# A Multitier Approach for Dynamic and Partially Observable Multiagent Path-Finding

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Abstract: This paper introduces a novel Dynamic and Partially Observable Multiagent Path-Finding (DPO-MAPF) problem and presents a multitier solution approach accordingly. Unlike traditional MAPF problems with static obstacles, DPO-MAPF involves dynamically moving obstacles that are partially observable and exhibit unpredictable behavior. Our multitier solution approach combines centralized planning with decentralized execution. In the first tier, we apply state-of-the-art centralized and offline path planning techniques to navigate around static, known obstacles (e.g., walls, buildings, mountains). In the second tier, we propose a decentralized and online conflict resolution mechanism to handle the uncertainties introduced by partially observable and dynamically moving obstacles (e.g., humans, vehicles, animals, and so on). This resolution employs a metaheuristic-based revision process guided by a consensus protocol to ensure fair and efficient path allocation among agents. Extensive simulations validate the proposed framework, demonstrating its effectiveness in finding valid solutions while ensuring fairness and adaptability in dynamic and uncertain environments.

## 1 INTRODUCTION

As defined by (Stern et al., 2021), Multiagent Path-Finding (MAPF) involves the allocation of conflict-free paths to rational agents, facilitating their travel from initial to target locations and is crucial for tackling various real-world challenges, including logistics and robot rescue operations. Due to its practical applications, it has attracted significant attention from researchers, leading to the exploration of various problem variants over time. While some researchers focus on the MAPF with fixed (i.e., static) obstacles for the agents to avoid along their paths (Sharon et al., 2015), others have introduced uncertainty by incorporating partial observability where agents lack all relevant information regarding the presence/location of those static obstacles (Shofer et al., 2023).

Besides, uncertainty in real-world scenarios arises from the stochastic or unpredictable behaviors of other entities in the environment. For instance, birds in a multi-drone path-finding problem contribute to uncertainty since agents may not be able to predict their presence or movement patterns, nor can they communicate with them. Similarly, the movement of pedestrians and animals or sudden blockage on the road, such as car accidents or building crashes, pose dynamic and unpredictable challenges for autonomous driving. These entities and events do not only create uncertainty but also result in dynamically *unavailable* regions in the environment, acting as obstacles that can potentially lead to conflict. To the best of our knowledge, none have comprehensively addressed the MAPF problem involving partially observable and dynamically moving obstacles that also exhibit unpredictable behaviors.

In this study, we extend the classical MAPF problem by introducing *partially observable and dynamic obstacles* (e.g., humans, vehicles, animals, and non-rational entities) that move unpredictably over time, while agents have limited vision to detect them. In our formulation, agents cannot communicate with these obstacles at any point and have no prior knowledge of their existence, positions, or velocities until they enter

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their field of view (i.e., partial observability). Consequently, our formulation introduces the challenge of navigating through both *static* and *dynamic* obstacles while coping with *limited knowledge* about environment. We refer to this problem as the *Dynamic and Partially Observable Multiagent Path-Finding* (DPO-MAPF) problem.

Regarding the potential solution approaches, two categories emerge: *centralized* and *decentralized*. Centralized approaches employ a single authority that uses all available knowledge to optimize paths for all agents, which can be effective for global optimization but faces scalability challenges as the number of agents increases. Decentralized approaches, on the other hand, grant autonomy to individual agents, offering scalability benefits but may struggle to find optimal solutions in complex environments. Hence, we propose a *multitier strategy* combining both approaches to leverage their strengths for effectively tackling the DPO-MAPF problem.

In both tiers, the primary objective remains guiding agents from their initial locations to their goal destinations while minimizing total travel time and avoiding collisions. In the first tier, we employ a centralized and offline pathfinding strategy, particularly in regions where static obstacles (e.g., walls, mountains, buildings) are pre-known and globally observable. This tier can integrate well-known centralized algorithms, such as (Sharon et al., 2015; Li et al., 2021b; Okumura et al., 2022; Lam et al., 2022).

To address the uncertainty introduced by DPO-MAPF problem, the second tier applies a decentralized and online conflict resolution approach, which includes a revising strategy along with a consensus protocol. The second tier primarily aims for decentralized execution of the centralized plan in a DPO-MAPF environment. If any conflicts arising from the uncertainty are detected in the current plan, the revising strategy employing a metaheuristic method (i.e., Ant Colony Optimization) resolves them. Furthermore, a consensus protocol inspired by (Eran et al., 2021) manages conflict resolution among agents by prioritizing them based on specific criteria, such as urgency in reaching their goals, objective functions, fairness among them, or specific task requirements.

Our proposed strategy encourages collaboration, information sharing, fairness, and self-organization among agents in dynamic and uncertain environments. Consequently, this study seeks to develop adaptive and responsive path-finding techniques that incorporate both offline and online decision-making processes, effectively addressing the challenges posed by static and dynamic obstacles and enhancing multi-agent coordination in real-world scenarios.

The rest of the paper is organized as follows: Section 2 reviews related work, and Section 3 defines the problem. Section 4 presents the mathematical formalization, and Section 5 outlines the proposed multitier approach. Section 6 introduces proposed decentralized and online conflict resolution. Section 7 details the experimental setup and the performance evaluation results. Finally, Section 8 discusses the main contributions and future work directions.

## 2 RELATED WORK

The MAPF problem has been widely studied, generally categorized into *centralized* and *decentralized* approaches. Centralized approaches utilize comprehensive knowledge and coordination, employing optimal solvers that consider all agents' paths for efficient conflict resolution and optimization. The A* algorithm is foundational in MAPF solvers, known for its efficiency in finding shortest paths (Ryan, 2008; Standley, 2010). Building on A*, (Sharon et al., 2015) introduce Conflict-Based Search (CBS), which coordinates multiple agents and is widely used in MAPF. CBS operates as a two-level search algorithm, resolving conflicts by branching into subproblems and replanning paths when needed. Subsequent enhancements like Meta-Agent CBS (MA-CBS) improve performance in conflict-heavy scenarios by coupling agents into meta-agents based on conflict frequency (Sharon et al., 2015). (Lam et al., 2022) advance CBS through the Branch-and-Cut-and-Price (BCP) algorithm, integrating mixed-integer programming (MIP) for better performance. (Li et al., 2021a) further enhance CBS with CBSH2-RTC, effectively resolving symmetric conflicts and reducing node expansions, significantly improving scalability.

As the number of agents increases, the state space expands exponentially, leading to the utilization of suboptimal solvers to handle this growth. In such cases, suboptimal solvers prove beneficial for finding paths quickly. (Semiz and Polat, 2021) introduce the Incremental MAPF (I-MAPF) problem and propose CBS-D*-lite, which combines CBS with D*-lite. This approach efficiently updates only affected agents' paths in dynamic environments without replanning from scratch, enhancing adaptability and computational efficiency over traditional CBS, although it may not yield optimal solutions in rapidly changing scenarios. (Li et al., 2021b) present Explicit Estimation CBS (EECBS), which employs online learning to obtain inadmissible heuristics of each high-level node and uses Explicit Estimation Search to select which high-level node to expand next. (Oku-

Table 1: Comparison of the Most Promising Multiagent Path-Finding Approaches.

| | **Approach** | **Optimality** | **Completeness** | **Environment** | **Uncertainty** | **Observability** | **Solver Type** |
|---|---|---|---|---|---|---|---|
| CBS (Sharon et al., 2015) | Centralized | Optimal | Complete | Static | No | Fully | Rule-based |
| CBSH2-RTC (Li et al., 2021a) | Centralized | Optimal | Complete | Static | No | Fully | Rule-based |
| EECBS (Li et al., 2021b) | Centralized | Suboptimal | Complete | Static | No | Fully | Rule-based |
| PIBT (Okumura et al., 2022) | (De)centralized | Suboptimal | Incomplete | Static | Yes | Partially | Priority/Rule-based |
| PRIMAL (Sartoretti et al., 2019) | Decentralized | Suboptimal | Incomplete | Static | Yes | Partially | Learning-based |
| BCP (Lam et al., 2022) | Centralized | Optimal | Complete | Static | No | Fully | Systematic |
| I-MAPF (Semiz and Polat, 2021) | Centralized | Optimal | Complete | Static | Yes | Partially | Systematic |
| **Our Approach** | **Mixed** | **Suboptimal** | **Complete** | **Dynamic** | **Yes** | **Partially** | **Multitier** |

mura et al., 2022) introduce Priority Inheritance with Backtracking (PIBT), a two-level decoupled approach that uses dynamic prioritization to manage agent movements, ensuring reachability in large, real-time environments. Despite its scalability and efficiency, PIBT may struggle in dense settings.

Although CBS and its variants excel at providing promising solutions, they struggle in highly dynamic environments with unpredictable movements. Most studies assume dynamic settings but typically focus on partial observability (Shofer et al., 2023) or the change in the number of agents in a team of cooperative and collaborative agents as a dynamic environment (Wan et al., 2018). The static assumption in many MAPF models can lead to inefficiencies and suboptimal performance when dealing with such dynamic elements. Additionally, their centralized nature demands high computational resources and full observability, often impractical in real applications. In contrast, this study addresses dynamic environments with partially observable and unpredictable entities, offering a more realistic perspective.

On the other hand, decentralized approaches in multi-agent pathfinding have gained significant attention, particularly for managing dynamic and large-scale environments by distributing path planning tasks among local decision-makers. These approaches offer a flexible and scalable solution when centralized coordination becomes impractical or computationally intensive. Due to limited knowledge, decentralized approaches do not guarantee optimality but offer reasonable solutions in a shorter time. Consequently, various techniques have been developed to address the challenges of decentralized multi-agent pathfinding (DMAPF) and coordination in dynamic environments. DMAPF requires multiple agents to coordinate their movements to avoid collisions and achieve their individual goals (Verbari et al., 2019). (Desaraju and How, 2012) utilize rapidly exploring random tree (RRT), a popular technique used in DMAPF to quickly explore the search space and find a feasible path for each agent.

Moreover, decentralized approaches offer diverse applications for MAPF beyond traditional methods. (Morag et al., 2023) introduce a learning-based motion planning framework where each agent constructs a graph of boundary value problems based on constraints. (Peng, 2023) combine heuristic search, empirical rules, and multiagent reinforcement learning with real-time and heuristic planners. (Netter and Vamvoudakis, 2023) present a decentralized motion planning algorithm that leverages game theory for online path updates in dynamic environments, mitigating the freezing robot problem while considering kinodynamic constraints. (Kasaura et al., 2023) propose periodic MAPP, using constraint relaxation and optimization for periodic agent appearances. Lastly, (Keskin et al., 2024) suggest a decentralized negotiation strategy using token protocols and path-aware/heatmap in grid environments.

Given the inherent complexities and dynamic nature of MAPF, bioinspired algorithms have emerged as a promising approach to enhance adaptability and collision avoidance in complex and dynamic settings (Aljalaud et al., 2023). For example, (Dai et al., 2019) introduces an improved Ant Colony Algorithm (ACO) for MAPF, combining features of the A* algorithm and the MAX-MIN Ant System to handle intricate environments. (Huang et al., 2021) optimize the ACO algorithm by refining parameters and addressing vertex conflicts. Their approach introduces adaptive pheromone intensity, reduction factors, and an initial pheromone distribution, which help to prevent the algorithm from getting stuck in local optima.

Finally, Table 1 summarizes the cutting-edge approaches mentioned above for the MAPF problem. In contrast to these studies mentioned above, our study proposes a novel multitier approach, leveraging the advantages of centralized and decentralized processes and integrating ACO into a decentralized and online framework to address the challenges of DPO-MAPF.

## 3 PROBLEM DEFINITION

MAPF problem involves the allocation of conflict-free paths to agents, guiding them from their initial locations to their goal locations. In the context of the MAPF problem, we consider $m$ agents represented by $a = \{a_1, a_2, ..., a_m\}$ navigating within a graph $G = (V, E)$. The initial and goal locations for an agent $a_i$ are denoted as $s_i \in V$ and $g_i \in V$, respectively, and an agent can traverse from vertex $v \in V$ to vertex $v' \in V$ if there is an edge such that $(v,v') \in E$. Time is considered discrete, and at each time step, an agent can occupy only one vertex of the graph and execute a single action for moving to an adjacent vertex. The path of every agent $a_i$ is represented by a sequence of vertices ($v_i \in V$) identified at each time step $t$ and denoted as $\pi_i$ (single-agent plan). At a given time step, agents cannot be located in the same vertex, $\pi_i^t \neq \pi_j^t\ \forall\ i \neq j$, and cannot traverse the same edge, $(\pi_i^t = \pi_j^{t+1}) \implies (\pi_i^{t+1} \neq \pi_j^t)\ \forall\, i \neq j$, where $\pi_i^t$ is the location of $a_i$ at $t$ (Stern, 2019). A valid solution for the MAPF problem is a set of single-agent plans without any conflict with the obstacles and each other.

As in the classical MAPF problem, the environment contains static obstacles, such as walls, buildings, or permanent structures, that remain in fixed and known locations throughout the path-finding process. Our DPO-MAPF problem also has partially observable and dynamically moving obstacles with unknown positions and behaviors. Let's assume that $n$ dynamic obstacles represented by $o = \{o_1, o_2, ..., o_n\}$ moving within the graph $G = (V,E)$. Like agents, each dynamic obstacle ($o_j$) has an initial location ($s_j^o \in V$) and can traverse from vertex $v \in V$ to vertex $v' \in V$ if there is an edge such that $(v,v') \in E$. The dynamic obstacles can only be positioned in one of the graph's vertices at each time step (see the red cells in Figure 1) and execute a single action at each time step $t$. The agents cannot be located in the same vertex and traverse the same edge with dynamic obstacles as well ($\pi_{a_i}^t \neq \pi_{o_j}^t$ and $\pi_{a_i}^t = \pi_{o_j}^{t+1} \implies \pi_{a_i}^{t+1} \neq \pi_{o_j}^t\ \forall a_i \in a \wedge o_j \in o$, where $\pi_{a_i}^t$ is the location of $a_i$ and $\pi_{o_j}^t$ is the location of $o_j$ at $t$). It is worth noting that the agents cannot communicate with these dynamic obstacles. Agents also disappear when they reach the goal position to prevent post-goal collisions, as addressed in (Sharon et al., 2015). Additionally, the agents have a limited vision to detect dynamic obstacles. Agents can observe dynamic obstacles only if they fall within their field of view. This means that agents are not provided with any information about the existence, position and velocity of dynamic obstacles before the agents' observation. Finally, the maximum total time step to complete the mission is limited.

To address this, we developed a simulation framework[1] for the DPO-MAPF task visualized in Figure 1. This framework enables grid-like DPO-MAPF scenarios, including different densities of dynamic and static obstacles, map sizes and number of agents. Each agent can move between neighboring cells based on available actions such as “Up”, “Down”, “Right”, “Left”, and “Wait”. Note that the grid environment, commonly used for path-finding problems, can be equivalently represented as an undirected graph where the connectivity between cells (i.e., vertices) is defined by their adjacency in the grid. This structured representation simplifies certain operations while maintaining the general properties of graph-based representations, thus ensuring that the classical MAPF problem defined on graphs is consistent with our grid-based implementation. Besides, inspired by (Eran et al., 2021), each agent's field of view is represented by a fixed-sized square (i.e., $5 \times 5$), with the agent always positioned at the center of this square. Consequently, this simulation framework facilitates the generation of environments, evaluation of proposed approaches, and visualization of agent movements within the DPO-MAPF environment.

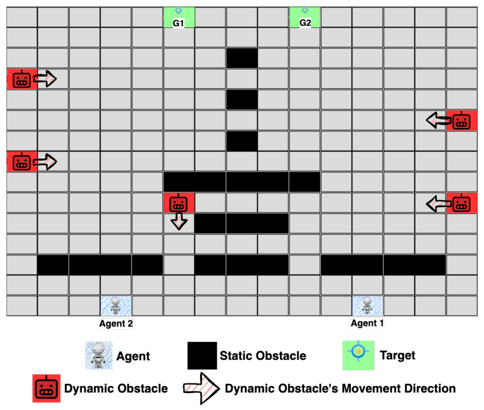


Figure 1: Illustration of the Simulation Framework.

## 4 LP FORMALIZATION

The linear programming (LP) approach has already been used to find optimal solutions for classical MAPF problems. However, it struggles with scalability issues due to the $\mathcal{NP}$-Hard nature of the MAPF problem, highlighting the necessity for alternative approaches such as heuristic algorithms.

In our context, we introduce an LP formalization of the DPO-MAPF problem for validation during the scenario-making process. Since LP models must be deterministic and solvable by commercial solvers, the uncertainty introduced by partial observability makes the LP approach less suitable for DPO-MAPF. In other words, the mathematical model requires complete information about the environment, including

[1]To ensure the reproducibility and transparency of our study, the corresponding GitHub repository is publicly available: https://github.com/aniltrue/DPO_MAPF_ICAART_2025

both dynamic and static obstacles. Nonetheless, it still provides a solid baseline for evaluation purposes.

The parameters and decision variables are summarized below, followed by a formulation of the DPO-MAPF problem:

***Indices:***

| | | |
|---|---|---|
| $i$ | = | Index representing a node in the graph |
| $t$ | = | Discrete time-step |
| $m$ | = | Index of an agent |

***Sets:***

| | | |
|---|---|---|
| $\mathcal{T}$ | = | The set of time-steps, $[0, T_{\max}]$. |
| $I$ | = | The set of all nodes in the graph. |
| $\mathcal{M}$ | = | The set of agents, indexed from 1 to $m$. |
| $\mathcal{E}$ | = | The set of all edges, where each edge is a pair $(i, i')$ indicating a direct connection between nodes $i$ and $i'$. |
| $\mathcal{N}(i)$ | = | The set of neighboring nodes of node $i$, defined as $\mathcal{N}(i) = \{i' \in I \mid (i, i') \in \mathcal{E}\}$. |

***Parameters:***

| | | |
|---|---|---|
| $o_{it}$ | = | Indicates whether node $i$ is occupied by an obstacle at time-step $t$. |
| $s_{im}$ | = | Indicates whether node $i$ is the starting position of agent $m$. |
| $g_{im}$ | = | Indicates whether node $i$ is the goal position of agent $m$. |
| $t^m_{\max}$ | = | The maximum available time step for agent $m$ to complete its travel. |
| $d_{ii'}$ | = | The distance between node $i$ and node $i'$ in the graph. |

***Decision Variables:***

| | | |
|---|---|---|
| $X_{itm}$ | = | Indicates whether agent $m$ is located at node $i$ at time-step $t$. |
| $A_{tm}$ | = | Indicates whether agent $m$ has reached its goal position at time-step $t$. |
| $Y_{ii'tm}$ | = | Indicates whether agent $m$ moves from node $i'$ to node $i$ at time-step $t$. |

***Mathematical Model***:

$$\text{minimize} \quad \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} \sum_{i\in I} \sum_{i'\in\mathcal{N}(i)} d_{ii'} \cdot Y_{ii'tm} \tag{1}$$

subject to:

$$X_{itm} \leq 1 - o_{it}, \qquad \forall m,\, t,\, i \tag{2}$$

$$X_{i0m} = s_{im}, \qquad \forall m,\, i \tag{3}$$

$$\sum_{i\in I} X_{itm} = 1 - A_{tm}, \qquad \forall m,\, t \tag{4}$$

$$\sum_{t\in\mathcal{T}} X_{itm} \geq g_{im}, \qquad \forall m,\, i \tag{5}$$

$$X_{itm} \leq \sum_{i'\in\mathcal{N}(i)} X_{i'(t-1)m}, \qquad \forall m,\, i,\, t \in \mathcal{T}\setminus\{0\} \tag{6}$$

$$\sum_{t\in\mathcal{T}} \sum_{i\in I} X_{itm}(1 - g_{im}) \leq t^m_{\max}, \qquad \forall m \tag{7}$$

$$A_{(t+1)m} \leq A_{tm} + \sum_{i\in I} \sum_{j\in\mathcal{J}} X_{ijtm} \cdot g_{ijm}, \quad \forall m,t \tag{8}$$

$$Y_{ii'tm} \leq X_{i'tm}, \qquad \forall m,\, i,\, t \in \mathcal{T}\setminus\{0\},\, i' \in \mathcal{N}(i) \tag{9}$$

$$Y_{ii'tm} \leq X_{itm}, \qquad \forall m,\, t,\, i,\, i' \in \mathcal{N}(i) \tag{10}$$

$$Y_{ii'tm} \geq X_{itm} + X_{i'(t-1)m} - 1, \tag{11}$$

$$\forall m,\, i,\, t \in \mathcal{T}\setminus\{0\},\, i' \in \mathcal{N}(i)$$

$$X_{itm} \in \{0,1\}, \qquad \forall m,\, t,\, i \tag{12}$$

$$Y_{ii'tm} \in \{0,1\}, \qquad \forall m,\, t,\, i,\, i' \tag{13}$$

$$A_{tm} \in \{0,1\}, \qquad \forall m,\, t \tag{14}$$

The objective function (1) aims to minimize the total travel distance of all agents across the graph for the agents from their starting positions to their respective goal positions while ensuring collision-free paths. Constraint (2) prevents agents from entering nodes blocked by any obstacle type at any time step. Constraint (3) guarantees that agents start their journey from their predefined starting points. Constraint (4) reinforces the condition that an agent is either present at a node or inactive once it reaches its goal, preventing collisions. Constraint (5) indicates whether agents have reached their predefined goals. Constraint (6) restricts agents to move only to adjacent nodes, ensuring valid movements within the graph. Constraint (7) sets a maximum time limit for each agent to complete its path. Constraint (8) specifies that an agent must actively move toward its goal if not yet reached, but once at the goal, it may remain stationary. Constraints (9 & 10 & 11) ensure the flow variable accurately represents agent movements between nodes only when both nodes are occupied consecutively. Finally, Constraints (12 & 13 & 14) enforce that all decision variables are binary.

## 5 PROPOSED MULTITIER RESOLUTION APPROACH

In the context of MAPF solvers, valid paths for agents are determined by either a single decision-maker (i.e., centralized) or multiple decision-makers (i.e., decentralized) during travel (i.e., online) or before travel (i.e., offline). Each approach has advantages and disadvantages, and the choice must be made carefully, considering the requirements, limitations, and attributes of the problem. The primary aim of this study is to introduce a practical approach to tackle the challenges of DPO-MAPF as outlined above. Our DPO-MAPF problem involves both static and dynamic obstacles. While static obstacles are known beforehand and can be efficiently handled using centralized and offline solvers, dynamic obstacles introduce partial observability, requiring a decentralized and online approach. To handle both dynamic and static obstacles, we introduce a multitier approach comprising the following key components and leveraging the advantages of various MAPF solver approaches:

1. **Centralized and Offline Path Finding.** The first tier, illustrated at the top of Figure 2, generates initial paths for each agent by considering only static obstacles (i.e., ignoring uncertainty) in a centralized and offline manner. This tier can employ either an optimal centralized and offline algorithm or a suboptimal one, each with advantages and disadvantages depending on the problem specifications and requirements. It is important to emphasize that the proposed approach is designed to be independent of the specific choice of MAPF solver.
2. **Decentralized and Online Conflict Resolution.** This tier independently executes each agent's path generated in the first tier until potential collisions caused by uncertainty are detected. When such conflicts arise, a revising strategy following a consensus protocol is employed to adjust their paths, dynamically preempting potential collision in real-time as shown at the bottom of Figure 2.

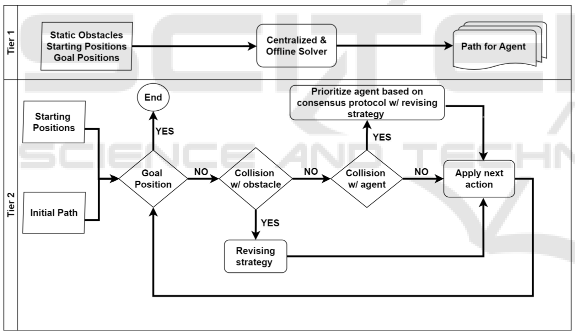


Figure 2: Illustration of the Proposed Multitier Approach.

## 6 CONFLICT RESOLUTION

Dynamic obstacles are unknown in advance and only become apparent when observed, which makes them challenging to manage in centralized approaches. To address this limitation, we propose integrating a decentralized and online conflict resolution method. In our proposed approach, agents independently handle any conflict while following their path. Recall the field of view concept where agents can dynamically observe a part of the graph on their route. When an obstacle-based conflict is foreseen, the agent revises its path accordingly via a revising strategy.

The proposed revising strategy firstly applies the "WAIT" action to update the current path of the corresponding agent. However, the waiting tactic may not always resolve some situations, such as corridor cases (Sharon et al., 2015) and the irrational behavior of dynamic entities. For such cases, a new path must be reconstructed for that agent during travel. To achieve this, we adapt the Ant Colony Optimization (ACO) algorithm to generate a new conflict-free path. ACO has proven effective for graph-based optimization (Stützle and Dorigo, 2004), particularly in minimizing total travel time in complex path-finding tasks (Dai et al., 2019; Huang et al., 2021). However, applying the standard ACO directly to dynamic and multiagent environments requires adaptation, also incorporating improvements in exploration - exploitation, and hyperparameter tuning (Algorithm 1).

Algorithm 1: Proposed ACO for Revising Strategy.

```
Input: Graph $G = (V, E)$, unavailable edges
Output: Conflict-free path for the agent
1  $\tau_{ij} \leftarrow 1 \; \forall \; (i, j) \in E$;
2  $\beta \leftarrow \beta_{initial}$;
3  while termination condition not met do
4      for each ant do
5          repeat
6              Calculate transition probabilities for
                 available edges (Eq. 15);
7              Randomly select an edge;
8              Traverse on the chosen edge;
9          until ant reaches the goal or exceeds
             travelling time limit;
10     Determine the incumbent path ;
11     Calculate reinforcement levels (Eq. 16);
12     Update pheromones (Eq. 17);
13     Decrease $\beta$ linearly from $\beta_{initial}$ to $\beta_{final}$;
14     if early-stopping condition is met then
15         Break the loop;
16 return The incumbent conflict-free path;
```

In our version, each agent independently executes ACO from their current position to their goal. Pheromone levels ($\tau$), representing historical desirability, are initially set to 1.0 for all edges (Line 1). The algorithm stops when it reaches either the iteration limit (e.g., 150) or an early-stopping condition, where the threshold is set to $1/3$ of the iteration limit (Lines 3-15). In each iteration, multiple ants (e.g., 75) are deployed, exploring within their travel time limits (Lines 5-9). For an adaptation of ACO to DPO-MAPF, a mechanism is essential to prevent conflicts by prohibiting traversal through occupied or reserved vertices and edges. For this purpose, the agent must integrate environmental knowledge by marking certain edges as "unavailable", considering (i) static obstacles, (ii) positions of other agents at the current and subsequent time steps, and (iii) current ($t$) and next

$(t+1)$ positions of observed dynamic obstacles.

The ants randomly select edges based on transition probabilities defined by Equation 15 (Line 6) while unavailable edges are always assigned a probability of zero. For the remaining edges, the transition probability is influenced by two factors: the current pheromone level, $\tau_{ij}(t)$, and heuristic information, $\eta_{ij}$, which corresponds to the inverse Manhattan distance to the goal. The balance between exploration and exploitation is controlled by the parameters $\alpha$ and $\beta$. At the beginning, exploration is prioritized by setting $\beta$ to a high value (e.g., $\beta_{initial} = 5.0$, Line 2). As the algorithm progresses, $\beta$ decreases linearly to $\beta_{final}$ (e.g., 0.5), encouraging exploitation of known paths (Line 13). To maintain stable probabilities, $\alpha$ is fixed at 1.0 throughout the process.

$$P_{ij}(t) = \frac{\tau_{ij}(t)^{\alpha} \times \eta_{ij}^{\beta}}{\sum_{l}^{\mathcal{N}(i)} \tau_{il}(t)^{\alpha} \times \eta_{il}^{\beta}} \tag{15}$$

Once all ants complete their journeys, the best path found so far (i.e., the incumbent path), is determined (Line 10). Pheromone levels are then updated based on the ants' performance. Edges traversed by ants receive pheromone reinforcement, with the magnitude determined by the ratio of the maximum travel capacity to the travel cost (Equation 16, Line 11). In contrast, non-traversed edges decay at a predefined evaporation rate, $\rho$ (e.g. 0.1), gradually reducing their influence over time (Equation 17, Line 12).

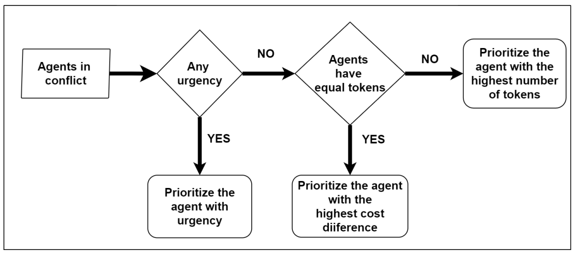
$$\Delta\tau_{ij}(t) = \begin{cases} t_{max}^{m}/L_k & \textbf{if } \text{ant } k \text{ uses edge } (i,j) \\ 0 & \textbf{otherwise} \end{cases} \tag{16}$$

$$\tau_{ij}(t+1) = (1-\rho) \times \tau_{ij}(t) + \sum_{k} \Delta\tau_{ij}(t)^{k} \tag{17}$$

This enhanced ACO approach ensures robust path revision by dynamically adapting to changing environments. If the initial revision fails, the agent continues along its current path, monitoring for new opportunities to resolve conflicts as conditions evolve.

Conflicts can arise not only from dynamic obstacles but also from interactions with other agents. In such cases, while the prioritized agent determined by a consensus protocol continues along its path, the others are expected to concede by revising their paths. To determine which agent receives priority, we introduce two consensus protocols:

- **Fair Token Protocol:** Initially, all agents have an equal number of tokens, and the agent who obtains more tokens will be prioritized, as illustrated in Figure 3. The prioritized agent then loses one token while the other agents gain one token, ensuring fairness among the agents. Since, the


Agents in conflict
Any urgency
NO
Agents have equal tokens
NO
Prioritize the agent with the highest number of tokens
YES
YES
Prioritize the agent with urgency
Prioritize the agent with the highest cost diiference
Figure 3: Fair Token Protocol.

obtained tokens will be considered in subsequent conflicts. The protocol first considers urgent conditions (e.g., reaching the end of travelling time limit or failure in revising strategy). To evaluate urgency, hypothetical revisions are applied to all agents' paths to assess if an urgent condition would arise. Based on this evaluation, the agent who has an urgency is prioritized. When urgency is detected for more than one agent, prioritizing is arbitrarily assigned. If no urgency is detected, the token difference is considered. When agents hold the same number of tokens, the difference in cost between their current and their hypothetical revised paths is used as a tiebreaker. This token-based protocol, inspired by (Eran et al., 2021), ensures feasibility and fairness. Unlike their negotiation-based strategy, we implement a rule-based token allocation approach.

- **Probability-Based Protocol:** This protocol uses a probabilistic approach to determine the likelihood of other agents conceding to a particular agent by considering factors such as the number of agents involved ($N$), the remaining travel time limit ($C_m = t_{max}^{m} - t_{current}^{m}$) and remaining distance to the goal of agent $m$, $D_m$. The risk factor associated with failing to reach the goal if agent $m$ will concede is computed using Equation 18. The priority score helps agents efficiently resolve conflicts, as indicated in Equation 19. Subsequently, $P_{concede}^{m}$ is determined by considering the normalized priority score and the risk factor, as outlined in Equation 20, and calculates the probability of other agents conceding to agent $m$. Consequently, this protocol prioritizes the agents randomly based on the assigned probabilities.

$$R_m = \hat{D}_m \cdot (1 - \hat{C}_m) \quad (18) \qquad PS_m = C_m - D_m \quad (19)$$

$$P_{\text{concede}}^{m} = (\hat{PS}_m \cdot R_m)/(\sum_{l=1}^{L} \hat{PS}_l \cdot R_l) \tag{20}$$

## 7 EXPERIMENTAL EVALUATION

As DPO-MAPF is a novel challenge, no benchmark algorithms are available for direct comparison. Instead, we investigate what extent we have solved this problem. For this purpose, we assess various aspects of the proposed algorithm across multiple scenarios and configurations to demonstrate the impact and performance of key components.

### 7.1 Experimental Setup

To facilitate experiments, we developed a custom map generator to create diverse MAPF simulation setups. This generator produces environments tailored to different scenarios using unique random seeds, allowing for variations in static and dynamic obstacle densities, the number of agents, and map sizes. We designed 132 different map configurations with dimensions of $10 \times 10$, $15 \times 15$, $20 \times 20$, and $25 \times 25$, accommodating 3 to 12 agents and dynamic obstacles. These configurations included static obstacle densities of 5%, 10%, 15%, and 20%, with three random seeds assigned to each scenario. The maximum total time step was set to twice the *Manhattan distance* between each agent's initial and target locations.

For the evaluation, the first tier uses either an optimal (CBSH2-RTC (Li et al., 2021a)) or a sub-optimal (EECBS (Li et al., 2021b)) solver. Additionally, we run 9,504 scenarios, including different algorithm configurations with three different random seeds due to the non-deterministic nature of ACO. We also utilize five evaluation metrics: *success rate*, *runtime*, *concession difference*, *Earth Mover's Distance (EMD)*, and *optimality gap*. The success rate assesses how effectively the algorithms handle uncertainty, while runtime measures the processing time. The number of concessions made by each agent at the end of a scenario is recorded, and the maximum concession difference among agents is obtained, reflecting how fairly the protocols prioritize agents during conflicts. Finally, the EMD quantifies changes in a path after applying the revision process (Rubner et al., 1998; Wiedemann et al., 2021). The average EMD for all agents at the end of each scenario is considered, making it a key metric for assessing the impact of the revision process. The optimality gap measures the difference in the objective value between found solution and optimal solution.

For each randomly generated scenario, we use a commercial solver (e.g., GUROBI) to obtain optimal solutions and validate the existence of feasible solutions for a more reliable evaluation. Through our LP formalization, we calculate the optimality gap and ensure that only operationally feasible and solvable scenarios are included in the evaluation. However, the Certainty Assumption (Winston, 2022) does not hold due to the inherent uncertainty in DPO-MAPF. Therefore, dynamic obstacles are treated as fully observable and known in advance for the LP solution.

Furthermore, Section 7.2 and Section 7.3 evaluate the distinct configurations of the proposed multitier approach. It is worth noting that ideal results aim for a higher success rate, a lower optimality gap, a shorter runtime and a lower concession difference. For each evaluation metric, normality tests and statistical group comparisons are applied to identify significant differences. If a significant difference is observed within a group, corresponding pairwise statistical tests are conducted for post-hoc analysis, with detailed explanations provided in the Appendix.

### 7.2 Evaluation of Multitier Components

This section evaluates the performance of the proposed multitier approach by analyzing the impact of the utilization of a centralized solver (i.e., first tier) and various consensus protocols (i.e., second tier). Besides primary protocols, we include a random strategy that randomly prioritizes agents. Furthermore, we compare the performance of optimal (CBSH2-RTC) and sub-optimal (EECBS) solvers in the first tier.

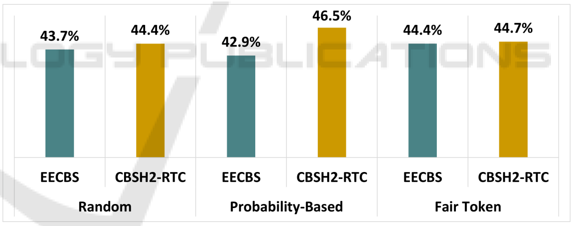


Figure 4: Success Rate of Multitier Components.

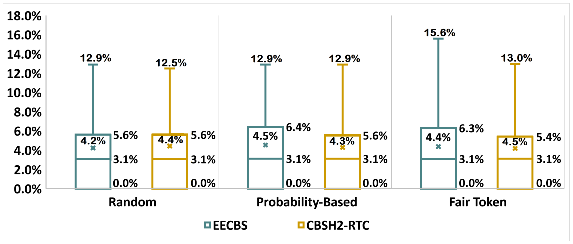


Figure 5: Optimality Gap of Multitier Components.

Figures 4 and 5 illustrate the performance of the proposed approach across different configurations in terms of success rate and optimality gap, respectively. The Probability-Based protocol with the optimal solver achieves the highest success rate (46.5%), while the same protocol with the sub-optimal solver records the lowest success rate (42.9%). Although the optimal solver generally shows a slightly higher success rate, no statistically significant difference is observed in the group test ($p \approx .313 > .05$). Additionally, the Probability-Based protocol with the sub-optimal solver exhibits the worst optimality gap (4.5%), while the Random protocol with the sub-optimal solver shows the best optimality gap (4.2%), again with no statistically significant difference in the group test ($p \approx .994 > .05$).

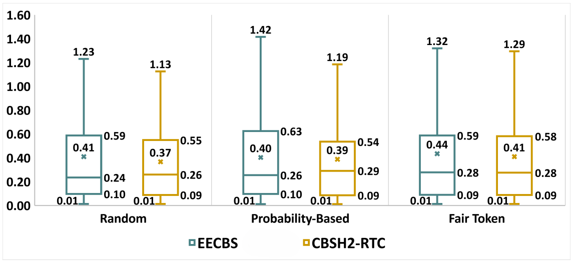


Figure 6: Runtime (sec) of Multitier Components.

In terms of runtime, the Fair-Token protocol with the sub-optimal solver has a statistically significant longer runtime, as confirmed by the group test ($p \approx .003 < .05$) and displayed in Figure 6. This is because, unlike other protocols that only revise the paths of non-prioritized agents, the Fair-Token protocol evaluates potential urgency by considering hypothetical revisions for all agents, including the prioritized one. This additional evaluation step increases the runtime.

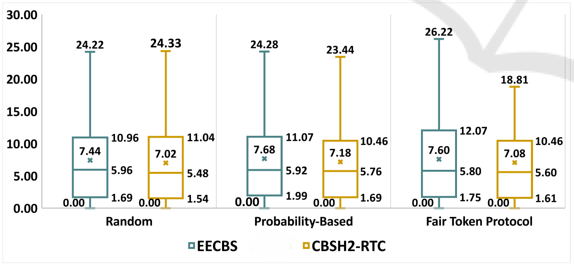


Figure 7: Average EMD of Multitier Components.

Moreover, Figure 7 presents the average EMD, which measures the magnitude of path changes for each configuration. No significant differences, as indicated by the group test ($p \approx .168 > .05$). This analysis indicates that consensus protocols and the centralized solver have no substantial impact on the success rate, optimality gap, or runtime, as they do not directly affect the revision of the paths taken by the agents in DPO-MAPF.

Furthermore, Figure 8 shows the maximum concession difference among agents at the end of a scenario. The Fair-Token protocol, with both optimal

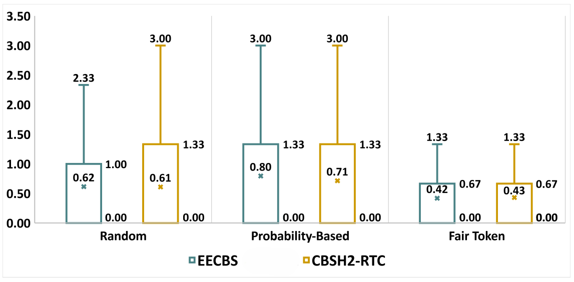


Figure 8: Max. Concession Diff. of Multitier Components.

and suboptimal solvers, achieves the greatest fairness (0.43 and 0.42, respectively), while the Probability-Based protocol with the both solvers records the worst fairness (0.71 for the optimal solver and 0.80 for the suboptimal one). Statistically significant differences are observed across all combinations in the group test ($p \approx .000 < .05$). The Fair-Token protocol, specifically designed to promote fairness, results in the lowest concession difference overall. On the other hand, the Probability-Based protocol tends to frequently prioritize a single agent with the highest risk of failing to complete its path, potentially leading to unfair situations.

## 7.3 Evaluation of Revising Strategy

The Fair-Token protocol with the optimal solver (CBSH2-RTC) is the best performing configuration for our multitier approach based on the evaluation results. Using this configuration, this section evaluates the performance of proposed revising strategy by examining different aspects, including:

- **No-Revising:** A baseline scenario where no revising strategy is applied.
- **Only Waiting:** A configuration that employs only the “Waiting” strategy as a revising method.
- **Basic ACO:** Our basic variant of ACO adapted to DPO-MAPF, introducing “unavailable edges” (areas ants cannot traverse) and limiting ants’ travel based on the maximum time step of the agent.
- **Enhanced ACO:** Our enhanced variant of Basic ACO incorporating the following proposed improvements: (i) applying the “Waiting” strategy before ACO, (ii) implementing an early-stopping condition, and (iii) dynamically $\beta$ adjustment.

Figure 9 shows the success rate of the revising strategies, indicating that Enhanced ACO statistically significantly outperforms No-Repairing, Only Waiting, and Basic ACO (44.7% vs. 18.9% & 23.5% & 39.9%). Note that the group test shows the statistically significant difference among revising strategies

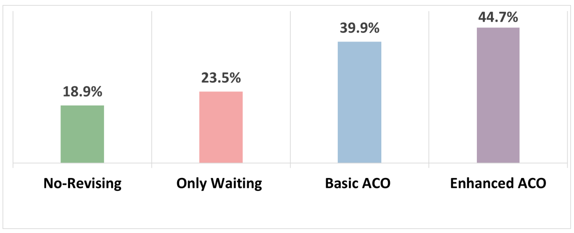


Figure 9: Success Rate for Revising Strategies.

($p \approx .000 < .05$). Basic ACO also statistically outperforms No-Repairing and Only Waiting, indicating that the traditional Only Waiting strategy struggles to handle the non-rational behavior of dynamic obstacles, underscoring the importance of path reconstruction.

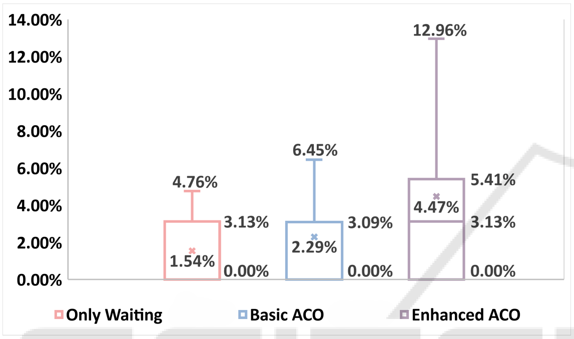


Figure 10: Optimality Gap for Revising Strategies.

Furthermore, Figure 10 presents the optimality gap among the strategies, where the difference in the group test is statistically significant ($p \approx .000 < .05$). Despite the increase in success rate, Enhance ACO has the worst optimality gap (4.47%), indicating a trade-off in the proposed enhancements. Note that the optimality gap can be only calculated for successfully completed scenarios, meaning that the gap between strategies becomes less relevant when the difference in success rates is statistically significant.

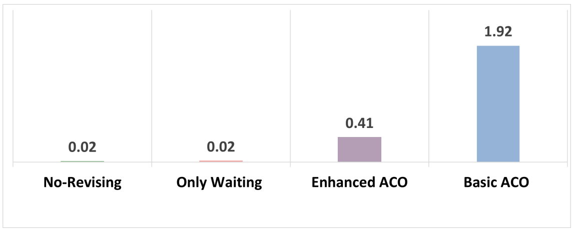


Figure 11: Runtime (sec) for Revising Strategies.

Moreover, Figure 11 illustrates the runtime for each revising strategy, with a statistically significant difference in the group test ($p \approx .00 < .05$). The Basic ACO strategy yields the slowest runtime (1.92 sec) while the No-Revising and Only Waiting achieve the fastest one (0.02 sec). This result denotes that ACO

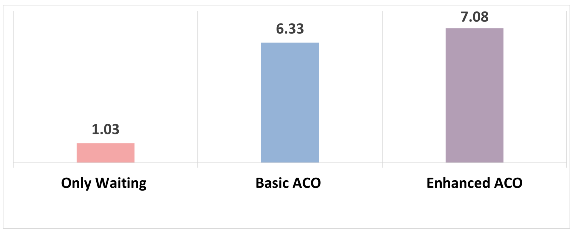


Figure 12: Average EMD for Revising Strategies.

process increases the runtime to resolve conflicts but achieves a higher success rate. However, the early-stopping condition and the use of waiting strategy enhancements successfully reduce the redundant processing time without compromising the success rate.

Figure 12 presents the average EMD for each revising strategy, with a statistically significant difference in the group test ($p \approx .000 < .05$). Regarding to average EMD, Enhanced ACO revises paths more extensively than the other versions (7.08 vs. 6.33 and 1.03). The results suggest that more substantial updates tend to increase the success rate, the optimality gap, and the runtime. The correlations between average EMD and optimality gap (0.106), success rate (0.550), and runtime (0.504) are also observed.

Notably, the LP formulation overlooks the uncertainty challenges of the DPO-MAPF problem. This means that both the lower bound (i.e., optimal value) and the feasibility check provided by LP for randomly generated scenarios may not be achievable due to the lack of situational awareness. While this limitation complicates analysis and reasoning, the LP solution still serves as a valuable baseline for evaluation. Our proposed approach might already be achieving the best possible performance under ideal conditions, but this remains uncertain due to the inherent unpredictability of DPO-MAPF.

## 8 CONCLUSION

Uncertainty in real-world scenarios arises from the stochastic or unpredictable behaviors of other entities and events in the environment. To better account for such uncertainties, this study extends the traditional MAPF problem by introducing partially observable and dynamically moving obstacles, referred to as DPO-MAPF. To address this challenge, we propose a multitier solution approach. In the first tier, a centralized and offline path planning algorithm is employed to find conflict-free paths for each agent, considering only static and fully observable obstacles. The second tier involves a decentralized and online conflict resolution process that employs a heuristic-based revising strategy (i.e., ACO) and a consensus protocol to resolve conflicts arising from the uncertainty challenges in DPO-MAPF.

For evaluation, we developed a grid-based framework, which enables the random generation of scenarios with varying map sizes, agent counts, and obstacle densities. Additionally, a linear programming approach using a commercial solver was developed to validate the feasibility of the randomly generated scenarios. The evaluation results show that the proposed revising strategy improves the success rate under the uncertainty of DPO-MAPF, although it slightly increases the optimality gap and extends the runtime. Furthermore, the results indicate that the magnitude of revision is correlated with success rate, runtime, and optimality gap. While the first tier employs state-of-the-art approaches (CBSH2-RTC and EECBS), using an optimal solver yields better outcomes. For consensus protocols, Fair Token protocol provides the greatest fairness among agent without compromising performance. In summary, the decentralized and online conflict resolution approach enables agents reach their goals under uncertainty and enhances fairness among agents, which is crucial in certain real-world applications.

Our problem definition closely mirrors real-life scenarios by incorporating uncertainty, making it applicable for applications such as autonomous vehicle navigation, drone fleet coordination, and warehouse robotics. Future work could focus on refining the revising strategy, including (i) experimenting with different field-of-view ranges, (ii) utilizing initial paths instead of constructing them from scratch, and (iii) optimizing the balance between success rate, optimality gap, and runtime to develop more robust solutions for DPO-MAPF. Further analysis should expand the variety of scenarios (e.g., larger map sizes, a higher number of agents, or existing benchmark scenarios from (Stern et al., 2021)) by identifying alternative approaches for validation and optimal values rather than LP due to the scalability and uncertainty issues. Additionally, this study establishes a benchmark for performance comparison in future research, as DPO-MAPF is a newly defined problem with no alternative algorithms available thus far.

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## APPENDIX

The components of proposed approaches (mentioned in 7.2) are evaluated through a series of statistical analyses to ensure their reliability and consistency during the evaluation process. Firstly, normality tests using the Shapiro-Wilk test were conducted, and all evaluation metrics (average EMD, optimality gap, runtime, and max. concession difference) failed across all groups. Consequently, non-parametric tests were applied for further comparison.

For success rate, the Cochran's Q test was used and indicated no significant difference ($p \approx .313 > .05$). Similarly, the average EMD was evaluated using the Friedman test, which showed no significant difference ($p \approx .168 > .05$). Lastly, the optimality gap was analyzed with the Kruskal-Wallis test, yielding no significant difference ($p \approx .994 > .05$). Additionally, for both runtime and max. concession difference, the Friedman test was applied and showed significant differences for both metrics ($p \approx .003 < .05$ for runtime, $p \approx .000 < .05$ for max. concession Difference), prompting pairwise comparisons using the Wilcoxon signed rank test (Table 2).

Table 2: Pairwise Comparisons (p-values) for Runtime and Max. Concession Difference.

| **Comparison** | **Runtime** | **Max. Concession Difference** |
|---|---|---|
| Random (EECBS) vs Fair Token (EECBS) | **.030** | **.002** |
| Probability-Based (CBSH2-RTC) vs Fair Token (EECBS) | **.019** | **.000** |
| Fair Token (EECBS) vs Random (CBSH2-RTC) | **.005** | **.012** |
| Fair Token (EECBS) vs Probability-Based (EECBS) | **.008** | **.000** |
| Random (EECBS) vs Probability-Based (CBSH2-RTC) | .330 | .179 |
| Random (EECBS) vs Random (CBSH2-RTC) | .085 | .957 |
| Random (EECBS) vs Fair Token (CBSH2-RTC) | .829 | **.003** |
| Probability-Based (CBSH2-RTC) vs Random (CBSH2-RTC) | .543 | .092 |
| Probability-Based (CBSH2-RTC) vs Probability-Based (EECBS) | .510 | .519 |
| Probability-Based (CBSH2-RTC) vs Fair Token (CBSH2-RTC) | .274 | **.000** |
| Fair Token (EECBS) vs Fair Token (CBSH2-RTC) | .340 | .900 |
| Random (CBSH2-RTC) vs Probability-Based (EECBS) | .232 | .074 |
| Random (CBSH2-RTC) vs Fair Token (CBSH2-RTC) | .178 | **.003** |
| Probability-Based (EECBS) vs Fair Token (CBSH2-RTC) | .513 | **.001** |

Now, the revising strategies (mentioned in 7.3) are evaluated through a series of statistical analyses. For all evaluation metrics (optimality gap, runtime, average EMD) failed the normality test (i.e., Shapiro-Wilk), leading to the use of non-parametric tests again, illustrated in Table 3. The Friedman test was applied for runtime and average EMD all show significant differences ($p \approx .000 < .05$), followed by the Wilcoxon signed rank test for pairwise comparisons. For the optimality gap, the Kruskal-Wallis test indicated significant differences ($p \approx .000 < .05$), prompting Conover-Iman test for pairwise comparisons. Finally, success rate were analyzed using Cochran's Q test, which also showed significant differences ($p \approx .000 < .05$), followed by McNemar's test for pairwise comparisons.

Table 3: Pairwise Comparisons (p-values) for Revising Strategies.

| **Comparison** | **Optimality Gap** | **Average EMD** | **Runtime** | **Success Rate** |
|---|---|---|---|---|
| Enhanced ACO vs Basic ACO | **.000** | **.014** | **.000** | **.023** |
| Enhanced ACO vs Only Waiting | **.000** | **.000** | **.000** | **.000** |
| Basic ACO vs Only Waiting | .402 | **.000** | **.000** | **.000** |
| Basic ACO vs No-Repairing | - | - | **.000** | **.000** |
| Enhanced ACO vs No-Repairing | - | - | **.000** | **.000** |
| Only Waiting vs No-Repairing | - | - | **.000** | **.005** |