Towards Enhanced Decision Making: Integrating Weighted Expected Solution Points in Multi-Criteria Analysis

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Abstract: Multi-Criteria Decision Analysis (MCDA) addresses complex problems across various domains by considering multiple decision criteria. This interdisciplinary field offers a systematic approach to decision-making, accommodating contradictory criteria and non-linear factors. Reference points are crucial in MCDA, facilitating a nuanced understanding of decision interrelationships and outcomes. While classic MCDA methods rely on static reference points, recent advances introduce manual allocation mechanisms, such as the Stable Preference Ordering Toward Ideal Solution (SPOTIS) and Characteristic Objects Method (COMET). However, incorporating reference points alone may overlook the significance of individual criteria, leading to the paradox of equal evaluations. To address this issue, an extension of the COMET method, Expected Solution Point (ESP-COMET), introduces weighted considerations to accurately reflect experts' preferences. This paper proposes a methodology to integrate weights into ESP-COMET, enhancing its efficacy in decision modeling. We applied the proposed approach in the case study focused on the evaluation of hydrogen-fueled vehicles. Identifying the decision model and considering both the expected solution point and the relevance of the criteria to it, we demonstrated the utility of weighted ESP in improving decision-making processes.

1 INTRODUCTION

Multi-Criteria Decision Analysis (MCDA) is an interdisciplinary field that analyzes decisions in the context of multiple criteria. It is used when faced with complex problems, considering various factors. These criteria may frequently conflict with one another, as they embody various priorities like efficiency, profit, or other considerations. An essential element of MCDA is the consideration of non-linear criteria that may be relevant to the analysis. MCDA is used in various areas, from energy to medicine (Saraji et al., 2023; Kizielewicz et al., 2020), logistics to sustainability issues (Moslem, 2023; Więckowski et al., 2024).

MCDA, using a variety of mechanisms, is an effective tool to solve such problems. Approaches relying on pairwise comparisons prove advantageous when experts aim to identify the model based on their knowledge. Alternatively, methods dependent on the coefficients of individual components facilitate adaptable decision making by permitting dynamic adjustments to these coefficient values. In addition, methods that use reference points efficiently represent the decision grid by focusing solely on these reference points.

Using reference points makes it possible to better understand the relationships between different decision criteria and assess how changes in these criteria affect the outcome of a decision. Reference points form the basis of many classic MCDA methods, such as the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) (Aldino et al., 2023) and the Viekriterijumsko Kompromisno Rangiranje (VIKOR) (Nath et al., 2023). In addition, the logic of reference points is also reflected in newer approaches such as Compromise Ranking of Alternatives from Distance to Ideal Solution (CRADIS) (Chakraborty et al., 2024).

However, the methods above operate on statically defined reference points based on a decision matrix.

With reference points, decision experts can better reflect their preferences, which has resulted in the development of methods that allow experts to allocate

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these points manually. The logic behind such manual mechanisms assumes that the decision expert for a given problem assigns his ideal values to specific criteria. Thus, to normalize the decision space, these very objects are used. In addition, using fixed values for these objects with fixed boundaries of the decision problem allows the method to be immune to the rank-reversal paradox. Examples of these methods are methods that will allow reference points to be used in this way include Stable Preference Ordering Toward Ideal Solution (SPOTIS) (Dezert et al., 2020) and Characteristic Objects Method (COMET) (Shekhovtsov et al., 2023).

In the SPOTIS approach methods, an expert determines a single point that forms the basis for a decision grid. The COMET approach, on the other hand, is based on fuzzy logic, specifically the Mamdani model, where characteristic objects are created based on specific characteristic values. This raises a dimensionality problem, where the expert is forced to compare characteristic objects among themselves to get the right preference. To solve this problem, (Shekhovtsov et al., 2023) proposed an extension of the COMET method with the Expected Solution Point (ESP-COMET). With this approach, an expected point is assigned based on which the preference values of the characteristic objects are determined. This eliminates the difficulty of an expert evaluating a high-dimensional problem and facilitates comparing objects.

However, with this solution, there is another problem related to the relevance of the criteria. Using ESP alone allows for grid modeling to evaluate alternatives according to a declared reference point. However, it does not consider the relevance of individual criteria to that point. Therefore, this paper will focus on a proposal to add weight to ESP-COMET to better reflect the preferences of experts. In the remainder of the paper, we will present the process of identifying such a model and its practical implementation, using the example of evaluating hydrogen-powered vehicles.

The remainder of the paper is organized as follows. Section 2 presents related work in relation to the topic of decision making. Section 3 presents the methodology, with the initial assumptions related to the COMET method, the expanded COMET method with ESP and correlation coefficients. Section 3.2 discusses the proposed approach. Section 4 presents a simple example related to weighting the expected solution point. Section 5 presents research on a practical problem related to hydrogen-powered cars. Section 6 presents conclusions and future research propositions.

2 RELATED WORKS

Methods based on reference objects have gained significant attention and are widely applied across various fields due to their robust ability to handle complex multi-criteria decision-making processes. For example, (Awodi et al., 2023) developed a fuzzy TOPSIS-based risk assessment model to effectively manage the risks associated with nuclear decommissioning. This approach allowed for a more nuanced assessment by incorporating uncertainty through fuzzy logic. Similarly, (Aldino et al., 2023) used the TOPSIS method with an alternative weighting procedure to identify the highest performing graduates, demonstrating the versatility of the method in educational evaluation.

In the context of sustainable technologies, (Więckowski et al., 2024) applied the RANking COMparison (RANCOM) method in combination with ESP-SPOTIS to optimize decision making for the selection of electric vehicles, highlighting the flexibility of SPOTIS in handling adaptive systems.

Moreover, (Nath et al., 2023) proposed a VIKOR framework for biodiesel production using heterogeneous agricultural waste-based catalysts. Their work underscores the adaptability of VIKOR in sustainable energy research. (Saraji et al., 2023) utilized the Fermatean CRITIC-VIKOR approach to assess the challenges in implementing renewable energy technologies in rural areas, demonstrating the applicability of the method to assess complex technological adoption scenarios.

On the other hand, pairwise comparison methods, such as the Analytic Hierarchy Process (AHP) and ELimination Et Choix Traduisant la REalité (ELECTRE), compare alternatives in pairs, judging which of the two performs better relative to specific criteria. These techniques are instrumental when the direct comparison between multiple criteria is complex, allowing for a more gradual and structured evaluation process.

In his work, (Romero-Ramos et al., 2023) integrated a GIS-AHP approach to assess the potential of solar energy to meet the demand for heat in industrial areas in the south-eastern part of Spain. This study demonstrates the effectiveness of AHP in spatial decision-making, combining geographical data with multi-criteria evaluation. Similarly, (Ahadi et al., 2023) used the AHP method to determine the optimal site for a solar power plant in Iran, highlighting the utility of AHP in planning energy infrastructure, mainly when multiple conflicting criteria such as land use, environmental impact and cost are involved.

While the discussed methods provide valuable frameworks for multi-criteria evaluations, they often lack a comprehensive approach that enables decision-makers or experts to express their preferences and insights in a clear and transparent manner. Addressing this limitation we propose a novel weighted ESP-COMET approach. This method aims to offer a more holistic solution, enhancing the ability of decision-makers to provide their judgments while maintaining the robustness of the decision-making process.

3 METHODOLOGY

In this paper, we focus on our proposed weighting method within the ESP-COMET approach introduced by (Shekhovtsov et al., 2023). To this end, we developed a framework consisting of five stages. The first stage involved creating research data to evaluate the new approach. For this purpose, we utilized an environment created in Python, and the data was generated from a uniform distribution. In the next stage, we implement the proposed weighted ESP-COMET approach. After its implementation, we moved on to the third stage, where we compared the unweighted ESP-COMET approach with our proposal. Once the differences between these approaches were determined, a simulation was created in which ESP-based approaches with weighting options, such as SPOTIS and our proposed ESP-COMET, were applied. In the final fifth stage, we analyze the approaches using the weighted Spearman correlation coefficient (r_w). The entire framework of the work is presented in Figure 1, while the techniques used and the details of our proposed approach are discussed in subsequent sections.

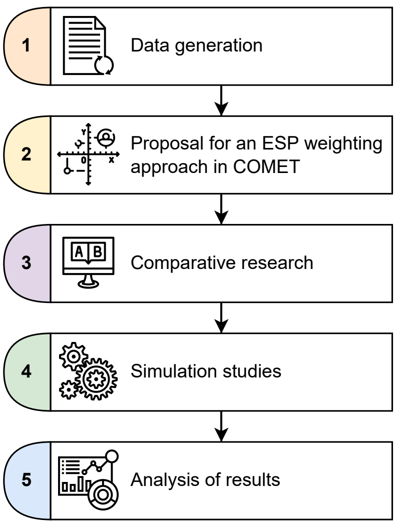

Figure 1: The framework of the proposed approach.

3.1 Preliminaries

3.1.1 Characteristic Objects Method

The Characteristic Objects Method (COMET) is a distinctive Multiple Criteria Decision Analysis (MCDA) approach that is completely resistant to the ranking reversal phenomenon. This method ensures consistent and unequivocal results regardless of changes in the set of alternatives (Kizielewicz et al., 2021). The main steps of the COMET algorithm are as follows:

Step 1. Identify the criteria of the decision problem and represent each criterion using fuzzy numbers.

Step 2. Create a set of Characteristic Objects by applying the Cartesian product of fuzzy numbers, representing all potential combinations.

Step 3. Conduct pairwise comparisons of the Characteristic Objects based on expert judgment, summarize these judgments, and compute preference values.

Step 4. Transform each characteristic object and its associated preference value into a fuzzy rule.

Step 5. Utilize the fuzzy rule base along with Mamdani's inference method to assess and rank alternatives, where higher preference values indicate better alternatives.

The complete algorithm of the COMET method can be found in (Kizielewicz et al., 2021), while the implementation of this method is presented (Kizielewicz et al., 2023).

3.1.2 Expected Solution Point COMET

Expected Solution Point COMET was created as an answer to the dimensionality problem in the standard COMET procedure (Shekhovtsov et al., 2023). This problem appears when the Matrix of Expert Judgements (MEJ) should be identified by an expert. Suppose that we have t characteristic objects, therefore, the identification of the MEJ matrix of size $t \times t$ will require $\frac{t(t-1)}{2}$ pairwise comparisons. This makes it difficult to identify the decision model due to the large number of pairwise comparisons required.

To answer that, ESP-COMET was proposed, creating the alternative way to identify the pairwise comparison matrix. To use it, first, the expert or decision maker should define the n Expected Solution Points based on their preferences and domain knowledge. Each of the ESP vectors consists of r values, where r is a number of criteria in the decision problem (1).

$$ESP = \left\{esp_{ij}\right\}_{n\times r} \tag{1}$$

ESP-COMET uses Equation (2), which incorporates the function f_{ESP}. This function computes an aggregated normalized distance from predefined Expected Solution Points concerning a characteristic object. A smaller distance indicates a preferable characteristic object.

$$\alpha_{ij} = \begin{cases} 1.0, & f_{ESP}(CO_i) < f_{ESP}(CO_j) \\ 0.5, & f_{ESP}(CO_i) = f_{ESP}(CO_j) \\ 0.0, & f_{ESP}(CO_i) > f_{ESP}(CO_j) \end{cases}, \quad (2)$$

The function $f_{ESP}(CO_i)$ is defined as (3).

$$f_{ESP}(X) = min_i \sqrt{\sum_{j=1}^{r} (x'_j - esp'_{ij})^2} \quad (3)$$

In Equation (3), X represents the abstract Characteristic Object, comprising the values x_j, where $j \in 1, 2, \ldots, r$, and esp_{ij} denotes the expected value i for the criterion j. The values x'_j and esp'_{ij} represent the normalized counterparts of x_j and esp_{ij}, respectively, computed using Equation (4). This normalization process relies on the values $c_j^{(min)}$ and $c_j^{(max)}$, which denote the smallest and largest characteristic values for criterion j. The same normalization procedure outlined in Equation (4) is applied to each ESP.

$$x'_j = \frac{x_j - c_j^{(min)}}{c_j^{(max)} - c_j^{(min)}}, \quad (4)$$

where $c_j^{(min)}$ and $c_j^{(max)}$ denote the minimum and maximum characteristic values within criterion C_j.

3.1.3 Similarity Coefficients

In this work, we use two similarity coefficients to measure the differences between the rankings and weights.

The Weighted Spearman's correlation coefficient (Dancelli et al., 2013) extends the traditional Spearman coefficient by incorporating weights, emphasizing changes at the top of the rankings. It evaluates the correlation between two rankings, with values ranging from -1 (reversed rankings) to 1 (identical rankings), and 0 indicating no correlation.

The Weights Similarity Coefficient (Shekhovtsov, 2023) provides a robust measure for comparing differences in sets of criteria weights. It uses the Manhattan distance, normalized for cross-set comparability, to quantify dissimilarities, offering a similarity value scaled between 0 and 1.

3.2 Proposed Approach

The paper which introduced the ESP-COMET approach focuses mainly on the idea of the approach and the influence of the selection of characteristic objects on the identification of the preference function and therefore the accuracy of the model (Shekhovtsov et al., 2023). However, this paper missed the important point of weighting the distances to ESP during the aggregation process. This paper addresses this issue by introducing the weighting mechanism to the ESP-COMET method and investigates it's properties.

To introduce the weights into the ESP-COMET method, we modify Equation (3), adding the weight vector to it (5):

$$f_{ESP}(X) = min_i \sqrt{\sum_{j=1}^{r} w_j \cdot (x'_j - esp'_{ij})^2}, \quad (5)$$

Where w_j is the importance weight for the jth criterion. Such modification of the algorithm allows us to simply manipulate the importance of the weights by the expert if needed. This paper focuses on this version. However, it is possible to extend the approach further, introducing weight sets for different ESP, and also providing ability to weight ESP points.

4 SIMPLE EXAMPLE

To demonstrate our proposed approach, we first want to apply it to a simple two-criterion problem. This simple example contains two criteria, each in the range $[0, 10]$, with chosen ESP $\{3, 3\}$ as shown on the leftmost part of Figure 3. All of those values were arbitrary selected, as they will best illustrate the changes of the preference function.

In this example we compare unweighted version of the ESP-COMET (which can be considered weighted with equal weights) with weighted ESP with the weights vector $w_{ESP} = \{0.01, 0.99\}$.

Such extreme change in the weights will in our opinion show how big can be change in the preference function when weights are introduced.

The comparison results are presented in Table 1, which contains the indices of the characteristic objects (COs) evaluatedCO_i as well as the criteria values for these characteristic objects (C_1 and C_2). Next, we include preferences for these COs for normal P_U and weighted P_W algorithms, as well as respective rankings of characteristic objects R_U and R_W.

Analyzing Table 1 one can notice that in the case of the unweighted method there are several links, such as the link between CO_2 and CO_4, CO_6 and CO_8 and others. This can be easily explained by the fact that those characteristic objects have the same distance to the ESP and are therefore treated equally. In case of weighted ESP however, we can see that weights work correctly, introducing the change in the ranking. CO_5 which has the same criteria values as ESP is placed first in both rankings, and COs such CO_9 and CO_3 that are most distant from ESP are ranked worst.

Table 1: Preferences and rankings of unweighted (P_U, R_U) and weighted (P_W, R_W) ESP models.

CO_i	C_1	C_2	P_U	P_W	R_U	R_W
CO_1	0	0	0.6000	0.5000	4	5
CO_2	0	3	0.8000	0.8750	2.5	2
CO_3	0	10	0.2000	0.1250	7.5	8
CO_4	3	0	0.8000	0.6250	2.5	4
CO_5	3	3	1.0000	1.0000	1	1
CO_6	3	10	0.4000	0.2500	5.5	7
CO_7	10	0	0.2000	0.3750	7.5	6
CO_8	10	3	0.4000	0.7500	5.5	3
CO_9	10	10	0.0000	0.0000	9	9

In addition, we calculate several numerical coefficients to show how different those rankings and preferences are. First, the value of the coefficient WSC_2 between equal weights and the vector $w = \{0.01, 0.99\}$ used in this example is 0.5100, which is the maximum value of WSC_2 which can be obtained by measuring the difference with equal weights. Next, we evaluate preferences and CO ranking, obtaining Weighted Spearman's correlation r_w 0.8752 and Mean Absolute Error (MAE) 0.1222. Both these values point that these rankings and preferences are similar, and of order of the characteristic objects in ranking are generally preserved, however, weighted version has no ties.

In Figure 2 we present two identified MEJ matrices, respectively, for unweighted, weighted ESP-COMET models and the absolute difference between those MEJ matrices. As can be seen, the introduction of weights mostly influences ties, but there are also changes (i.e., in the 8 th row, for CO_8), which explain the change in the preference and ranking position for CO_8.

Next, in Figure 3, the preference functions are presented for both the unweighted and the weighted ESP-COMET models. We also present the difference between them, calculated as values of the preference function for the unweighted model reduced by the corresponding values of the weighted model. In case of chosen weights, it can be seen that a darker region with higher preference values is stretched among C_1 values, instead of concentrating around the ESP as was in case of unweighted ESP. From the difference, it can be seen that the preferences for $CO_8 = \{10, 3\}$ and for the region adjusted to it change the most. The preference value of CO_8 changed from 0.400 to 0.750 after weighting was introduced, implying such a large change in the preference function. The Characteristic Objects for $CO_4 = \{3, 0\}$ and $CO_6 = \{3, 10\}$ are removed after the weights are introduced, which can also be seen in the visualization of the differences. Only the most distant characteristic object $CO_9 = \{10, 10\}$ does not change its preference value, which remains 0 for both weighted and unweighted models.

5 CASE STUDY

However, since a simple example does not present real data, we decided to present our approach to the real-world data collected for the purpose of this research. With the increasing popularity of alternative fuel for cars, we present a case study of choosing the most suitable hydrogen car, based on the data collected from the manufacturers' web pages.

The cars are evaluated using the criteria presented in Table 2. The minimum and maximum values are arbitrarily chosen based on the collected data to build a model that fits all the alternatives chosen. The weights of the criteria were calculated using the RANCOM method (Więckowski et al., 2023) and the ESP was arbitrary chosen by the decision maker, based on their needs and expectations for the car. According to the input of the decision maker, the most important criterion for them is price, followed by range and tank capacity. The least important criterion is year, which is expected, because the range for years is quite small.

Table 2: Criteria description, as well as the weights determined using the RANCOM method.

C_i	Crit. names	Units	w_j	ESP	Min	Max
C_1	Year	–	0.012	2023	2015	2024
C_2	Est. Price	$k	0.210	30	30	100
C_3	Range	km	0.173	800	400	800
C_4	Power output	KW	0.111	200	100	400
C_5	Tank cap.	kg	0.173	6	4.4	6.5
C_6	Horse power	–	0.136	130	130	550
C_7	Max torque	Nm	0.062	300	260	410
C_8	0 to 100 time	s	0.050	8	4.5	12.5
C_9	Max speed	km/h	0.074	150	130	250

Data for the hydrogen cars chosen for the evaluation are presented in Table 3. It can be seen that most of the cars are actually new, with only A_6 being an outlier in the year criterion. The price and other criterion values are different, and there is no simple way to decide which one is will fit the decision-maker's expectations best. The data of the alternatives was collected from the manufacturers' web pages.

We evaluated the data presented in Table 3 with both ESP-COMET and weighted ESP-COMET using the weights shown in Table 2. The results of the com-

Figure 2: Identified MEJ for unweighted (left) and weighted (center) ESP-COMET and difference between these two functions (right), characteristic objects are represented by the black dots.

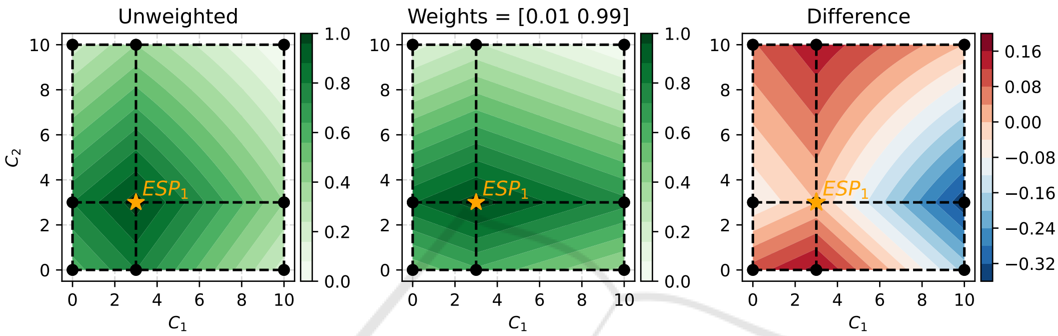

Figure 3: Contour representation of the preference functions for unweighted (left), weighted (center) ESP-COMET and difference between these two functions (right), characteristic objects are represented by the black dots.

Table 3: Data of the alternatives.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
A_1	2023	50.0	646	128	5.6	182	406	9.2	161
A_2	2023	60.0	612	135	6.3	161	394	9.2	179
A_3	2021	59.0	579	103	5.5	174	299	9.0	165
A_4	2019	45.0	437	155	4.4	211	364	5.8	160
A_5	2022	30.0	400	150	4.4	134	260	7.8	130
A_6	2015	66.5	594	100	5.6	136	406	12.5	160
A_7	2024	90.0	504	295	6.0	401	347	6.0	180
A_8	2023	45.0	590	134	5.6	182	300	7.8	170
A_9	2022	68.0	800	220	5.4	300	347	6.5	200
A_{10}	2022	100.0	800	400	5.4	550	347	4.5	250

Table 4: Case study result.

A_i	P_U	P_W	R_U	R_W
A_1	0.8367	0.8275	3	2
A_2	0.8111	0.7974	5	3
A_3	0.8193	0.7767	4	6
A_4	0.5798	0.6218	7	8
A_5	0.8493	0.7942	2	4
A_6	0.5404	0.7182	8	7
A_7	0.5271	0.4408	9	9
A_8	0.9054	0.8556	1	1
A_9	0.7803	0.7855	6	5
A_{10}	0.2907	0.3294	10	10

putations are presented in Table 4. We first present preferences for unweighted (P_U) and weighted (P_W) models, as well as respective rankings R_U and R_W. Change in weights introduces several changes in the ranking, however, alternative A_8, which has the best preference, and alternatives A_7 and A_{10} which perform the worst preserve their positions in the rankings. Three alternatives worsen their position in the ranking, and four alternatives improved. For example, alternatives A_1 and A_2 moved from third and fifth positions to second and third, because they have a better price to other criteria ratio.

The differences in both rankings are clearly visible in the visualization in Figure 5. In this visualization, every dot represents an alternative, and the x coordinate of the dot represents its position in the R_U ranking and the y coordinate represents the position in the R_W ranking. It can be seen that deviations from the diagonal are not big, implying that rankings are rather similar. This is also confirmed by the Weighted Spearman correlation coefficient, which is equal to 0.9012 for these two rankings. Similarly, WSC_2 value between equal weights and weights computed using the RANCOM methods is 0.7531, which show that these weight vectors are similar. For characteristic objects, we provide only the value r_w between their classifications, which is 0.8843. The number of characteristic objects for this problem is 729, and, therefore, it can be represented or visualized in readable form.

Results obtained in this case study shows, that in this particular problem weights does not strongly in-

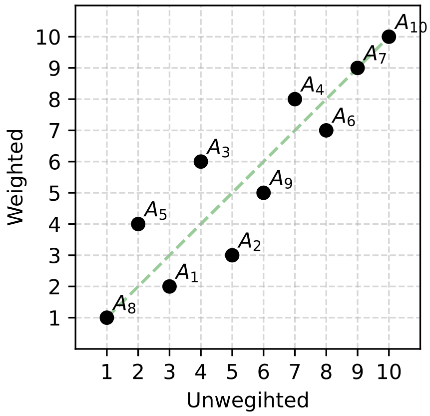

Figure 4: Visual difference of the rankings for the weighted and unweighted ESP-COMET models.

fluence the final ranking, however this needs to be investigated further.

5.1 Simulation Experiment

In order to investigate how strong the influence of the weights is in the ESP-COMET approach, we design a simple simulation experiment. This experiment is based on the data derived from the case study of hydrogen cars previously presented. We use the same criteria and ESP as provided in Table 2, and then change the weights to check how much different results we can obtain (compared to the unweighted ESP-COMET).

We also include the Stable Preference Ordering Towards Ideal Solution (SPOTIS) method in the simulation, as this method is ESP-capable and therefore can be compared with the COMET method. The simulation experiment is described with Algorithm 1. For this experiment, we first need to define the decision problem consists of characteristic values cv and the decision matrix X, as well as ESP and the number of iterations to make in the simulation. Next, we calculate the rankings R_U for alternatives from X and the rankings of the characteristic objects using the standard ESP-COMET procedure (line 1). Additionally, in line 2 we calculate the ranking of the alternatives R_{SE} using the SPOTIS method and defined ESP. Next, in lines 3-10 we repeat $n = 10000$ calculations of these rankings, but with randomly generated weights $\mathbf{w}_R$. The sum of random weights $\mathbf{w}_R$ is ensured to be equal to 1. Next, with this weight, we calculate the ranking of alternatives R_W and the ranking of characteristic objects R_W^{CO} using the weighted ESP-COMET algorithm. Next, we also calculate the ranking R_{SW} using the SPOTIS approach and the same random weight vector. Finally, we memorize all the results for further analysis.

Algorithm 1: Algorithm of the simulation research.

Input: Number of iterations $n \leftarrow 10000$
Input: Decision matrix X
Input: Expected Solution Point ESP
Input: Characteristic values cv

```
1:  R_U, R_U^CO ← ESP_COMET(X, ESP)
2:  R_SE ← SPOTIS(X, ESP, w_E)
3:  for i ← 1, 2, ..., n do
4:      w_R ← random_weights()
5:      R_W, R_W^CO ← wESP_COMET(X, ESP, w_R)
6:      R_SW ← SPOTIS(X, ESP, w_R)
7:      Write r_w between R_U and R_W
8:      Write r_w between R_SE and R_SW
9:      Write r_w between R_U^CO and R_W^CO
10: end for
```

Output: Collected r_w values for different rankings.

The results of the simulation are presented in Figure 5 in the form of box plots. This visualization presents the distributions of the r_w values between the classification calculated with equal weights and the weights generated randomly. It can be seen that the values of both methods r_w (named COMET and SPOTIS in the visualization) have a similar range $[0.5, 1.0]$, however, the quantiles and the average values are different. When weighted ESP-COMET is used, the average r_w values between the classification build with equal weights and random weights is ≈ 0.92, however, in the case of the SPOTIS method the average r_w is ≈ 0.85, which is lower. The average value of r_w is also higher in case of ranking of the Characteristic objects ("COs" label). Such good correlations can be explained with the fact that alternatives in the COMET methods are evaluated based on COs and not based on the weights, and because the distances between characteristic objects are usually larger than between alternatives, it is less possible to have changes in order of alternatives, when weights and order of characteristic objects have some small changes. This implies that ESP-COMET is a more robust and stable approach.

6 CONCLUSIONS

This paper presents an essential aspect of the ESP-COMET algorithm, introducing the weighting mechanism and examining changes in the preference function and the method's stability. We present the application of the method to a simple example, consisting of two criteria, where we observe changes in the adaptation of the decision map and the effect of ESP weights on its formation. In addition, the paper

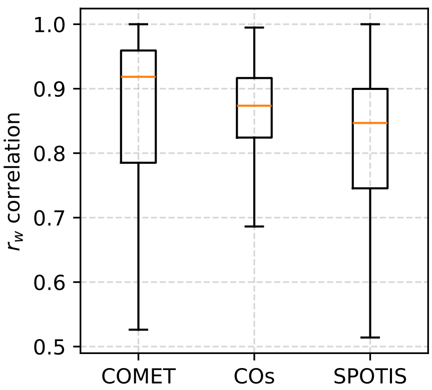

Figure 5: Results of the comparison of 10000 models with random weights and unweighted ESP-COMET on alternatives and characteristic objects rankings.

presents a practical example related to the evaluation of hydrogen cars. Based on the study, the proposed approach shows high application potential. Moreover, comparing it with the SPOTIS method, it turns out that the influence of the weights on the final ranking is more limited, which translates into obtaining stable rankings, resistant to slight deviations in the weights.

However, this method needs future investigations. One of the feature of this methods is a possibility for providing several ESP points, which can be used to introduce more complex weighting algorithms. Such algorithms are useful in group decision making and other complex decision scenarios. In addition, consideration should be given to integrating this tool for possible re-identification of MCDA models, where the research direction may be a Stochastic Identification of Weights (SITW) - ESP-COMET hybrid (Kizielewicz et al., 2024).

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