Comparative Analysis of Simulated Annealing and Particle Swarm Optimization for Multi-Robot Task Allocation in ROS

Dhruv Kumar Sharma[a], Ujjwal Singh[b], Snehal Nalawade[c] and Pratik Shah[d]
Indian Institute of Information Technology Vadodara, Gujarat, India
{202151049, 202151158, 202151160, pratik}@iiitvadodara.ac.in

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Abstract: A comparative analysis of two prominent optimization techniques—simulated annealing (SA) and particle swarm optimization (PSO)—is conducted within the framework of multi-robot systems (MRS). The research investigates how each algorithm effectively allocates tasks among multiple robots, focusing on performance metrics, convergence speed, and robustness in dynamic environments. Through extensive simulations in ROS, utilizing a dedicated testbed for real-world scenario emulation, distinct advantages and limitations of both algorithms are revealed across various setups. The testbed integrates realistic garbage generation, dynamic obstacles, and robot interactions, allowing for detailed empirical evaluations. The study highlights the practical implications of using SA and PSO for multi-robot coordination, laying the groundwork for future research on hybrid approaches and algorithmic enhancements in complex robotic applications.

1 INTRODUCTION

In recent years, the integration of multi-robot systems has gained significant attention due to their potential to execute complex tasks more efficiently than single-robot counterparts. These systems leverage the collaborative capabilities of multiple robots to achieve objectives that would be infeasible for a single entity, thereby enhancing both robustness and scalability.

Latest studies have focused on multi-robot task allocation strategies in uncertain environments for cleaning. (Wang et al., 2023) developed a robust optimization model using mixed-integer linear programming, comparing techniques like simulated annealing, genetic algorithms, particle swarm optimization, and deep reinforcement learning (DRL) for task distribution. (Jeon et al., 2015) studied the reallocation strategies for tasks. Performance Maximization was considered the most effective strategy to achieve cleanliness and efficiency. (Orr and Dutta, 2023) surveyed Applications of Multi-Agent DRL, which include GNNs, attention mechanisms and simulators for path planning as well as task allocation. (Le et al., 2020), and (Le et al., 2018) focused task assignment and reallocation task with efficiency and obstacle handling. The work of (Hong et al., 2021) introduced task reassignment efficiency under constraints when using replicator dynamics. (Elfakharany and Ismail, 2021) proposed decentralized task allocation under end-to-end DRL, optimizing coordination in the unstructured environment. (Park et al., 2022) demonstrated real-time cooperative task allocation using reinforcement learning, improving adaptability and performance.

While most research on multi-robot systems has focused on coverage path planning or task allocation, this paper presents an efficient algorithm for task allocation and scheduling in a multi-robot cleaning system. By leveraging the modular architecture of the Robot Operating System (ROS), we enable synchronized communication and coordination, enhancing overall system performance.

Our study highlights the potential of ROS-based multi-robot systems utilizing either SA or PSO to improve efficiency and reliability. Comprehensive simulations compare the performance of these algorithms, demonstrating their effectiveness and applicability to real-world scenarios.

[a] https://orcid.org/0009-0007-8928-6947
[b] https://orcid.org/0009-0001-4828-3834
[c] https://orcid.org/0009-0002-6325-7201
[d] https://orcid.org/0000-0002-4558-6071

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2 PROBLEM STATEMENT

We designed a multi-robot cleaning system for large areas, featuring drones for image capture, ground cleaning robots, and a central control system coordinating all operations. Drones, equipped with sensors, send environmental data to the central system, which updates a map and dynamically assigns cleaning tasks based on real-time garbage and obstacle detection.

We developed a testbed to evaluate robot configurations, task allocation, and scheduling strategies in realistic scenarios. It includes a garbage generation routine and uses ROS2 for robot control. The *slam_toolbox* (Macenski, 2019), (Macenski and Jambrecic, 2021) package handles dynamic obstacle detection, while *nav2* (Macenski et al., 2020) ensures efficient navigation. Simulations are conducted in Gazebo for realistic and flexible testing.

2.1 Problem Formulation

In a large public area with randomly generated garbage and obstacles such as vehicles and pedestrians, the goal is to ensure continuous cleaning while optimizing robot utilization. The area is represented as an environment map—a numerical matrix encoding objects and conditions—to facilitate algorithmic processing.

The system employs N cleaning robots, M drones, and a central management unit. Drones survey the area at set altitudes, capturing ground images and transmitting them to the central unit. A YOLOv8 model, trained on the area, processes these images to identify garbage and obstacles, updating the environment map accordingly. This map informs a task scheduling algorithm that assigns cleaning tasks to robots, optimizing their movement across regions and dynamically adjusting to changes in garbage distribution. Task schedules are communicated to robots via the central unit, which continuously monitors and updates the map, reassigning tasks as needed.
Key variables:

- $N \rightarrow$ Total number of robots
- $K \rightarrow$ Total number of tasks
- $n_r \rightarrow$ Number of tasks allocated to robot r
- $P \rightarrow$ Precedence adjacency list; where P_r represents a list of tasks assigned to robot r, and task P_{ri} precedes P_{rj} $\forall\ i < j$
- $D \rightarrow$ Distance matrix. A $K \times K$ matrix containing distances between tasks.
- $litter[i] \rightarrow$ Amount of litter present at task i
- $l_r \rightarrow$ Total litter collected by robot r after task allocation
- $t_r \rightarrow$ Time taken by robot r to complete its assigned tasks
- $L_r \rightarrow$ Litter capacity of robot r
- $T_r \rightarrow$ Time capacity of robot r
- $\alpha \rightarrow$ Litter penalty constant
- $\beta \rightarrow$ Time penalty constant
- $C_r \rightarrow$ Cost associated with robot r after task allocation

System performance is evaluated by the ability to maintain cleanliness, with robots efficiently covering regions and removing garbage to ensure optimal area cleanliness. The objective is to minimize the average waiting time, reducing the mean interval between garbage accumulation and its removal.

2.2 Constraints

Several constraints are taken into consideration while devising the task assignment & scheduling algorithm.

1. The robot's battery level (*time_remaining*) is checked before assigning new tasks to ensure sufficient charge.
2. The robot's garbage load (*current_dump*) is monitored, and it is directed to the dumping ground if the load approaches the tank's capacity (*dump_capacity*) before resuming collection tasks.
3. Both robots and the central system are informed of the locations of charging stations and dumping grounds.

All the constraints and objective functions involved are as follows:

$$l_r = \sum_{i=1}^{n_r} litter[P_{ri}] \quad (1)$$

Equation (1) determines the total amount of litter collected by robot r following the task allocation process. The tasks assigned to the robot are retrieved from the Precedence list P_r, and the litter corresponding to each task is accessed from $litter[P_{ri}]$. This is then summed across all tasks to yield the total litter collected by robot r.

$$t_r = \sum_{i=1}^{n_r} \left(\frac{D[P_{r(i-1)}][P_{ri}]}{s} + litter[P_{ri}] \times t \right) \quad (2)$$

Equation (2) calculates the time taken by robot r to complete its assigned tasks. The time includes

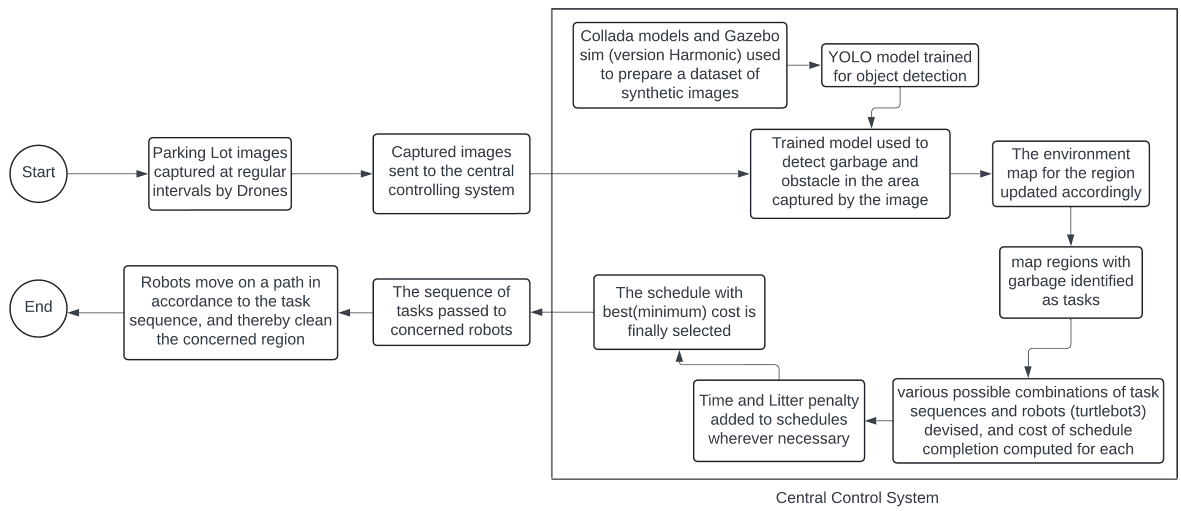


Figure 1: Multi-Robot garbage detection & task allocation system: The termination conditions for the algorithm includes completion of all tasks, i.e. the algorithm terminates when all garbage and obstacle tasks in the mapped region are assigned and completed by the robots as per the computed optimal schedule.

both the travel time from task $P_{r(i-1)}$ to task P_{ri} and the time required to clean the litter at task P_{ri}. $D[P_{r(i-1)}][P_{ri}]$ represents the distance between the two tasks, and s is the speed of the robot. $litter[P_{ri}]$ represents the amount of litter present at task P_{ri}, and t represents the time required to clean unit litter.

$$C_r = t_r + max(0, \alpha \times (l_r - L_r)) + max(0, \beta \times (t_r - T_r)) \tag{3}$$

Equation (3) calculates the total cost associated with robot r after task allocation. The cost includes the time taken to complete the tasks, the litter penalty (if applicable), and the time penalty (if applicable). This cost function is critical in evaluating the efficiency of each robot.

$$C_{max} \geq t_r + max(0, \alpha \times (l_r - L_r)) + max(0, \beta \times (t_r - T_r)) \tag{4}$$

Equation (4) determines the maximum cost C_{max} among all robots. It represents the worst-case scenario in terms of task allocation and is essential for identifying bottlenecks in the system.

$$\min_{x \in X} C_{max}(x) \tag{5}$$

Finally, The goal of the optimization process is to minimize the maximum cost C_{max} among all robots. By achieving this, the system ensures that tasks are allocated in a manner that optimizes the overall performance, leading to the best possible outcome.

3 ENVIRONMENT-AGENT FORMULATION

The environment is represented by a matrix A with values $a_{ij} \in \{0, 1, 2\}$, where 0 denotes a clean region, 1 denotes waste, and 2 indicates an obstruction.

The cost-function-driven task scheduler assigns tasks according to the current map and robot state in real time, ensuring efficiency. After that, robots clean the designated regions, and constant updates on robot conditions and environmental factors ensure seamless operations. This well-coordinated strategy, which combines robots to carry out tasks and drones to gather data, guarantees an effective cleaning procedure.

On a testbed, environment photos were manually labeled on the Roboflow platform and divided into training, testing, and validation sets to create the dataset used to train the YOLOv8 garbage detection model. Because of its excellent garbage detection accuracy, the trained YOLOv8 model is an essential part of our system.

A cost optimization function is critical in task scheduling and resource allocation, aiming to minimize operational costs by considering factors like time, energy consumption, and resource utilization. It evaluates various candidate solutions to balance performance metrics with system constraints, ensuring efficient resource allocation while reducing costs. In research involving robotic systems or complex environments, such functions enhance system efficiency and scalability by guiding real-time decision-making

Data: Robots, Distance matrix, P matrix, Dump capacity, Speed, Dump location, Penalty coefficients α, β, Time limit T_0
Result: Total cost *total_cost*
$total\_cost \leftarrow 0$
for *each robot i* **do**
  Initialize $travel\_time \leftarrow 0$, $total\_litter \leftarrow 0$, $previous\_task \leftarrow depot$
  for *each task* $j \in P_i$ **do**
    if $total\_litter + litter[j] > dump\_capacity$ **then**
      $travel\_time$ += time to dump and back to next task
      $total\_litter \leftarrow litter[j]$
    end
    else
      $total\_litter$ += $litter[j]$
    end
    $travel\_time$ += time to next task
    $previous\_task \leftarrow j$
  end
  $total\_time \leftarrow travel\_time + total\_litter \times t$
  $robot\_cost \leftarrow total\_time$
  Add penalties if applicable:
  if $total\_litter > dump\_capacity$ **then**
    $robot\_cost$ += $(total\_litter - dump\_capacity) \cdot \alpha$
  end
  if $total\_time > T_0$ **then**
    $robot\_cost$ += $(total\_time - T_0) \cdot \beta$
  end
  $total\_cost \leftarrow \max(total\_cost, robot\_cost)$
end
return *total_cost*

Algorithm 1: Calculate Cost.

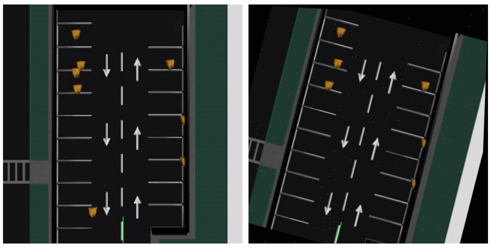
based on dynamic factors like workload, task priority, and resource availability. The cost function used is given in Algorithm 1. In our study, we employ Simulated Annealing (SA) and Particle Swarm Optimization (PSO) within the cost optimization framework to explore and exploit the solution space, enabling the reduction of costs while satisfying system constraints.

For more details on the SA and PSO algorithms, refer to APPENDIX A and APPENDIX B.

4 TESTBED DEMONSTRATION

The testbed developed consists of three main components: a 3D model representing the cleaning environment, models of the cleaning robots, and models of surveying drones. The communication between the simulated environment and the central control system, as well as the tasks of path planning and navigation, are managed using ROS (Robot Operating System) along with its associated packages, specifically *slam_toolbox* and *nav2*.

We consider a generic parking lot as the testbed environment. The 3D model is sourced from Sketchfab. This environment model is used to generate a dataset, which is essential for training the garbage detection model. Sample augmented images from the simulation are provided in Figure 2, illustrating the environment used for model training.



(a) (b)

Figure 2: Augmented images from the simulation used for training the garbage detection model.

To simulate the generation of garbage in a dynamic environment, a garbage spawning routine (Algorithm 2) is used, relying on three probabilistic components: time intervals (modeled by an exponential distribution), the number of garbage bags (following a Poisson distribution), and their spatial coordinates (determined by a uniform distribution). These probabilistic models enable varied and realistic scenarios.

Data: Initial spawn rate λ
Result: Garbage bags spawned at random coordinates
initialize λ;
while *true* **do**
  draw time interval T from exponential distribution;
  schedule the next garbage spawn event after T seconds;
  draw the number of garbage bags N from the Poisson distribution, at scheduled time;
  for $i = 1$ **to** N **do**
    draw x and y coordinates from the uniform distribution;
    spawn garbage bag i at the specified coordinates (x, y);
  end
end

Algorithm 2: Garbage Spawning Routine.

The cleaning robots in the simulation are based on the *TurtleBot3*, a ROS-enabled mobile robot platform. For the surveying tasks, an X3 UAV model, obtained from Gazebo Fuel, is used. Modifications were made to the drone model, including the addition of a LiDAR sensor for altitude measurement and adjusting the onboard camera to capture images of the ground beneath the drone. These modifications enhance the drone's capability for environmental monitoring and data collection.

The implementation of the described testbed is available at this GitHub repository.

5 EXPERIMENT AND SIMULATIONS

5.1 Performance Analysis with Varying Robot Counts and Task Complexity

This study evaluates how performance metrics—cost, time elapsed, and energy—vary with the number of robots and garbage spawning rates. The number of robots is varied from 1 to 10, and the Poisson distribution parameter (λ), governing garbage generation rates, is varied from 1 to 10. For each combination of robot count and λ, metrics are calculated as the mean of 10 independent observations to ensure reliability.

Results demonstrate that Particle Swarm Optimization (PSO) outperforms Simulated Annealing (SA) across all metrics. PSO consistently achieves lower costs, faster task completion, and reduced energy consumption. For example, with $\lambda = 2$ and 8 robots, PSO incurs a cost of 134.2 compared to SA's 168.6, while for $\lambda = 9$ and 9 robots, PSO's cost is 403.2 compared to SA's 556.2. Similarly, in terms of time and energy, PSO completes tasks more efficiently. For $\lambda = 4$ and 8 robots, PSO reduces time to 211.4 units compared to SA's 268.8, and for $\lambda = 9$ and 9 robots, PSO uses 317.0 energy units, significantly lower than SA's 3170.0. These findings highlight PSO's superior optimization capability in dynamic, resource-intensive environments.

The penalties (α and β) play a critical role in influencing the total cost of the system. Robots that exceed their litter capacity or battery limitations incur these penalties, which substantially increase the overall costs. This effect is especially pronounced in scenarios with fewer robots, as each robot must cover a larger area, leading to greater chances of surpassing these thresholds. Generally, the total cost, time, and energy required to complete cleaning tasks decrease as the number of robots increases. This is because a larger number of robots can share the workload, allowing tasks to be completed more quickly and with lower energy consumption. However, an excessive number of robots may introduce coordination challenges, reflecting the "law of diminishing returns," where the benefits of adding more robots are offset by the overhead of managing them.

Performance metrics improve with an increasing number of robots, as task distribution reduces individual workload. However, excessive robots lead to diminishing returns due to coordination challenges. Higher λ values increase costs, time, and energy, particularly with fewer robots, as each must cover larger areas. Efficient task allocation requires balancing robot count with task complexity, ensuring minimized costs and energy consumption. Tables 1 and 2 summarize the optimal configurations for SA and PSO.

Table 1: Summary of optimal Number of Robots, Lambda Poisson, Cost, Time, and Energy for Different Configurations (SA).

Robots	Lambda Poisson	Cost	Time	Energy
3	1	68.5	68.5	169.0
8	2	168.6	168.6	546.4
4	3	252.4	252.4	730.8
9	4	268.8	268.8	1314.6
8	5	319.4	319.4	1486.2
6	6	395.3	395.3	1602.6
8	7	493.4	493.4	2344.0
8	8	565.1	565.1	2782.8
9	9	556.2	556.2	3170.0

Table 2: Summary of optimal Number of Robots, Lambda Poisson, Cost, Time, and Energy for Different Configurations (PSO).

Robots	Lambda Poisson	Cost	Time	Energy
3	1	35.4	36.2	35.4
8	2	134.2	134.6	457.2
8	3	289.8	294.8	871.2
10	4	246.4	271.6	942.4
9	5	351.8	385.4	548.8
5	6	282.6	345.2	423.6
10	7	341.8	475.2	341.8
9	8	402.4	550	402.4
10	9	403.2	563.8	317

5.2 Impact of Robot Battery Capacity on Cleaning Cost Efficiency

This experiment focuses on a controlled setup where several parameters are fixed, and the focus is placed on the effect of time capacity. The parameters involved are:

Poisson Distribution (fixed): λ is set to a constant value of 4. This reflects a moderate rate of task arrival (garbage generation) in the system.

Number of Robots (fixed): Set to 3 robots, providing a baseline capacity for task allocation.

Dump Capacity (fixed): Set to a high value of 100,000 units to avoid penalties associated with exceeding the dump capacity. This ensures that the experiments focus on other factors without the influence of dump-related penalties.

Time Capacity (variable): The time capacity is varied in this setup. By modifying the time capacity, the system is tested under different operational constraints, allowing an exploration of how time restrictions affect the overall performance.

The rationale behind this setup is to isolate the impact of time capacity on the task allocation process, while minimizing other variables (such as penalties related to dump capacity). This controlled experiment provides insights into the algorithms' efficiency when time becomes the primary limiting factor.

Figure 3 and Figure 4 indicate the relationship of time-capacity for the set of batteries and cost incurred by the multi-robot system. Cost decreases steeply with the increase in battery capacities primarily because of decreased charging demands, or time and energy spent going to the charging station and back to cleaning zones, after which it reaches a cost range. This indicates that optimizing battery capacity can significantly reduce operational costs up to a certain point, after which the cost stabilizes.

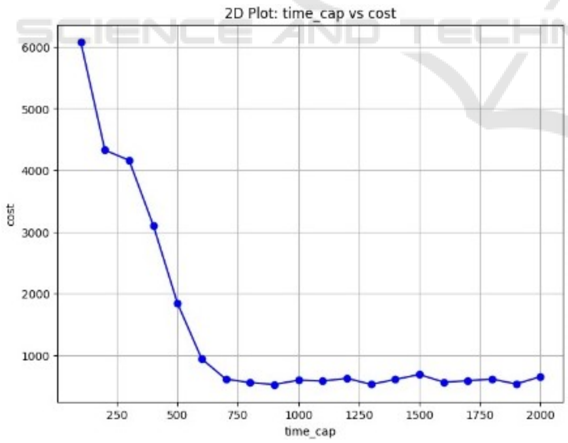


Figure 3: Effect of Robot Battery Capacity(time_cap) on System Cost [SA] (Time Capacity vs. Cost). The parameters kept constant here are: λ = 4, and num_robots = 3.

Figure 3 represents a system where costs are initially much higher and more significantly affected by increasing the time_cap. The model stabilizes after some point, indicating that increasing the time_cap further offers diminishing returns.

Figure 4 represents a system where the cost is much lower from the start, but there is less stability over time, with more fluctuations as the time_cap increases.

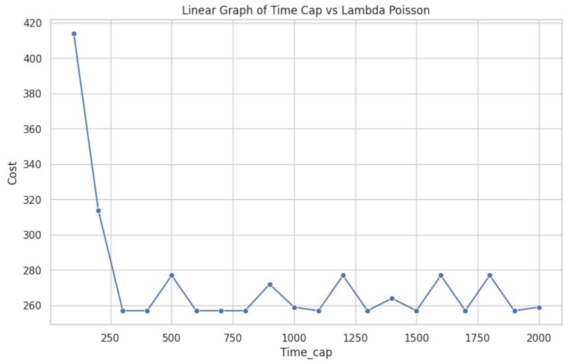


Figure 4: Effect of Robot Battery Capacity(time_cap) on System Cost [PSO] (Time Capacity vs. Cost). The parameters kept constant here are: λ = 4, and num_robots = 3.

5.3 Limitations

The study assumes homogeneous abilities of ground robots and uniform environments, which simplifies simulations but may not accurately capture the com plexities of real-world scenarios. Additionally, the use of fixed robot speeds and constant garbage gener ation rates does not account for dynamic factors, such as varying terrain conditions or human activity, which could significantly impact performance in practical applications.

6 CONCLUSION

This research underscores the efficacy of multi-robot systems when integrated with the ROS framework and an optimized algorithm for task scheduling. The implementation of algorithms like simulated annealing (SA) and particle swarm optimization (PSO) within the ROS ecosystem has demonstrated significant improvements in task coordination, resource utilization, and operational efficiency. By facilitating seamless communication and synchronization between multiple robots, our system effectively addresses the complexities associated with dynamic task environments.

In conclusion, the findings from this study contribute valuable insights into the design and deployment of multi-robot systems, offering a viable solution for complex tasks in diverse domains. Future research may focus on further refining the task scheduling algorithms, expanding the system's capabilities, and exploring its application in more challenging and dynamic environments. The advancements achieved here lay the groundwork for more sophisticated multi-robot applications, paving the way for broader adoption in both industrial and research settings.

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APPENDIX A

Simulated Annealing

Simulated Annealing (SA) is an optimization algorithm that navigates the solution space by allowing both upward and downward adjustments, helping escape local minima and move toward a global solution. It gradually reduces a "temperature" parameter to balance exploration and refinement, making it effective for complex problems.

Cooling Rate: The cooling rate (α) is defined as $T_{\text{new}} = \alpha \cdot T_{\text{old}}$, where it controls the rate of temperature decrease. Its value varies between 0 and 1. A higher α (closer to 1) promotes broader exploration, reducing the risk of getting trapped in local minima, while a lower α focuses on refining the current solution but risks suboptimal outcomes. Proper tuning ensures an optimal balance between exploration and exploitation. The employed algorithm is given in Algorithm 3.

Data: Initial temperature T_0, Cooling rate r
Result: Best solution and Cost
Initialize $T \leftarrow T_0$, random P_matrix, $current\_cost \leftarrow$ cost of P_matrix
$best\_cost \leftarrow current\_cost$, $best\_solution \leftarrow P\_matrix$
while $T > T_s$ **do**
  for $i \leftarrow 1$ **to** L_k **do**
    Apply a random operation: $swap()$, $invert()$, or $move()$; Compute $\Delta \leftarrow$ cost change
    if $\Delta < 0$ *or* $random() < exp(-\Delta/T)$ **then**
      Update $current\_solution$, $current\_cost$
      if $current\_cost < best\_cost$ **then**
        Update $best\_solution$, $best\_cost$
      end
    end
  end
  $T \leftarrow T \cdot r$
end
return $best\_solution, best\_cost$

Algorithm 3: Simulated Annealing (SA).

APPENDIX B

Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a collaborative optimization algorithm where a swarm of particles explores the solution space, adjusting their positions based on their own experience and the best solutions found by others. This information sharing enables an efficient balance between exploration of new areas and exploitation of promising regions, making PSO effective for complex problems.

Inertia Factor (ω): The inertia factor (ω) governs the influence of a particle's current velocity on its next velocity. A higher ω encourages broad exploration, while a lower ω focuses on refining around the best-known solutions. The velocity update formula is:

$$v_i(t+1) = \omega \cdot v_i(t) + c_1 \cdot r_1 \cdot (p_i - x_i) + c_2 \cdot r_2 \cdot (g - x_i),$$

where $v_i(t)$ is the current velocity of particle i, p_i is its personal best position, and g is the global best position. A decaying ω is often used to start with global exploration and gradually shift to local exploitation for better convergence. The employed algorithm is given in Algorithm 4.

```
Data: N: Number of particles, T: Max
      iterations, w, c1, c2: PSO factors
Result: global_best: Optimal solution,
        global_best_cost: Optimal cost
Initialize global_best_cost ← ∞, particles with
  random positions and velocities
for each particle do
    if particle.personal_best_cost <
      global_best_cost then
        Update global_best and
          global_best_cost
    end
end
for t = 1 to T do
    for each particle do
        Update velocity:
          particle.velocity ← w · velocity + c1 ·
          rand · (personal_best − position) +
          c2 · rand · (global_best − position)
        Update position using velocity
        new_cost ← particle.calculate_cost()
        if new_cost <
          particle.personal_best_cost then
            Update personal best
            if new_cost < global_best_cost
              then
                Update global_best and
                  global_best_cost
            end
        end
    end
end
return global_best, global_best_cost
```

Algorithm 4: Particle Swarm Optimization (PSO).