{q\in \mathbf{N}_\rho} \varpi_{gq}\bar{P}_pP_q\bar{P}_p\right).\end{aligned} \tag{22}$$

Moreover, Lemma 2.1 allows that

$$-\bar{P}_pP_q\bar{P}_p \le \beta^2 P_q^{-1} - 2\beta\bar{P}_p. \tag{23}$$

Based on (23) and (18) ensuring $\bar{P}_pP_q\bar{P}_p \le Q_{pq}$, (22) can be bounded by

$$I_N\otimes \bar{P}_p\mathbf{P}^{\times}_g\bar{P}_p \le \sum_{h\in \mathbf{N}_\phi\setminus\{g\}} \pi_{gh}(t)(I_N\otimes Q_q(\beta)) \tag{24}$$

where

$$Q_q(\beta) = \sum_{q\in \mathbf{N}_\rho} \varpi_{hq}Q_{pq} + \sum_{q\in \mathbf{N}_\rho} \varpi_{gq}\beta^2\bar{P}_q - 2\beta\bar{P}_p.$$

Thus, by combining (24) and recalling $\Gamma_t = R + \Delta_t S$, (21) can be ensured by

$$0 > \left[\begin{array}{c|ccc} \Omega_{11,\Delta} & (*) & (*) & (*) \\ \hline (I_N\otimes E^T) & -\gamma^2 I_{N\cdot n_w} & 0 & 0 \\ (I_N\otimes \varepsilon\bar{P}_p) & 0 & -I_{N\cdot n_x} & 0 \\ (I_N\otimes C\bar{P}_p) & 0 & 0 & -I_{N\cdot n_z}\end{array}\right] \tag{25}$$

where

$$\begin{aligned}\Omega_{11,\Delta} = I_N \otimes \left(\mathbf{He}\{A\bar{P}_p\} + DD^T\right) \\ + \mathbf{He}\{(L_g + M_g) \otimes BR\bar{F}_p\} \\ + \mathbf{He}\{(L_g + M_g) \otimes B\Delta_t S\bar{F}_p\} \\ + \left(\sum_{h \in \mathbf{N}_\phi \setminus \{g\}} \pi_{gh}(t)(I_N \otimes Q_q(\beta))\right).\end{aligned}$$

Under the help of Lemma 2.2 and the usage of $\chi = \left[\begin{array}{c|ccc|c} I_{N \cdot n_x} & 0 & 0 & 0 & 0 \end{array}\right]$, (25) is equivalent to

$$0 > \Omega + \chi^T \left(\sum_{h \in \mathbf{N}_\phi \setminus \{g\}} \pi_{gh}(t)(I_N \otimes Q_q(\beta))\right) \chi \quad (26)$$

where

$$\Omega = \left[\begin{array}{c|ccc|c} \Omega_{11} & (*) & (*) & (*) & (*) \\ \hline (I_N \otimes E^T) & -\gamma^2 I_{N \cdot n_w} & 0 & 0 & 0 \\ (I_N \otimes \varepsilon \bar{P}_p) & 0 & -I_{N \cdot n_x} & 0 & 0 \\ (I_N \otimes C\bar{P}_p) & 0 & 0 & -I_{N \cdot n_z} & 0 \\ \hline (I_N \otimes S\bar{F}_p) & 0 & 0 & 0 & -\alpha_g I_{N \cdot n_u} \end{array}\right]$$

$$\begin{aligned}\Omega_{11} = I_N \otimes \left(\mathbf{He}\{A\bar{P}_p\} + DD^T\right) \\ + \mathbf{He}\{(L_g + M_g) \otimes BR\bar{F}_p\} \\ + \alpha_g \left((L_g + M_g) \otimes B\right) \left((L_g + M_g) \otimes B\right)^T.\end{aligned}$$

Next, let us denote some useful matrices as follows:

$$\zeta_1(t) = \left[\pi_{gh}(t)\right]_{h \in \widetilde{\mathbf{N}}_g \setminus \{g\}}, \ \zeta_2(t) = \left[\pi_{gh}(t)\right]_{h \in \mathbf{N}_g^\times \setminus \{g\}}$$

$$\Phi^T(t) = \left[I \mid \left(\zeta_1(t) \otimes \chi\right)^T \mid \left(\zeta_2(t) \otimes \chi\right)^T\right].$$

Based on $\mathbf{N}_\phi = \mathbf{N}_g^\star + \widetilde{\mathbf{N}}_g + \mathbf{N}_g^\times$, (26) is rewritten by

$$\begin{aligned}0 > \Omega + \chi^T \Bigg(\sum_{h \in \mathbf{N}_g^\star \setminus \{g\}} \pi_{gh}(I_N \otimes Q_q(\beta)) \\ + \sum_{h \in \widetilde{\mathbf{N}}_g \bigcup \mathbf{N}_g^\times \setminus \{g\}} \pi_{gh}(t)(I_N \otimes Q_q(\beta))\Bigg) \chi \\ = \Omega_{(1)} + \mathbf{He}\left\{\left(\zeta_1(t) \otimes \chi\right)^T \Omega_{(2)}\right\} \\ + \mathbf{He}\left\{\left(\zeta_2(t) \otimes \chi\right)^T \Omega_{(3)}\right\}. \quad (27)\end{aligned}$$

Further, it can be referred from (26) that $0 > \mathbb{T}_{(3)}$ ensuring $0 > T_{gh}$ for all $h \in \widetilde{\mathbf{N}}_g \setminus \{g\}$. Then, it follows from (3) that

$$\begin{aligned}0 \leq \sum_{h \in \widetilde{\mathbf{N}}_g \setminus \{g\}} (\pi_{gh}(t) - \underline{\pi}_{gh})(\pi_{gh}(t) - \overline{\pi}_{gh}) \chi^T \left(I_N \otimes T_{gh}\right) \chi \\ = \mathbb{T}_{(1)} + \mathbf{He}\left\{\left(\zeta_1(t) \otimes \chi\right)^T \mathbb{T}_{(2)}\right\} \\ + \left(\zeta_1(t) \otimes \chi\right)^T \mathbb{T}_{(3)} \left(\zeta_1(t) \otimes \chi\right). \quad (28)\end{aligned}$$

Especially, if $g \in \mathbf{N}_g^\star$, (4) leads to

$$\begin{aligned}0 = \left(\sum_{h \in \mathbf{N}_g^\star} \pi_{gh} + \sum_{h \in \widetilde{\mathbf{N}}_g \setminus \{g\}} \pi_{gh}(t) + \sum_{h \in \widetilde{\mathbf{N}}_g^+ \setminus \{g\}} \pi_{gh}(t)\right) \\ \times \chi^T \mathbf{He}\left\{\left(I_N \otimes X_g\right)\right\} \chi \\ = \mathbb{X}_{(1)} + \mathbf{He}\left\{\left(\zeta_1(t) \otimes \chi\right)^T \mathbb{X}_{(2)}\right\} \\ + \mathbf{He}\left\{\left(\zeta_2(t) \otimes \chi\right)^T \mathbb{X}_{(3)}\right\}. \quad (29)\end{aligned}$$

Hence, according to the S-procedure and the definition of $\Phi(t)$, combining (27) with (28) and (29) results in

$$0 > \Phi^T(t) \left[\begin{array}{c|c|c} \Omega_{(1)} + \mathbb{T}_{(1)} + \mathbb{X}_{(1)} & (*) & (*) \\ \hline \Omega_{(2)} + \mathbb{T}_{(2)} + \mathbb{X}_{(2)} & \mathbb{T}_{(3)} & 0 \\ \hline \Omega_{(3)} + \mathbb{X}_{(3)} & 0 & 0 \end{array}\right] \Phi(t)$$

which directly leads to (17). □

4 NUMERICAL EXAMPLE

Let us consider the multi-agent system featuring three followers and one leader, as discussed in (He et al., 2020):

$$A = \begin{bmatrix} -3 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, E = \begin{bmatrix} 1.0 \\ 0.1 \\ 1.0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.2 & 0.8 & 1.0 \end{bmatrix}, D = \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.1 \\ 0.1 & 0 \end{bmatrix}. \quad (30)$$

Fig. 1 shows the directed graph of the multi-agent system (30) with the nonhomogeneous Markov network topology. As shown in Fig. 1, one can establish the network-topology-dependent leader adjacency matrices M_{ϕ_t} and the network-topology-dependent Laplacian matrices L_{ϕ_t} as follows, for $\mathbf{N}_\phi = \{1,2,3,4\}$:

$$M_1 = \mathbf{diag}(1,1,1), \ M_2 = \mathbf{diag}(1,1,0)$$
$$M_3 = \mathbf{diag}(1,0,1), \ M_4 = \mathbf{diag}(1,0,1)$$

$$L_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, L_2 = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}, L_4 = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The actuator fault level is given by $(\underline{\Gamma}^1, \overline{\Gamma}^1 = (0.8, 1.0)$. Further, the bounds of $\pi_{gh}(t) \in [\underline{\pi}_{gh}, \overline{\pi}_{gh}]$

and condition probability ϖ_{gp} are:

$$[\pi_{gh}]_{g,h\in\mathbf{N}_\phi} = \begin{bmatrix} -2.0 & 0.1 & \times & \times \\ \times & \times & 0.1 & 0.1 \\ 0.2 & \times & -2.0 & \times \\ 0.1 & \times & \times & \times \end{bmatrix}$$

$$[\overline{\pi}_{gh}]_{g,h\in\mathbf{N}_\phi} = \begin{bmatrix} -1.0 & 0.5 & \times & \times \\ \times & \times & 0.6 & 0.8 \\ 0.9 & \times & -1.0 & \times \\ 0.8 & \times & \times & \times \end{bmatrix}$$

$$[\varpi_{gp}]_{g\in\mathbf{N}_\phi,p\in\mathbf{N}_\rho} = \begin{bmatrix} 0.5 & 0.2 & 0.1 & 0.2 \\ 0.2 & 0.5 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.5 & 0.2 \\ 0.2 & 0.2 & 0.1 & 0.5 \end{bmatrix}$$

where "$\times$" denotes the entirely unknown transition rate. Based one the availability of transition rates $\pi_{gh}(t)$, one can obtain the following network topology sets in Table 1. By applying Theorem 3.1, Table 2 displays the comparison between the minimum $\mathcal{H}_\infty$ performance level γ obtained by (He et al., 2020, Theorem 2) and Theorem 1. As shown in Table 2, the reliable $\mathcal{H}_\infty$ control synthesis condition outlined in Theorem 3.1 attains better performance compared to (He et al., 2020, Theorem 2). Especially, different from (He et al., 2020, Theorem 2), Theorem 3.1

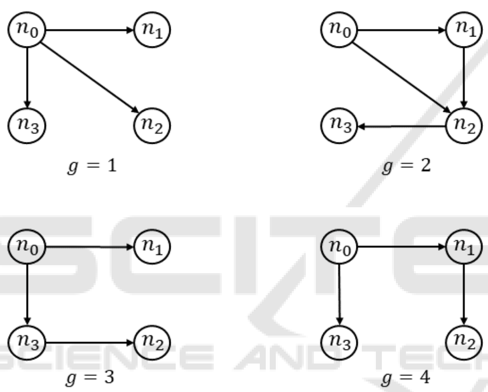

Figure 1: Network-topology-dependent digraphs for $g \in \mathbf{N}_\phi = \{1,2,3,4\}$.

Table 1: Network topology mode subsets for each network topology mode g.

g	Network topology mode subsets
1	$\mathbf{N}_1^\star = \emptyset$, $\widetilde{\mathbf{N}}_1 = \{1,2\}$, $\mathbf{N}_1^\times = \{3,4\}$
2	$\mathbf{N}_2^\star = \emptyset$, $\widetilde{\mathbf{N}}_2 = \{3,4\}$, $\mathbf{N}_2^\times = \{1,2\}$
3	$\mathbf{N}_3^\star = \emptyset$, $\widetilde{\mathbf{N}}_3 = \{1,3\}$, $\mathbf{N}_3^\times = \{2,4\}$
4	$\mathbf{N}_4^\star = \emptyset$, $\widetilde{\mathbf{N}}_4 = \{1\}$, $\mathbf{N}_4^\times = \{2,3,4\}$

Table 2: Comparison of $\mathcal{H}_\infty$ performance levels.

	Theorem 3.1	(He et al., 2020, Theorem 2)
γ	1.4068	1.5139

even opens up the possibility to deal with the appearance of actuator faults. Then, for $\beta = 1.0$ and $\varepsilon = 0.5$, the asynchronous control gains are determined as follows:

$$F_1 = \begin{bmatrix} -2.1250 & -12.2710 & -11.0770 \end{bmatrix}$$
$$F_2 = \begin{bmatrix} -2.1250 & -11.7660 & -10.5670 \end{bmatrix}$$
$$F_3 = \begin{bmatrix} -2.0630 & -11.4580 & -10.2930 \end{bmatrix}$$
$$F_4 = \begin{bmatrix} -2.2540 & -12.8940 & -11.6260 \end{bmatrix} \quad (31)$$

where $f(x_t^i) = \begin{bmatrix} \sin(x_{1,t}^i) & \sin(x_{2,t}^i) \end{bmatrix}^T$. Based on (31), Figs. 2-(a), (b), and (c) demonstrate the state responses of (30) with $w_t \equiv 0$, where $x_0^0 = [0.2 \ \ -0.1 \ \ 0.1]^T$, $x_0^1 = [0.2 \ \ -0.3 \ \ 0.2]^T$, $x_0^2 = [0.1 \ \ -0.3 \ \ -0.2]^T$, and $x_0^3 = [-0.1 \ \ 0.1 \ \ -0.1]^T$. Moreover, Figs. 3-(a), (b), and (c) display the error system of three followers. Thus, as illustrated in Fig. 2 and Fig. 3, the leader-following consensus is successfully achieved, even in the presence of asynchronous control topologies and actuator faults. In addition, for

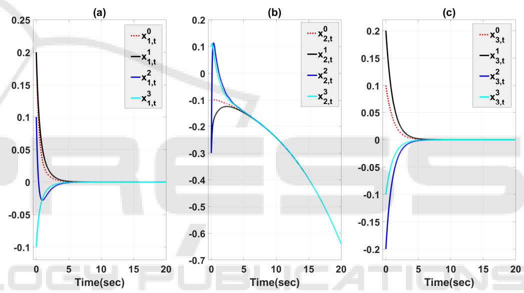

Figure 2: State response of (30) with $w_t \equiv 0$.

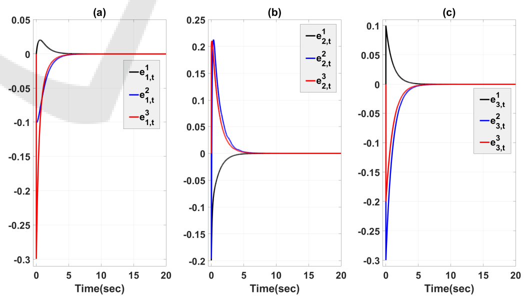

Figure 3: Error tracking of (30) with $w_t \equiv 0$.

$x_0^0 = x_0^1 = x_0^2 = x_0^3 = 0$, Figs. 4-(a), (b), and (c) demonstrate the error responses $\bar{x}_t^i = x_t^i - x_t^0$ of (30) with $w_t^1 = 1, w_t^2 = -1, w_t^3 = 2$, for $t \in [0,3)$; $w_t^1 = -1, w_t^2 = 1, w_t^3 = -2$, for $t \in [3,6)$; and $w_t^1 = w_t^2 = w_t^3 = 0$, otherwise. Fig. 4 illustrates the state trajectories of each agent, highlighting their achievement of robust $\mathcal{H}_\infty$ leader-following consensus. Furthermore, Figs. 5-(a), (b), and (c) display the error system of three followers in this case, Fig. 5-(d) demonstrates the control input applied to each follower, Fig. 5-(e)

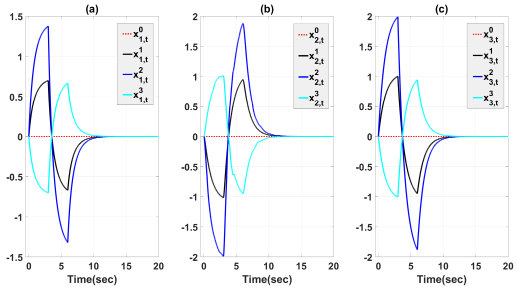

Figure 4: State response of (30) with $w_t \neq 0$.

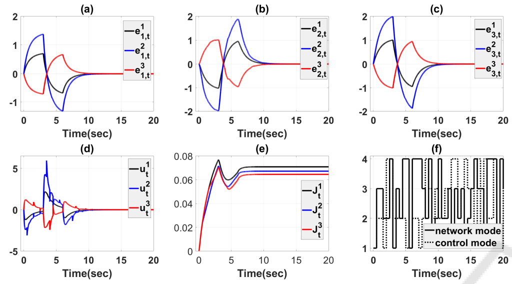

Figure 5: (a), (b), (c): Error tracking of (30) with $w_t \neq 0$; (d): the control input of (30); (e): the real value of $J^i_t = \int_0^t \|z^i_\tau\|^2 d\tau / \int_0^t \|w^i_\tau\|^2 d\tau$; (f): mode evolution of network topology and control topology.

shows the value of $J^i_t = \int_0^t \|z^i_\tau\|^2 d\tau / \int_0^t \|w^i_\tau\|^2 d\tau$ for each follower, and Fig. 5-(f) displays the evolution of the generated topologies for both the system network and control input. As depicted in Fig. 5, the error responses of a group of agents are approaching to zero despite the occurrence of asynchronous control topologies, actuator faults, and non-zero disturbances. Additionally, Fig. 5-(e) reveals that the actual $\mathcal{H}_\infty$ performance levels are lower than those derived using Theorem 3.1.

5 CONCLUDING REMARKS

This paper tackles the challenge of ensuring reliable and asynchronous leader-following consensus in multi-agent systems with nonhomogeneous Markov network topologies. To simplifies transition rate-dependent and conditional probability-dependent consensus conditions into a finite set of solvable linear matrix inequalities, the proposed method i) introduces an asynchronous control-topology-dependent Lyapunov function, ii) develops an innovative relaxation technique distinct from exiting ones, which is based on entirely unknown transition rates, and iii) addresses the non-convexity arising from multiple varying-time parameters. Looking ahead, our future research will explore control strategies for leaderless consensus in nonlinear multi-agent systems, particularly in the context of denial-of-service attacks.

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REFERENCES

He, M., Mu, J., and Mu, X. (2020). $\mathcal{H}_\infty$ leader-following consensus of nonlinear multi-agent systems under semi-Markovian switching topologies with partially unknown transition rates. *Information Sciences*, 513:168–179.

Li, W., Xie, L., and Zhang, J. (2015). Containment control of leader-following multi-agent systems with markovian switching network topologies and measurement noises. *Automatica*, 51:263–267.

Nguyen, N. and Kim, S. (2019). Relaxed robust stabilization conditions for nonhomogeneous Markovian jump systems with actuator saturation and general switching policies. *International Journal of Control, Automation and Systems*, 17(3):586–596.

Nguyen, N. and Kim, S. (2021). Asynchronous $\mathcal{H}_\infty$ observer-based control synthesis of nonhomogeneous Markovian jump systems with generalized incomplete transition rates. *Applied Mathematics and Computation*, 411:126532.

Sakthivel, R., Sakthivel, R., Kaviarasan, B., and Alzahrani, F. (2018). Leader-following exponential consensus of input saturated stochastic multi-agent systems with Markov jump parameters. *Neurocomputing*, 287:84–92.

Takaba, K. (1998). Robust $\mathcal{H}_2$ control of descriptor system with time-varying uncertainty. *International Journal of Control*, 71(4):559–579.

Valentine, F. (1945). A lipschitz condition preserving extension for a vector function. *American Journal of Mathematics*, 67(1):83–93.

Wang, Y., Xia, J., Wang, Z., Zhou, J., and Shen, H. (2019). Reliable consensus control for semi-Markov jump multi-agent systems: A leader-following strategy. *Journal of the Franklin Institute*, 356(6):3612–3627.

Xie, L. and de Souza, C. (1990). Robust $\mathcal{H}_\infty$ control for linear time-invariant systems with norm-bounded uncertainty in the input matrix. *Systems & Control Letters*, 14(5):389–396.

Yang, H., Zhang, H., Wang, Z., and Yan, H. (2023). Reliable leader-following consensus of discrete-time semi-Markovian jump multi-agent systems. *IEEE Transactions on Network Science and Engineering*, 10(6):3505–3518.