Applying Quantum Tensor Networks in Machine Learning: A Systematic Literature Review

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Abstract: Integrating quantum computing (QC) into machine learning (ML) holds the promise of revolutionizing computational efficiency and accuracy across diverse applications. Quantum Tensor Networks (QTNs), an advanced framework combining the principles of tensor networks with quantum computation, offer substantial advantages in representing and processing high-dimensional quantum states. This systematic literature review explores the role and impact of QTNs in ML, focusing on their potential to accelerate computations, enhance generalization capabilities, and manage complex datasets. By analyzing 23 studies from 2013 to 2024, we summarize key advancements, challenges, and practical applications of QTNs in quantum machine learning (QML). Results indicate that QTNs can significantly reduce computational resource demands by compressing high-dimensional data, enhance robustness against noise, and optimize quantum circuits, achieving up to a 10-million-fold speedup in specific scenarios. Additionally, QTNs demonstrate strong generalization capabilities, achieving high classification accuracy (up to 0.95) with fewer parameters and training data. These findings position QTNs as a transformative tool in QML, bridging critical limitations in current quantum hardware and enabling real-world applications.

1 INTRODUCTION

Quantum computing (QC) is a groundbreaking computational paradigm that leverages the principles of quantum mechanics to process information. Its fundamental unit, the qubit, can exist in a superposition of states, representing both 0 and 1 simultaneously. This property allows quantum computers to encode and manipulate significantly more information with fewer units compared to classical systems. One of QC's most promising applications lies in accelerating the processing of large datasets and complex algorithms, particularly in machine learning (ML). The integration of quantum computing and ML, known as quantum machine learning (QML), has the potential to revolutionize fields such as pattern recognition, optimization, and data analysis (Tychola et al., 2023). By utilizing quantum algorithms, QML aims to enhance the efficiency and accuracy of tasks like classification and regression, paving the way for transformative advancements in data-driven technologies (Tychola et al., 2023) (Manjunath et al., 2023).

Tensor Networks (TNs) are versatile mathematical frameworks widely used in physics and computer science to model and analyze complex systems, such as quantum many-body systems and neural networks (Azad, 2024). A tensor network is composed of interconnected tensors and multi-dimensional arrays that encode numerical data (Azad, 2024). Each tensor corresponds to a specific component or site within the system, representing its states and capturing all possible configurations or conditions of that component (Biamonte and Bergholm, 2017b).

This structured representation significantly simplifies computational challenges by reducing the complexity of operations such as evaluating the partition function, a key measure summarizing the energy configurations of a system in thermal equilibrium (Liu et al., 2021). TNs are particularly well-suited for systems with numerous degrees of freedom, where traditional methods often struggle with scalability and efficiency (Kardashin et al., 2021).

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Quantum Tensor Networks (QTNs) extend classical tensor networks, enabling the efficient representation and manipulation of high-dimensional quantum states (Rieser et al., 2023) (Hou et al., 2024). They offer a robust framework for compressing and simplifying large quantum systems, a capability that is essential for managing the complexity inherent in quantum computations (Biamonte and Bergholm, 2017a) (Azad, 2024) (Liu et al., 2021).

Additionally, quantum computers operate within the Noisy Intermediate-Scale Quantum (NISQ) era, which is defined by significant hardware constraints such as environmental noise, imperfect quantum gates, and limited qubit availability. QTNs provide a pathway to address these challenges by optimizing quantum state representations, minimizing computational resource demands, and enhancing quantum error correction techniques. These advances facilitate more efficient utilization of the constrained resources available in NISQ-era quantum hardware.

In the context of quantum machine learning (QML), QTNs present promising solutions to foundational challenges, including reducing representation complexity for quantum systems and efficiently handling large datasets. By leveraging QTNs, researchers aim to enhance scalability and computational efficiency, overcoming critical limitations in quantum computing performance while unlocking new opportunities for advancements in machine learning applications (Biamonte and Bergholm, 2017a) (Liu et al., 2021). Consequently, developing QTNs represents a pivotal step in integrating quantum computing (QC) with machine learning (ML), fostering progress at the intersection of these transformative fields.

Despite its potential, there is a lack of understanding of QTNs practical applications due to its recent emergence, which demands a proper systematic literature review (SLR) to explore its benefits and potential uses. This review might reveal effective methods, address challenges, and highlight progress in quantum computing, offering valuable insights for researchers aiming to fully harness QTNs potential (Guala et al., 2023).

As far as we know, there is no SLR on Quantum Tensor Networks and their applications in Quantum Machine Learning. This paper aims to fill this gap by investigating and summarizing the main applications and challenges of QTN. By doing so, we seek to provide a clearer understanding of the potential of QTN in QML and guide future research in this emerging field. This paper is organized as follows: **Section 2** details the definition of Tensor Networks, QTN, and QML; **Section 3** summarizes the SRL steps; **Section 4** describes the key observations obtained from the SRL and finally **Section 5** brings this research main conclusion and the necessary future steps.

2 BACKGROUND

2.1 Tensor Networks

Tensor Networks (TN) operate using tensors, an extension of traditional data structures like matrices and vectors (Bridgeman and Chubb, 2016) (Bañuls, 2023), representing systems through interconnected smaller tensors (nodes) connected by edges (Sengupta et al., 2022). This structure allows TN to effectively represent multidimensional real-world data, making them suitable for modeling complex phenomena across various disciplines (Cores et al., 2024). Additionally, TNs excel in handling structured and hierarchical data, providing comprehensive, contextual representations that reveal complex patterns and subtle interactions between variables (Cores et al., 2024), making them invaluable for solving computational challenges across different research fields.

There are various types of TNs, but three are especially important: **Matrix Product States (MPS)**, the most famous example of TN states due to its underlying powerful methods for simulating many-body quantum systems in one dimension, such as the Density Matrix Renormalization Group (DMRG) algorithm. (Orús, 2014); **Projected Entangled Pair States (PEPS)** it's defined by a network of three-dimensional tensors similar to the MPS network, but with the addition of a set of binding tensors that connect them (Orús, 2014). PEPS is used for two-dimensional systems, which extends the efficiency and power of MPS to higher-dimensional quantum systems and is, therefore, relevant for describing complex physical phenomena; and finally **The Multi-scale Entanglement Renormalization Ansatz (MERA)**, a tensor network framework designed to represent strongly correlated quantum many-body states efficiently. MERA uses a hierarchical organization of tensors to capture entanglement across multiple scales, making it especially useful for simulating quantum field theories. It has also been adapted for computational methods like variational Monte Carlo and applications in QML (Rieser et al., 2023) (Bhatia and Kumar, 2018).

Apart from these types, **Tree Tensor Network (TTN)**, is especially relevant for this SLR because it is a hierarchical model used to efficiently represent quantum states by decomposing them into a tree-like structure of tensors. Each tensor in the network represents a local part of the quantum state, and the connectivity between tensors reflects the correlations between those parts. This structure allows for the efficient representation and manipulation of high-dimensional quantum states, particularly in systems with strong local correlations (Rieser et al., 2023).

The structure of these TNs is detailed in Figure 1.

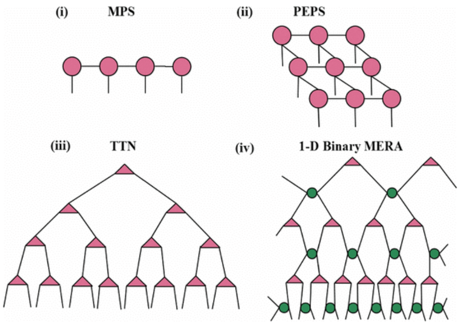

Figure 1: Types of TN (Bhatia and Kumar, 2018)).

2.2 Quantum Tensor Networks

Quantum Tensor Networks (QTNs) are an extension of classical tensor networks, specifically designed for quantum systems, enabling efficient representation and manipulation of complex quantum states (Biamonte and Bergholm, 2017a). They play a significant role in developing algorithms for small-scale quantum computers, especially those with limited qubit resources. Furthermore, while QTNs are not inherently resistant to noise, they provide a powerful framework that can be integrated with quantum error correction methods, which are crucial in the NISQ era, where quantum hardware is prone to noise and limited qubit coherence.

One key feature of QTNs is their ability to represent quantum circuits, where each qubit and quantum gate operation is associated with a specific tensor, with the tensor network capturing the interactions and entanglement between qubits. This allows the temporal evolution of a quantum system governed by quantum gates to be effectively expressed within a QTN framework. Figure 2 illustrates how quantum circuits and QTNs are closely related, providing a clear mapping between the two concepts.

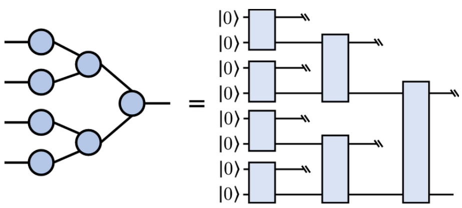

Figure 2: Quantum circuit based on QTN (Guala et al., 2023)).

Popular examples of QTN include **Quantum Matrix Product States (QMPS)** (Adhikary et al., 2021), which is a compact representation of often intractable wave functions of complex quantum systems, such as Quantum Tensor Train Networks. It is designed for efficiently describing two-dimensional systems; and **Quantum Multi-scale Entanglement Renormalization Ansatz (QMERA)**, a more general approach that represents quantum systems with multiple entanglement scales. These QTN approaches have become essential tools for simulating quantum systems, optimizing quantum algorithms, and exploring the structure of quantum states, underscoring their value in advancing the field of quantum computation.

2.3 Quantum Machine Learning

Quantum Machine Learning (QML) is a research field that integrates quantum computing principles with machine learning techniques to tackle computational challenges more efficiently. Quantum computing utilizes phenomena like superposition and entanglement to process and store information in fundamentally novel ways. By leveraging these quantum properties, QML aims to enhance tasks such as classification, regression, clustering, and optimization, particularly in scenarios involving large and complex datasets (Huang et al., 2021a).

3 METHODOLOGY

This research employs a systematic literature review (SLR) approach to investigate the state of the art in quantum computing (QC), QTN, and QML. Following the guidelines proposed by Xiao and Watson in (Xiao and Watson, 2019), this review addresses specific research questions through a documented and reproducible process, emphasizing methodological rigor in data collection and synthesis stages. The hypothesis *"The incorporation of Quantum Tensor Networks into Quantum Machine Learning algorithms has the potential to bring about significant advances in terms of computational speed, generalization capability, and efficient handling of complex data."* was developed to address the following research question:

- Can Quantum Tensor Networks (QTN) demonstrate tangible benefits in Quantum Machine Learning (QML) applications?

Table 1 describes the databases explored, the search string developed, and the number of papers obtained in each SRL stage (from the initial search to the selected papers). This search focused on databases highly impacting scientific research, quantum technologies, and gray literature. Initially, the search string focused only on the work's abstract and title. To refine the results, only articles written in English were considered, as English is a widely accepted universal language for research. Additionally, we limited the search scope to studies published since 2013 to ensure we included the most current technologies, approaches, and solutions in the field. The search was concluded on August 17th, 2024, so articles published after this date were not considered, aligning with the deadlines for writing and submitting this material. This initial search returned 7967 documents.

Table 1: Details on the databases and search string used in the SRL.

DATABASES	- Google Scholar - IEEE Xplore - DOAJ - SpringerLink - IOPscience - MDPI - ScienceDirect - Annual Reviews - Pennylane - Quantinuum
SEARCH STRING	ABS(("Abstract":"Quantum Computing" OR "Abstract":"Quantum Tensor Networks") AND ("Abstract":"Quantum Machine Learning" AND ("Abstract":"computational speed" OR "Abstract":"generalization capability" OR "Abstract":"handling of complex data")))
Initial Search	7967
FILTER I	346
FILTER II	121
FILTER III	72
Selected Documents	23

Inclusion and exclusion criteria for the document search were established to ensure the relevance and quality of the selected studies. Both criteria were applied with a detailed explanation of their meaning during the selection process, clearly specifying the acceptance and rejection factors. This strategy aims to enhance transparency and reduce subjectivity in each filter. For this search, three stages were defined to gradually filter the most relevant work for the SLR, each requiring different reading depths. Details are described as follows:

FILTER I: TITTLE - Aim to filter Academic Studies on Quantum Computing in Machine Learning or Tensor Networks by their titles.

- **Includes:** Studies specifically discussing Quantum Tensor Networks (QTNs) and their application in Quantum Machine Learning tasks like classification and regression.
- **Excludes:**
  Studies unrelated to quantum computing, quantum tensor networks or quantum machine learning, studies not focused on the specified tasks, or those focusing solely on classical applications.

After applying this filter, the number of works was reduced from 7,967 to 346.

FILTER II: ABSTRACT - Aims to filter Academic Studies on Quantum Computing in Machine Learning or Tensor Networks or Tensor Networks by their abstract.

- **Includes:** Research papers, reviews, theses, and articles discussing integrating quantum computing techniques and tensor networks into Quantum Machine learning algorithms.
- **Excludes:** Studies that do not address quantum computing, machine learning, tensor networks, or their combination.

By narrowing the selection to studies that discuss integrating quantum computing techniques into machine learning algorithms, the number of documents was further reduced from 346 to 121.

Filter III: TEXT Aims to filter Academic Studies on Quantum Computing in Machine Learning or Tensor Networks by reading the complete text.

- **Includes:** Full-text articles, papers, and reports accessible for review.
- **Excludes:** Texts that are inaccessible or require special permissions for access.

Finally, the selection was limited to full-text articles and reports accessible for review, resulting in 72 documents.

The number of returned documents was observed to ensure the validity of the systematic review. Ultimately, 23 articles were selected for the analysis after thoroughly reviewing the 72 filtered articles, eliminating those that did not directly answer the research question. We focused on papers that implemented and tested QTN in different scenarios, clearly showing through experiments and comparative analysis the many benefits of QTN in QML.

While there is no strict rule, more than 30% of the filtered articles are often considered adequate for comprehensive analysis in such reviews, indicating a robust selection process. This quantity not only enhances the reliability of the review but also supports the overall quality of the findings.

Data extraction is the final step following the search and filtering stages. It involves extracting the necessary information about computational speed, generalization capability, and complex data handling through detailed reading to validate or refute the hypotheses. The extracted information is organized into topics related to the inclusion criteria as shown in Table 2. The purpose of this list is to ensure that all relevant aspects are covered and to facilitate analysis.

To address the research question guiding this SLR, we seek specific insights from the analyzed papers. The exploration of QML application areas focuses on identifying domains such as supervised learning or classification, enabling a comprehensive understanding of QML's practical scope. Algorithm performance is assessed using metrics like accuracy and reliability to evaluate the effectiveness of QML models and pinpoint areas for improvement.

A critical aspect of the analysis is computational speed, which examines the time efficiency of QML algorithms compared to classical counterparts, highlighting a core advantage of QTNs. The generalization capacity of QML models, reflecting their ability to reliably perform on unseen data, is another key factor, as it determines scalability and suitability for real-world deployment.

Efficiency in handling complex data is also evaluated, emphasizing QML's capability to process large, high-dimensional datasets—a domain where QTNs could offer substantial benefits. The resource demands for model training are examined to understand the feasibility of implementing QML compared to classical methods. Lastly, post-implementation benefits assess practical experimentations, demonstrating how QML solutions can deliver meaningful outcomes such as improved decision-making or cost reductions, ultimately showcasing the practical value of incorporating QTN.

4 RESULTS AND DISCUSSIONS

In this section, we discuss the main findings of the SLR. As mentioned, after applying inclusion and exclusion criteria and reading the papers completely, 23 studies were selected due to their relevance to answering the research question. In the following sections, we highlight the observed advantages of QTN in efficiently dealing with high-level data and enhancing QML performance, as well as the current applications of these networks.

Table 2: Key information analyzed during data extraction.

Areas explored in QML	Focuses on the specific domains (e.g., supervised learning, classification) where QML is applied, providing insights into its broader applications.
Algorithm performance	Evaluates the effectiveness of QML algorithms through metrics like accuracy and reliability, revealing their strengths and limitations.
Computational Speed	Assesses the time efficiency of QML algorithms, especially compared to classical methods, to determine computational advantages.
Generalization Capacity	Examines how well QML models adapt to new data, which is essential for practical application and reliability.
Efficiency in Handling Complex Data	Looks at QML's ability to process large or high-dimensional datasets, a significant advantage over classical approaches.
Post-implementation benefits	Analyzes the advantages of QML in practical experimentations, such as improved decision-making and cost savings, supporting its practical value

4.1 QTN Advantages

QTN is particularly beneficial in QML because it can **compress high-dimensional data**, significantly reducing the computational and memory resources needed to represent and manipulate large datasets. Studies by (Araz and Spannowsky, 2022) (Orús, 2019) (Rieser et al., 2023) (Huggins et al., 2019) highlight how QTNs offer efficient data compression techniques that significantly reduce memory and computational requirements and excel at representing and manipulating high-dimensional datasets while maintaining computational feasibility. For example, (Huggins et al., 2019) indicates that using QTNs for processing data like an 8×8 pixel image for handwriting recognition tasks can allow the number of physical qubits needed to scale logarithmically with the size of the processed data, whereas previously it scaled linearly.

In addition to enhancing the representational efficiency of high-dimensional data, QTNs also help overcome other issues, such as **barren plateaus**, by employing local loss functions, which optimize the training landscape (Rieser et al., 2023). For example, in a scenario where a classical model faced a barren plateau, the loss function showed minimal gradient updates, leading to stagnation in training with a plateau observed at a loss value of 0.3. However, applying QTN with local loss functions did not encounter the same plateau effect, resulting in a more favorable training landscape and achieving a loss value improving down to 0.15 (Rieser et al., 2023).

Moreover, QTNs offer significant advantages in **simulating large-scale quantum circuits**. Unlike traditional state vector methods, which scale exponentially with the number of qubits, QTNs leverage tensor contractions that are optimized through rearranged contraction sequences, thereby reducing memory requirements and computational costs (Lykov, 2024). For example, applying a diagonal representation of quantum gates decreased tensor network contraction's complexity by one to four orders of magnitude. By optimizing the conversion of the QAOA circuit into a tensor network and using techniques like the greedy algorithm from the QTensor package, calculating a single amplitude of the QAOA ansatz state had a significant improvement in terms of **computational speed**, achieving a speedup of up to 10 million times. This optimization allowed for the simulation of larger QAOA circuits, increasing the number of qubits from 180 to 244 on a supercomputer (Lykov, 2024).

Another significant advantage of QTNs in QML is their ability to effectively model **effectively model complex quantum correlations**, a feature that enables improved generalization in QML models (Rieser et al., 2023). QTNs achieve this balance between complexity and tractability through adjustable bond dimensions, which define the maximum number of states that can be represented at the interfaces between tensors. These bond dimensions determine the network's capacity to represent quantum states while influencing the structure and variety of the tensor network (Bernardi et al., 2022). By tuning these parameters, QTNs mitigate risks associated with complex datasets, such as overfitting (Huggins et al., 2019).

As demonstrated in studies by Rieser, Köster, and Raulf (Rieser et al., 2023), QTNs not only **enhance generalization** to unseen data but also provide robustness against noise — a critical challenge in quantum computing. For instance, research in (Rieser et al., 2023) and (Huggins et al., 2019) highlights how QTNs enable higher accuracy with less training data: with the MNIST dataset and the model used for digit classification, the accuracy was 0.88 using the full dataset of 60,000 images. However, applying the model with QTN, the accuracy was reduced to approximately 0.806, still an high value, but with the dataset size reduced to 20%, all while demonstrating resilience to noise. Furthermore, the hierarchical structure of QTNs facilitates the representation of weight parameters in machine learning algorithms, making them a powerful and efficient tool for processing and analyzing complex datasets (Huggins et al., 2019). Moreover, (Rieser et al., 2023) indicated that QTN classifiers can achieve high classification accuracies ranging from 0.85 to 0.95 while utilizing a limited number of parameters and internal qubits and suggests that the structured nature of QTNs may facilitate easier training through local optimization routines QTNs can efficiently capture relevant patterns in data with a reduced amount of training data required.

4.2 QTN Applications

QTNs are primarily applied in QML for quantum classification and feature extraction tasks. The work in (Araz and Spannowsky, 2022) evaluated the performance of Classical Tensor Networks (TN) and their quantum counterparts (QTN) in classifying simulated Large Hadron Collider (LHC) data, focusing on top jet discrimination against the QCD background. The results demonstrated that QML models combined with QTNs outperformed classical TN-based approaches. For instance, classical TNs required significantly more parameters— up to 64,800 in some configurations — to achieve competitive performance. At the same time, QTNs attained superior AUC values (e.g., 0.914 for Q-MERA) with as few as 17 parameters. Moreover, QTNs achieved this efficiency using only 10,000 training events, compared to 50,000 events required by classical TNs (Araz and Spannowsky, 2022). These findings underscore the advantages of QML models leveraging QTNs in terms of parameter efficiency and performance, particularly in high-energy physics applications (Araz and Spannowsky, 2022).

Variational Quantum Tensor Networks (VQTN) are designed to operate effectively on near-term quantum processors (Huang et al., 2021b). The VQTN classifier excels in tasks like classification by combining quantum and classical processing. While traditional QML algorithms like quantum principal component analysis or quantum linear regression leverage quantum properties solely for speed, the VQTN's hybrid model incorporates a classical neural network for processing output from quantum tensor networks.

Additionally, the algorithm employs kernel encoding, circuit models, multiple readouts, and stochastic gradient descent to reduce quantum circuit complexity and improve performance.

Experimental results demonstrate that VQTN reduces the number of qubits needed while achieving higher accuracy rates than the traditional QTN algorithm. Tested on Iris and MNIST datasets, using TTN models in its architecture, VQTN uses half the number of qubits compared to QTN while maintaining an average accuracy of 93.72%. Compared with the QTN algorithm, the accuracy is improved by 7.71% (Huang et al., 2021b). Thus, VQTN serves as a bridge that effectively enhances computational speed and capacity for complex data analysis, which may not be achievable with isolated QML algorithms.

Another notable application of QTNs lies in Natural Language Processing (NLP). Researchers at Quantinuum achieved a significant breakthrough by successfully running scalable quantum natural language processing (QNLP) models on quantum hardware (Quantinuum, 2024). Their tensor-network-based approach incorporates syntax awareness into the models, offering enhanced interpretability while reducing the number of parameters and gate operations required. Experimental results indicate that QNLP models running on current quantum devices achieve prediction accuracy comparable to neural-network-based classifiers, demonstrating the practical viability of these models (Quantinuum, 2024).

The experiments further utilized advanced models, including the unitary structured tensor network (uSTN) and the relaxed, structured tensor network (rSTN), both of which delivered strong performance across various NLP tasks (Quantinuum, 2024). Notably, the implementation of qubit reuse strategies enabled the compression of a 64-qubit uCTN circuit into an 11-qubit circuit, significantly optimizing resource usage. This efficient utilization of quantum hardware underscores the potential of tensor network models to enable sequence classification tasks on near-term quantum devices, paving the way for more resource-efficient and scalable quantum NLP applications.

A novel approach has been developed for embedding continuous variables into quantum circuits using piecewise polynomial features. This method, called Piecewise Polynomial Tensor Network Quantum Feature Encoding (PPTNQFE), aims to enhance the capabilities of QML models by integrating continuous variables into quantum circuits (Ali and Kabel, 2024). This method leverages low-rank TN to create spatially localized representations, making it particularly suitable for numerical applications like partial differential equations (PDEs) and function regression.

The efficacy of PPTNQFE is demonstrated through two applications in one-dimensional scenarios (Ali and Kabel, 2024). Firstly, it enables efficient point evaluations of discretized solutions to PDEs; specifically, compared to traditional rotation-based encoding methods, which can require exponential resources for representing similar data, PPTNQFE significantly reduces complexity by leveraging tensor networks, thus improving the efficiency of quantum algorithms in PDE solving. A second application is in function regression tasks, particularly for modeling localized features like jump discontinuities. Conventional approaches often struggle with periodicity and noise sensitivity, leading to inaccurate approximations. In jump function tests, PPTNQFE provided more accurate approximations using fewer parameters, simplifying training and deployment processes (Ali and Kabel, 2024).

5 CONCLUSIONS

This paper presented an SRL on the use of QTNs in QML. The primary objective was to address the research question: Can Quantum Tensor Networks demonstrate tangible benefits in quantum machine learning applications? Through a comprehensive analysis of 23 selected studies, we validated the hypothesis that QTNs significantly enhance machine learning by leveraging the unique capabilities of quantum computing.

The findings reveal that QTNs offer several key advantages in QML. Firstly, they excel in compressing high-dimensional data, which significantly reduces computational and memory requirements. This capability is particularly beneficial for handling large datasets, enabling more efficient processing and storage. Secondly, QTNs enhance computational speed, achieving up to a 10-million-fold speedup in specific scenarios, such as simulating large-scale quantum circuits. Thirdly, QTNs improve the generalization capabilities of QML models, allowing them to perform reliably on unseen data with fewer parameters and less training data.

Moreover, QTNs have shown promise in modeling complex quantum correlations, which enhances their ability to capture patterns in data. This feature, combined with their hierarchical structure, makes QTNs a powerful tool for tasks such as classification, feature extraction, and NLP. Applications in high-energy physics, variational QTNs, and QNLP have demonstrated the practical viability of QTNs, showcasing their potential to outperform classical methods in terms of parameter efficiency and accuracy.

Despite these advancements, several challenges remain. The implementation of QTNs on near-term quantum devices requires further optimization to address hardware constraints, such as limited qubit coherence and noise. Additionally, more systematic benchmarking against classical and QML approaches is needed to fully understand the trade-offs and advantages of QTNs.

In conclusion, the incorporation of QTNs into QML has the potential to drive significant advancements in computational efficiency, generalization, and data handling. Continued development and exploration of QTNs will not only advance the field of quantum machine learning but also contribute to the broader landscape of computational science, paving the way for transformative quantum technologies.

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