Toward a Quantum Fuzzy Approach for Emotion Modeling in Parent-Child Interactivity

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Abstract: This study presents an integrated framework combining Quantum Fuzzy computing concepts with emotion modeling and simulations of intelligent agents. It explores the distinctions between Quantum Fuzzy and Classical Computing, focusing on parent-child relationships. Simulations performed on the Qiskit platform highlight significant differences in the results produced by these two approaches. The research emphasizes how membership degrees(MD) are represented in the quantum circuit model by interpreting fuzzy operations through unitary quantum transformations. Established fuzzy connectives, such as the exclusive OR, serve as an algebraic basis for constructing quantum operators and circuit representations. The algorithms demonstrate substantial potential for extension, allowing for modeling interactions among multiple agents using multi-dimensional quantum registers. Simulations within Qiskit offer a solid foundation for implementing these algorithms on real quantum platforms, paving the way for further exploration in this interdisciplinary field.

1 INTRODUCTION

Quantum Computing (QC) introduces a revolution for solving classical complex problems, offering an exponentially superior processing capability compared to classical computers. Using qubits, the fundamental units of information in quantum systems, this technology enables the simultaneous execution of multiple entangled and parallel operations. Based on the principles of Quantum Mechanics, such as entanglement and superposition, quantum algorithms promise to solve computational challenges more efficiently, transforming sectors such as finance, logistics, and artificial intelligence.

Fuzzy Logic (FL) handles concepts beyond binary values. By dealing with incomplete and imprecise information, FL enables the manipulation of multi-valued fuzzy sets, reflecting the complexity of natural language. Flexible algorithms provide a gradual representation of knowledge, facilitating flexibility in decision-making processes, especially in expert systems where ambiguity and uncertainty are frequent. These characteristics make fuzzy logic particularly effective for modeling human emotional interactions, as it captures the inherent ambiguity and fluidity of emotions. Unlike binary models, which overly reduce emotions into fixed states, fuzzy logic represents them as a continuum, allowing gradual transitions and intertwined states such as apprehension, fear, or terror. This allows for a more refined approach to dynamic and context-dependent relationships. Consequently, systems based on fuzzy rules promote the interpretability of computational outcomes.

The quantum technology market is expanding, with a projected compound annual growth rate of 25% between 2024 and 2034 (IDTechEx Research, 2023). This growth is driven by advancements in three main areas: computing, sensors, and quantum communications. The development of hardware for quantum computing is expanding in research centers and data centers, while quantum sensors are finding

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applications in sectors such as precision navigation and medicine. In this context, the interrelationship between quantum technologies, such as lasers, scanners, and sensors, and quantum-fuzzy applications becomes evident.

Following recent scientific literature, we can explore the interaction between QC and FL. One example is the fuzzy connectives modeling via the extension of quantum operators (de Avila et al., 2019). Quantum implementations of fuzzy connectives using multi-qubit gates have also been investigated (Yogeesh et al., 2023), along with studies discussing the use of quantum movements in representing emotions in humanoid robots modeling (Deng et al., 2021). Additionally, the concept of entropy has been applied in the analysis of inference systems based on the quantum-neuro-fuzzy perspective, demonstrating the relevance of this approach for complex data analysis related to emotion modeling (Ferreira and Almeida, 2020).

This work applies a Quantum Fuzzy strategy to interpret family interactions, focusing on modeling emotions through membership degrees represented by the U quantum gate. To implement this approach, we developed quantum circuits that utilize the principles of superposition and entanglement in a quantum simulation environment. The simulations formally represent the membership values, reflecting the intensity of the behaviors between the parent-child agents.

The paper is organized as follows: Section II explores the fundamental concepts of FL and how it relates to quantum computing. In Section III, we discuss the modeling and simulation of emotions in family dynamics using fuzzy logic applied to quantum circuits. Section IV describes a case study that models emotions in a family context. Lastly, we present the final considerations and proposals for future research in Section V.

2 PRELIMINARIES

2.1 Basic Concepts of Fuzzy Logic

FL is a mathematical extension of traditional Boolean logic, providing a logical foundation for dealing with imprecise or uncertain data. Fuzzy set theory generalizes the classical one, by smooth transitions between associated classes (Zadeh, 1965). Furthermore, their multivalued generalizations, later formalized in (Zadeh, 1975), enabled new applications in many fields.

Let $U \neq \emptyset$ be the universal set. Classical set theory is based on the characteristic function $f_A \colon U \to \{0,1\}$, where $f_A(x) = 1$ if $x \in A$, and $f_A(x) = 0$ if $x \notin A$, with U being the universal set. This function associates each element $x \in U \neq \emptyset$ with a value in the discrete set $\{0,1\}$.

A fuzzy set A in U is characterized by the membership function $f_A : U \to [0,1]$ where, for each $x \in U$, $f_A(x)$ indicates the MD of each element x in the fuzzy set A.

A fuzzy set A in U can also be described as a set of ordered pairs, where each element $x \in U$ is associated with its respective MD $f_A(x) \in [0,1]$, that is, $A = \{(x, f_A(x)) \mid x \in U\}$. Extending this context, a multivalued fuzzy set can be defined by n-tuples in the multivalued logic approach.

Let A and B be fuzzy sets in $U \neq \emptyset$, represented by the membership functions $f_A, f_B \colon U \to [0,1]$, respectively. Taking $f_\cup, f_\cap \colon U \to [0,1]$, the union and intersection between A and B are, respectively, given as:

$$A \cup B = \{(x, f_\cup(x)) \mid x \in U\},\ f_\cup(x) = \max\{f_A(x), f_B(x)\};$$

$$A \cap B = \{(x, f_\cap(x)) \mid x \in U\},\ f_\cap(x) = \min\{f_A(x), f_B(x)\}.$$

The operators $\max, \min \colon [0,1]^2 \to [0,1]$ represent triangular norms and conorms and can be replaced by other functions of the corresponding classes, as seen in (Klement and Navara, 1999).

Moreover, according to (Bustince et al., 2003), let $f_{A'} \colon U \to [0,1]$. The fuzzy set A' expresses the fuzzy complement of A in U considering the standard negation $N_S \colon [0,1] \to [0,1]$ given by $N_S(x) = 1 - x$, and is defined by:

$$A' = \{(x, f_{A'}(x)) \mid x \in U\}, \quad \text{and} \quad f_{A'}(x) = 1 - f_A(x).$$

A function $\mathsf{E} \colon [0,1]^2 \to [0,1]$ is called exclusive OR (or XOR) if, for all $x, y \in [0,1]$, it satisfies the properties:

- E1: $\mathsf{E}(0,0) = \mathsf{E}(1,1) = 0$ and $\mathsf{E}(1,0) = \mathsf{E}(0,1) = 1$ (boundary conditions);
- E2: $\mathsf{E}(x,y) = \mathsf{E}(y,x)$ (symmetry);
- E3: $x \le y \Rightarrow \mathsf{E}(0,x) \le \mathsf{E}(0,y)$ (0-partial isotonicity);
- E4: $x \le y \Rightarrow \mathsf{E}(1,x) \ge \mathsf{E}(1,y)$ (1-partial antitonicity).

Example 1. *Let* $\mathsf{E}_P : [0,1]^2 \to [0,1]$ *be the Xor class,*

$$\mathsf{E}_P(x,y) = x + y - 2xy, \tag{1}$$

extending the binary classical operation expressed as $A \otimes B = (A \cup B) - (A \cap B)$.

An aggregation function $A \colon [0,1]^2 \to [0,1]$ verifies, for all $x, y, x', y' \in [0,1]$, the following properties:

- A1 $A(0,0) = 0$ and $A(1,1) = 1$;
- A2 $x \le x'$ and $y \le y' \Rightarrow A(x,y) < A(x',y')$ (strict isotonicity).

Additional properties can also be demanded:

A3 $A(x,y) = A(y,x)$ (symmetry);

A4 $A(x,x) = x$ (idempotence);

A5 $A(0,1) = \frac{1}{2}$;

A6 $A(\lambda x, \lambda y) = \lambda A(x,y)$, for all $\lambda \in [0,1]$ (homogeneity);

A7 $A(\lambda + x, \lambda + y) = \lambda + A(x,y)$ for all $\lambda \in [0,1]$ (linearity).

Example 2. *The arithmetic mean, given by:*

$$A(x,y) = \frac{1}{2}(x+y), \tag{2}$$

verifies seven properties, from A1 *to* A7.

2.2 Basic Concepts of Quantum Computing

In QC, the qubit is the basic unit of information, defined by a two-dimensional unit state vector $\psi = (\alpha, \beta)^t$, usually described in Dirac's notation (Nielsen and Chuang, 2000) by the expression: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where the coefficients α and β are complex numbers corresponding to the amplitudes of their respective states, satisfying the normalization condition $|\alpha|^2 + |\beta|^2 = 1$. So, it ensures the system's state vector, represented by $(\alpha, \beta)^t$, is unitary. The amplitudes configure a state of quantum superposition, giving rise to the phenomenon of quantum parallelism.

The state space of a multidimensional quantum system is obtained by the tensor product of the state spaces of its component systems. Considering a quantum system of two qubits, $|\psi\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$ and $|\varphi\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$, the related state space is composed by the tensor product:
$|\psi\rangle \otimes |\varphi\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$.

A state change in a quantum system is performed via a unitary quantum transformation (QT), represented by orthogonal square matrices of order 2^N, where N is the number of qubits in the transformation. Taking $\theta \in [0, \frac{\pi}{2}]$, $\lambda, \phi \in [0, 2\pi]$, an one-dimensional QT is represented by:

$$F = \begin{pmatrix} \cos\frac{\theta}{2} & -e^{i\lambda}\sin\frac{\theta}{2} \\ e^{i\phi}\sin\frac{\theta}{2} & e^{i(\phi+\lambda)}\cos\frac{\theta}{2} \end{pmatrix}. \tag{3}$$

In particular, when $\theta = \frac{\pi}{2}$, $\lambda = \pi$, and $\phi = 0$, then $F = H$, known as the Hadamard gate. The application of the unitary gate $H \otimes H$ on a classical state $|01\rangle$ generates a superposition state mathematically described by:

$$H \otimes H|01\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}|01\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}.$$

And when $\theta = \lambda = \phi = 0$, then $F = Id$ represents the Identity. Furthermore, when $\theta = \pi$, $\lambda = \pi$, and $\phi = 0$, then $F = X$ represents the Not gate, referred to as the Pauli X gate. Additionally, let $j = \sqrt{-1}$ be the imaginary unit. The QT associated with the quantum gate V qubit ($\sqrt{X}$) is given by the matrix expression:

$$V = \frac{1}{2}\begin{pmatrix} 1+j & -1+j \\ -1+j & 1+j \end{pmatrix}.$$

The evident exponential growth in the spatial and temporal complexity of quantum algorithms justifies the use of simulators to assist in the interpretation and perform computed algorithms.

The amplitudes of multidimensional quantum states are governed by the normalization condition, which is not always achieved through the tensor product of the corresponding states of the qubits (the basic states of the computational basis). In this case, we have an entangled state (Nielsen and Chuang, 2000).

For a characterization of two-dimensional entangled states, we consider the classical states $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$ as basic vectors of a two-dimensional quantum state. The entangled states are the linear combinations $|s'\rangle = \alpha_1|00\rangle + \beta_1|11\rangle$ and $|s''\rangle = \alpha_2|01\rangle + \beta_2|10\rangle$, with $\alpha_1, \beta_1, \alpha_2, \beta_2$ being normalized complex amplitudes and $\alpha_1^2 + \beta_1^2 = \alpha_2^2 + \beta_2^2 = 1$.

Example 3. *The composition of one entangled state with another generates a new entanglement. See, e.g., the three-dimensional quantum state $|s_\gamma\rangle$, given as:*

$$|s_\gamma\rangle = |s'\rangle \otimes \left(\frac{\sqrt{2}}{2}(|0\rangle + |1\rangle)\right).$$

So, the entangled qubits are intertwined in such a way that their individual properties cannot be described independently. When an entangled qubit is subjected to a measurement and its state is determined, the state of the other entangled qubit is instantly affected, regardless of the distance between them, known as the "spooky action at a distance".

The measurement operation on the current state of a quantum system is defined by a set of linear projections M_m, acting on the quantum states (Nielsen and Chuang, 2000). Let the state be given by $|\psi\rangle$. After the measurement, the output probability is given by:

$$p(|\psi\rangle) = \frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}}.$$

The measurement operations satisfy the completeness relation given as: $\sum_m M_m^\dagger M_m = I$. In one-dimensional systems, we have:

$$M_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = M_0^\dagger; \text{ and } M_1 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = M_1^\dagger.$$

For a qubit $|\psi\rangle$, with $\alpha, \beta \neq 0$, we observe probabilities to measure $|0\rangle$ and $|1\rangle$, resulting in:

- $p(|0\rangle) = \langle\phi|M_0^\dagger M_0|\phi\rangle = \langle\phi|M_0|\phi\rangle = |\alpha|^2$.
- $p(|1\rangle) = \langle\phi|M_1^\dagger M_1|\phi\rangle = \langle\phi|M_1|\phi\rangle = |\beta|^2$.

Therefore, after measuring the $|\psi\rangle$ state, we have $|\alpha|^2$ as the probability of being in the classical state $|0\rangle$; and $|\beta|^2$ as the probability of being in the other state, $|1\rangle$.

The Parent-Child algorithm is based on the QC Model, considering sequential composition. In this quantum context, the significance of superposition and entanglement in the representation and the parents' dynamics of emotions will be discussed. Additionally, we will address the implementation of its quantum circuits using the Qiskit framework, which is integrated with the Python programming language, simulating and executing quantum circuits.

Next, we will describe the case study.

3 EMOTION MODELING VIA QUANTUM FUZZY APPROACH

This section discusses the theoretical foundation from previous sections for simulating fuzzy systems that represent the modeling of emotions of effective agents using quantum computing. The Parent-Child case study exemplifies this methodology. The principles of quantum computing, such as superposition and entanglement, are considered to model and analyze such complex emotions, considering interpretations of uncertainty through the application of fuzzy logic.

Let U be a universe with cardinality $\|U\| = n$ defined in the set of the first natural numbers in N , $N = \{1, 2, \ldots, n\}$. For each element x_i, we can associate its MD $f_A(x_i)$ and non-membership degree $1 - f_A(x_i)$ to a one-dimensional quantum register obtained from the following superposition state:

$$|S_{f_A(x_i)}\rangle = |X_i\rangle = [\sqrt{f_A(\mathrm{x}_i)}|1\rangle + \sqrt{1 - f_A(\mathrm{x}_i)}|0\rangle]. \tag{4}$$

Thus, applying the tensor product, a n-dimensional quantum state (n-qubits) represents all the elements x_i. Let $U \neq \emptyset, |U| = n$ and let A be a fuzzy set defined by the membership function $f_A \colon U \to [0, 1]$. For each $\mathrm{x}_i \in U$, a n-dimensional fuzzy state is given by the expression:

$$|S_{f_A(x_i)}\rangle = \bigotimes_{1 \leq i \leq n} [\sqrt{f_A(\mathrm{x}_i)}|1\rangle + \sqrt{1 - f_A(\mathrm{x}_i)}|0\rangle].$$

Highlighting the differences in relation to non-classical correlations, we can explore the emotional correlations, applying quantum states and operators.

This research explores the emotion intensities, expanding on the ideas seen in (Raghuvanshi and Perkowski, 2010). The angle of the qubit is illustrated as meridians in the Bloch sphere (Nielsen and Chuang, 2000) and the intensity of the emotion is shown as a point between the north and south poles, $|0\rangle$ and $|1\rangle$. We consider fuzzy interpretations to illustrate different types and intensities of emotions, such as the joy variable, which varies from serenity (low) to ecstasy (high).

Example 4. *In Eq. (3), if $\lambda = \pi$, $\phi = 0$ and*

$$\theta = 2arc\ tan \left(\frac{\sqrt{f_A(\mathrm{x}_i)}}{\sqrt{1 - f_A(\mathrm{x}_i)}} \right),$$

we have the QT F is given as given by

$$F_A = \begin{pmatrix} \sqrt{1 - f_A(\mathrm{x}_i)} & \sqrt{f_A(\mathrm{x}_i)} \\ \sqrt{f_A(\mathrm{x}_i)} & \sqrt{1 - f_A(\mathrm{x}_i)} \end{pmatrix}, \tag{5}$$

verifying the following properties:

- $F_A^\dagger = F_A^{-1} = F_A$;
- $F_A|0\rangle = \sqrt{1 - f_A(\mathrm{x}_i)}|0\rangle + \sqrt{f_A(\mathrm{x}_i)}|1\rangle$;
- $F_A|1\rangle = \sqrt{f_A(\mathrm{x}_i)}|0\rangle - \sqrt{1 - f_A(\mathrm{x}_i)}|1\rangle$;
- $f_A(\mathrm{x}_i) = 1 \Rightarrow F_A = X$;
- $f_A(\mathrm{x}_i) = \frac{1}{2} \Rightarrow F_A = H$.

The Parent-Child (PC) algorithm is graphically represented by a quantum circuit considering the sequential composition of unitary transformations performing superpositions and entanglements to represent the dynamics of emotional modeling. Additionally, we analyze the implementation of its quantum circuits using the Qiskit platform, exploring its features in Python library for designing, simulating, and executing quantum circuits.

4 CASE STUDY: PC-INTERACTIVITY

The PC-problem models the mood change of a child based on the level of interactivity of their parents. If both caregivers are interactive, the child will also be happy. In other cases, the child will be in a "half happy" and "half unhappy" state, interpreted as a superposition state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$.

The Parent-Child algorithm in (Raghuvanshi and Perkowski, 2010) is based on the QC model, where the emotion intensity is modeled as projections on the Bloch Sphere (Nielsen and Chuang, 2000), a stereographic representation of qubits, given by a point between the north and south poles, representing $|0\rangle$ and

$|1\rangle$, respectively. Thus, the type of emotion is modeled by the phase angle of the related qubit, geometrically represented by meridians on the Bloch Sphere. The most positive emotional activity is at $|1\rangle$, and at $|0\rangle$, the least positive.

In this work, a general interpretation of the Parent-Child algorithm based on fuzzy aggregations extends this interpretation to multiple agents, modeling more complex interactions within the family structure and considering, e.g., a stepfather and a stepmother.

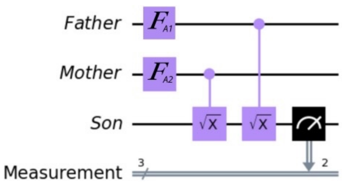

Figure 1: Fuzzy Modeling Circuit of the Parent-Child Interactivity.

Fig. 1 describes the $C1$ circuit and presents the fuzzy approach based on f_{A1} and f_{A2} membership functions, respectively modeled by F_{A_1} and F_{A_2} quantum gates, given as matrices obtained from Eq. (5) that consider controlled gates based on the "Square Root of the Not" gate $V = \sqrt{X}$.

The sequential composition via controlled operators includes the following description:

- CV_3^2, which executes the operator $\sqrt{X}$ (V) on the 3rd qubit (target) when the 2nd qubit is in state $|1\rangle$; and
- CV_3^1, which executes the operator $\sqrt{X}$ (V) on the 3rd qubit when the 2nd qubit is in state $|1\rangle$.

Now, the emotional modeling based on a fuzzy aggregation function is formalized in the next proposition.

Proposition 1. *The fuzzy arithmetic mean provides a behavioral interpretation for the* $C1$ *circuit in Fig. (1).*

Proof. Consider the fuzzy arithmetic mean operator, as given by Eq. (2). When $f_{A_1}(x_1) = x_1$ and $f_{A_1}(x_2) = x_2$, we have $F_{A_1}|0\rangle = \sqrt{1-x_1}|0\rangle + \sqrt{x_1}|1\rangle$ and $F_{A_2}|0\rangle = \sqrt{1-x_2}|0\rangle + \sqrt{x_2}|1\rangle$. Additionally, taking $x_1 + x_2 \neq 0$ and $S_1 = (F_{X_1} \otimes F_{X_1} \otimes Id)(|X_1\rangle \otimes |X_2\rangle \otimes |0\rangle)$ as given by Eq. (4), the resulting entangled state S_4, obtained from the temporal evolution, is summarized as follows:

$$\begin{aligned} S_4 &= (M_1^3 \otimes C_{V_3^1} \otimes C_{V_3^2})(S_1) \\ &= \frac{j-1}{\sqrt{2(x_1+x_2)}}\sqrt{(1-x_1)x_2}|011\rangle + \frac{j-1}{\sqrt{2(x_1+x_2)}}\sqrt{(1-x_2)x_1}|101\rangle + \\ &\quad - \frac{\sqrt{x_1x_2}}{\sqrt{2(x_1+x_2)}}|111\rangle \text{ and } p_1^{C_2} = \frac{x_1+x_2}{2}. \end{aligned} \tag{6}$$

Thus, based on the algebraic expressions extracted from the qFuzzyAnalyser Library (Buss et al., 2024) and, related to the above measurement, the results promote the quantum-fuzzy interpretation via the arithmetic mean, as given by Eq. (2). Thus, independently from the membership functions attributions F_{A_1} and F_{A_2}, it is interpreted by the quantum gate given by the $M_1^3(C_{V_3^1} \otimes C_{V_3^2})$ composition.

Therefore, the probability of the child's behavior changing from $|0\rangle$ to $|1\rangle$ is given by the arithmetic mean performed by the intensity of the behaviors between the parents-child agents. □

4.1 Classical PC-Interactivity

Table 1 presents an analysis based on classical inputs, fixing the 3rd qubit as $|0\rangle$. In addition, $\mathbf{S_0} = \mathbf{S_1}$ since $F_{A1} = F_{A2} = Id$. Note that the first two qubits, representing the parents' mood, do not change during the evolution from $\mathbf{S_0}$ to $\mathbf{S_4}$. Furthermore, measuring the 3rd qubit (related to the child's mood) the following holds:

- In the first row, MD $f_A(x_1) = f_A(x_2) = 0$ return the same initial classical states, with probability $p = 0$ for the 3rd qubit in $|1\rangle$. In these cases, the circuit interpretation guarantees that, like the parents, the child's mood remains in $|0\rangle$;
- In the last row, interpreting the parents' happy attitude as $f_A(x_1)=f_A(x_2)=1$ results in a change in the child's emotional behavior, from unhappy to happy;
- In the other rows, modeling only one of the parents as happy, the measure of the 3rd qubit always returns a state in superposition, with probability $p = 0.5$ to evaluate whether the child maintains the same mood or experiences a mood change.

Table 1: Temporal Evolution for the Classical Parent-Child Interactivity.

$\mathbf{S_1}$	$\mathbf{S_2}$	$\mathbf{S_3}$	$\mathbf{S_4}$
$\|000\rangle$	$\|000\rangle$	$\|000\rangle$	$p=0, S_f=\|001\rangle$
$\|010\rangle$	$\frac{j+1}{2}\|010\rangle+\frac{j-1}{2}\|011\rangle$	$\frac{j+1}{2}\|010\rangle+\frac{j-1}{2}\|011\rangle$	$p=\frac{1}{2}, S_f=\|011\rangle$
$\|100\rangle$	$\|100\rangle$	$\frac{j-1}{2}\|100\rangle+\frac{j+1}{2}\|101\rangle$	$p=\frac{1}{2}, S_f=\|101\rangle$
$\|110\rangle$	$\|111\rangle$	$\|110\rangle$	$p=1, S_f=\|111\rangle$

So, in the emotion modeling of parent-child interaction as described in Fig. (1), the parent's happiness influences the child to become (or remain) happy. However, this influence on mood change is not evident when both parents are unhappy. Whenever at least one of them is happy, there is a 50% chance of changing the child's mood, either to a happy mood (proactive attitude) or to an unhappy one (passive attitude).

Table 2: Temporal Evolution of the Fuzzy Model of the Parent-Child Interactivity.

$\mathbf{S_0}$	$\mathbf{S_1}$	$\mathbf{S_2}$	$\mathbf{S_3}$	$\mathbf{S_4}$
$\|000\rangle$	$\frac{1}{2}\|000\rangle+\frac{\sqrt{3}}{2}\|010\rangle)$	$\frac{1}{2}(\|000\rangle+\frac{\sqrt{3}}{4}(j+1)\|010\rangle+\frac{\sqrt{3}}{4}(j-1)\|011\rangle)$	$\frac{1}{2}(\|000\rangle+\frac{\sqrt{3}}{4}(j-1)\|010\rangle+\frac{\sqrt{3}}{4}(j+1)\|011\rangle)$	$p=\frac{3}{8}, S_f=\|011\rangle$
$\|010\rangle$	$\frac{\sqrt{3}}{2}\|000\rangle+\frac{1}{2}\|001\rangle$	$\frac{\sqrt{3}}{2}(\|000\rangle-\frac{j-1}{4}\|010\rangle+\frac{j-1}{4}\|011\rangle)$	$\frac{\sqrt{3}}{2}(\|000\rangle-\frac{j-1}{4}\|010\rangle+\frac{j-1}{4}\|011\rangle)$	$p=\frac{1}{8}, S_f=\|011\rangle$
$\|100\rangle$	$\frac{1}{2}\|100\rangle+\frac{\sqrt{3}}{2}\|110\rangle)$	$\frac{1}{2}\|100\rangle+\frac{\sqrt{3}}{4}(j+1)\|110\rangle+\frac{\sqrt{3}}{4}(j-1)\|111\rangle$	$\frac{j+1}{4}\|100\rangle+\frac{j-1}{4}\|101\rangle+\frac{2\sqrt{3}}{4}\|111\rangle$	$p=\frac{7}{8}, S_f=\frac{j-1}{4}\|101+\frac{2\sqrt{3}}{4}\|111\rangle$
$\|110\rangle$	$\frac{\sqrt{3}}{2}\|100\rangle+\frac{1}{2}\|110\rangle$	$\frac{\sqrt{3}}{2}\|100\rangle+\frac{j+1}{4}\|110\rangle+\frac{j-1}{4}\|111\rangle)$	$\frac{\sqrt{3}}{4}(j+1)\|100\rangle+\frac{\sqrt{3}}{4}(j-1)\|101\rangle+\frac{1}{2}\|111\rangle)$	$p=\frac{5}{8}, S_f=\frac{\sqrt{10}}{5}(\frac{\sqrt{3}(j-1)}{2}\|101\rangle+\frac{\sqrt{10}}{5}\|111\rangle)$

4.2 Fuzzy PC-Interactivity: Algebraic Discussion

In this case study, we consider quantum-fuzzy inputs for one of the parents interpreted by the 2$^{\text{nd}}$ qubit. The other agents, related to the qubits 1 and 3, remain in the classical state $|0\rangle$. Therefore, we consider $F_{A1} =$ Id and

$$F_{A2} = \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \quad (\theta = 2\frac{\pi}{3}\text{ rad} = 120^\circ).$$

Table 2 summarizes the results of the Parent-Child interactivity. In this case study, the resulting probability of a single measurement on the 3$^{\text{rd}}$ qubit in $|1\rangle$, according to the algebraic expression presented in Eq. (6), results in the following interpretations:

- In the 1st row, when the input variable received MD $f_A(x_1) = 0$ and $f_A(x_2) = 0.75$, the output variable received a membership degree $f_B(y) = \frac{3}{8} = 0.375$;
- In the 2nd row, when the input variable received MD $f_A(x_1) = 0$ and $f_A(x_2) = 0.25$, the output variable received the lowest membership degree: $f_B(y) = 0.125$;
- In the 3rd row, MD $f_A(x_1) = 1$, and $f_A(x_2) = 0.75$ returning the highest MD $f_B(y) = 0.875$;
- And finally, in the last row, the input variable related to MD $f_A(x_1) = 1$ and $f_A(x_2) = 0.25$, implies a MD $f_B(y) = 0.625$ for the output variable.

The probability distributions seen earlier can also be obtained through simulations on the Qiskit platform, as shown in the histograms in Fig. 2. Observe that the best option for changing the child's mood is related to the highest degree of the parents in the fuzzy set, interpreting happiness as the linguistic variable.

4.3 Fuzzy PC-Interactivity: Qiskit Simulations

The histograms in Fig. 2 illustrate the simulation via quantum-fuzzy inputs related to the execution of the quantum circuit. These histograms report results from Table 2, analyzing the temporal evolution of the family dynamics for the four initial states, and the 3$^{\text{rd}}$ qubit as $|0\rangle$.

The X-axis of the histogram depicts the possible states of the three qubits after executing the circuit and measurement. The Y-axis indicates the frequency of each state, based on a total of 1000 circuit executions. The frequencies of the $|110\rangle$ and $|010\rangle$ states are particularly important for our analysis, as they reflect distinct scenarios within the family dynamic in question.

The third qubit starts in the $|0\rangle$ state in the presented scenarios. The first row of Table 2 corresponds to the first histogram. After executing the circuit, the child represented by the third qubit becomes happy 387 times and remains in the initial mood 613 times. In the second histogram, the child becomes happy 123 times and remains sad 877 times. In the third histogram, the child changes the mood 873 times and stays in the initial state 127 times. Finally, the child's mood changes 625 times while keeping it 375 times. These simulations suggest that happy parents can influence the child to become or remain satisfied.

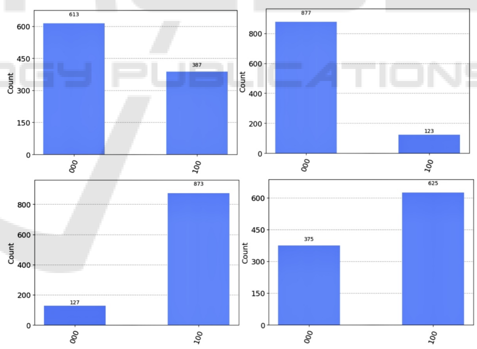

Figure 2: Histogram: Fuzzy Approach to the Parent-Child Interactivity.

5 FINAL CONSIDERATIONS

Based on algebraic expressions integrating fuzzy connectives, including the fuzzy exclusive "or" and fuzzy arithmetic means, we focused on the parent-child dilemma as a simple case to investigate the application of a generalized quantum gate in the quantum-fuzzy context. This allows more precise in the modeling three agents' emotions, in a 3-dimensional quantum system.

The simulation conducted on the Qiskit platform revealed distinct patterns in the probability distributions, providing perceptions of the emotional dynamics of family interactions. Additionally, the implementation of fuzzy operators, considering the model of Quantum Circuits, emphasizes the importance of superposition and entanglement in the emotional representation.

The quantum approach refines the modeling of interactions between multiple agents. The analysis of the histograms obtained during the simulation allowed valuable insights into the applicability of quantum computing to tackle complex real-world problems, especially in the modeling of emotions. Future works may explore the fundamentals of quantum-fuzzy theory in modeling emotions for human-like behavior in future intelligent robots. Moreover, the results can be extended for new research based on data fusion for artificial intelligent systems (Tiwari et al., 2024).

Expanding the dimensional model, the natural language related to emotional and social contexts can also essentially improve their practical applicability on real systems. Besides, applying quantum neural networks (QNN) and transferring the simulations performed in Qiskit to real quantum hardware (as the IBM quantum platform) is the next research step, validate our Quantum Fuzzy models in real-world scenarios. Thus, when more agents are involved, like restructured families (stepfather/stepmother and half brothers/systems) and the Parent-Children Interactive model will involve more than three agents, justify the use of quantum simulators. So, further work based on emerging technologies and combining the potentials of QC and FL, can model emotions collaborates in relevant scenarios as affective computing, social robotics, and neurorobotics research areas.

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REFERENCES

Buss, J., Novack, B., Botelho, C., Santos, H., Lucca, G., Cruz, A., Yamin, A., and Reiser, R. (2024). Fusion data on fuzzy modality: From algebraic interpretations to quantum simulations via qiskit platform. In Lesot, M.-J., Vieira, S., Reformat, M. Z., Carvalho, J. P., Slezak, D., Batista, F., Bouchon-Meunier, B., and Yager, R. R., editors, *Information Processing and Management of Uncertainty in Knowledge-Based Systems - 20th Int. Conference, IPMU 2024, Lisbon, Portugal, July 22-26, 2024, Proceedings*, volume 1176 of *Lecture Notes in Networks and Systems*, pages 1–6. Springer.

Bustince, H., Burillo, P., and Soria, F. (2003). Automorphisms, negations and implication operators. *Fuzzy Sets Syst.*, 134(2):209–229.

de Avila, A. B., Reiser, R., Pilla, M. L., and Yamin, A. C. (2019). Interpreting Xor intuitionistic fuzzy connectives from quantum fuzzy computing. In Guervós, J. J. M., Garibaldi, J. M., Linares-Barranco, A., Madani, K., and Warwick, K., editors, *Proc. of the 11th Intl Joint Conf. on Comp. Intelligence, 2019, Vienna*, pages 288–295. ScitePress.

Deng, R., Huang, Y., and Perkowski, M. A. (2021). Quantum motions and emotions for a humanoid robot actor. In *51st IEEE International Symposium on Multiple-Valued Logic, ISMVL 2021, Nur-Sultan, Kazakhstan, May 25-27, 2021*, pages 207–214. IEEE.

Ferreira, A. B. and Almeida, J. M. (2020). Entropic measures for quantum-neuro fuzzy inference systems.

IDTechEx Research (2023). Quantum technology market 2024-2034: Trends, players, forecasts. Available: https://www.idtechex.com/en/research-report/quantum-technology-market-2024-2034-trends-players-forecasts/1005. Accessed on Oct. 13, 2024.

Klement, E. P. and Navara, M. (1999). A survey on different triangular norm-based fuzzy logics. *Fuzzy Sets and Systems*, 101(2):241–251.

Nielsen, M. A. and Chuang, I. L. (2000). *Quantum Computation and Quantum Information*. Cambridge University Press.

Raghuvanshi, A. and Perkowski, M. (2010). Fuzzy quantum circuits to model emotional behaviors of humanoid robots. In *IEEE Congress on Evolutionary Computation*, pages 1–8.

Tiwari, P., Zhang, L., Qu, Z., and Muhammad, G. (2024). Quantum fuzzy neural network for multimodal sentiment and sarcasm detection. *Information Fusion*, 103:102085.

Yogeesh, N., Girija, D. K., Rashmi, M., and Divyashree, J. (2023). Quantum implementation of fuzzy logic conjunction and disjunction using multi-qubit gates. In *European Chemical Bulletin, vol. 12, no. 05, pp. 1795–1805*.

Zadeh, L. A. (1965). Fuzzy sets. *Inf. Control.*, 8(3):338–353.

Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning - I. *Inf. Sci.*, 8(3):199–249.