TRACKING OF EXTENDED CROSSING OBJECTS USING THE VITERBI ALGORITHM

Andreas Kräußling, Frank E. Schneider and Dennis Wildermuth
Research Establishment for Applied Sciences (FGAN)
Wachtberg, Germany

Keywords: Multiple target tracking, expanded objects, Viterbi algorithm, crossing targets.

Abstract: Tracking, which means determining the positions of the humans in the surrounding, is one of the goals in the field of mobile robots that operate in populated environments. This paper is concerned with the special problem of tracking expanded objects under such constraints. A solution in form of a Viterbi based algorithm, which can be useful for real–time systems, is presented. Thus a Maximum–a–posteriori (MAP) filtering technique is applied to perform the tracking process. The mathematical background of the algorithm is proposed. The method uses the robots' sensors in form of laser range finders and a motion and observation model of the objects being tracked. The special problem of the crossing of two expanded objects is considered. The mathematical background for this problem is enlightened and a solution for it in form of a heuristic algorithm is proposed. This algorithm is tested on simulated and real data.

1 INTRODUCTION

When tracking people and other expanded objects in densely populated surroundings with robot–borne devices like laser scanners or sonar the situation is usually as follows: there are a lot of readings from obstacles like walls or other targets and only some readings from the object itself. The problem of allocation of data obtained from the just now accounted target is referred to as the problem of data association in the literature. Therefore, the tracking algorithm uses a gate which separates the signals belonging to the current target from other signals as a solution to this problem. A second feature of all algorithms for tracking vast objects should be the description of shape and extension of the objects. Finally, an estimate for the centre of the object has to be computed. For this purpose two antithetic methods can be used. First, all points that have passed the gate successfully can be used for the calculation of the estimate in terms of a weighted mean. Second, only the point that fits best to the previous data can be used for the further calculations. In this paper we picture the Viterbi algorithm as an example for the latter of these two methods. The Viterbi algorithm has been introduced in (Viterbi, 1967), a good description of it is given in (Forney, 1973). It has already been recommended for tracking punctiform targets in clutter (Quach and Farrooq, 1994) and has also been introduced for tracking expanded objects and evaluated on real and simulated data in (Kräußling et al., 2004).

In this paper we focus on the special problem of tracking two expanded objects which are crossing. This means that the gates of the two objects are intersecting. The problem of two crossing targets is a common challenge in the field of multiple target tracking (MTT). In this case the Viterbi algorithm, as it has been proposed in (Kräußling et al., 2004), often fails to track the two objects when their paths get separated again. Our research has shown that this is an important drawback of all tracking algorithms introduced so far for expanded objects, for example Koch's EM algorithm (Koch and Stannus, 2003). This often occurs when one target is occluded at least partially by the other. Hence in this paper a heuristic algorithm is introduced, that is capable of handling the special problem of the crossing of two expanded objects, based on a special feature of our implementation of the Viterbi algorithm. The crossing of two objects is also a well known problem in people tracking which frequently occurs since persons move together, talk to each other, and then separate again.

This paper is organized as follows. In section 2 related work in the field of tracking using mobile robots

Kräußling A., E. Schneider F. and Wildermuth D. (2004).
TRACKING OF EXTENDED CROSSING OBJECTS USING THE VITERBI ALGORITHM.
In *Proceedings of the First International Conference on Informatics in Control, Automation and Robotics*, pages 142-149
DOI: 10.5220/0001129601420149

is presented. Section 3 describes the model used and gives details of the Viterbi algorithm, as we use it for tracking expanded objects. In section 4 a mathematical dicussion of the problem of tracking two crossing expanded targets is given. Furthermore, our heuristic solution is proposed. Section 5 describes the experiments and simulations. Finally, in section 6 the summarized results and an outlook on future work are presented.

2 RELATED WORK

Tracking people, e.g. for surveillance, is a well studied problem in machine vision (Sclaroff and Rosales, 1998). Common attempts to track people are inspired by the need in humanoid robotics to react to human beings. A new approach of tracking objects, that has rapidly gained popularity in mobile robotics during the last years, uses laser range finders. Earlier work uses occupancy grids and linear extrapolation of occupancy maps to estimate trajectories (Elfes et al., 1999). Newer methods involve the use of advanced trajectory estimation algorithms (Fod et al., 2001). Laser sensing differs significantly from vision in ways that can be exploited for tracking. In vision, variables like colour, intensity, and depth are available. In contrast, lasers are restricted to one plane of the observable space. Most of the useful information for tracking is in just one parameter – the range to the nearest obstacle over an arc. The range measurements are, however, of high accuracy, especially in comparison with other range sensors such as ultrasound or infra red. Thus, lasers have rapidly gained popularity for mobile robotic applications such as collision avoidance, navigation, localization, and map building (Thrun, 1999), (Thrun et al., 1999). The problem of estimating the position of moving objects is an important problem in mobile robotics. This ability allows a robot to adapt its velocity to the speed of people in the environment and to improve its collision avoidance behaviour in situations in which the trajectory of the robot crosses the path of a human (Sclaroff and Rosales, 1998). In this context the work of Schulz (Schulz et al., 2001) is very notable. He combined the ideas of Joint Probabilistic Data Association Filtering (JPDAF) (Bar-Shalom and Tse, 1975), (Fortmann et al., 1983) with Particle Filtering (Gordon et al., 1993), (Pitt and Shephard, 1999) and called his method Sample–based Joint Probabilistic Data Association Filtering (SJPDAF). Thereby, he is able to assign the measurements to the tracked objects and to reproduce multimodal densities, a major improvement for example when handling obstacles.

The problem of two crossing targets has been studied for the first time in (Fortmann et al., 1983). In this work the problem of tracking two punctiform crossing targets in clutter, as it occurs for example in the field of aerial surveillance, has been investigated. It has been shown there that with the classical PDAF algorithm introduced in (Bar-Shalom and Tse, 1975) or with the nearest neighbour algorithm the objects are not tracked correctly after the crossing. In the case of the PDAF algorithm both objects are tracked in the middle of the connection line of the true positions after the crossing. In the case of the nearest neighbour algorithm both objects are tracked close to the true positions of the same object after the crossing. Because of these difficulties in (Fortmann et al., 1983) a new algorithm in form of the Joint Probabilistic Data Association Filter has been proposed which performs well under the constraints of the crossing of two punctiform targets.

3 MATHEMATICAL BACKGROUND OF THE VITERBI ALGORITHM

3.1 The Model

The dynamics of the object to be observed and the observation process itself are modeled as customary by

$$x_k = Ax_{k-1} + w_{k-1} \tag{1}$$

and

$$z_k = Bx_k + v_k. \tag{2}$$

Thereby x_k is the object state vector at time k, A is the state transition matrix, z_k is the observation vector at time k and B is the observation matrix. Furthermore, w_k and v_k are supposed to be zero mean white Gaussian noises with $E(w_i(w_j)^\top) = Q\delta_{ij}$, $E(v_i(v_j)^\top) = R\delta_{ij}$ and $E(w_i(v_j)^\top) = 0$, which means the measurement noise and the process noise are uncorrelated. In these equations the object is supposed to be punctiform. Nevertheless, the model will also be helpful for the description of an expanded target. In this case x_k will get the state vector of the centre of the target at time k. Since the motion of a target has to be described a two dimensional kinematic model is used. Therefore it is

$$x_k = \begin{pmatrix} x_1 & x_2 & \dot{x}_1 & \dot{x}_2 \end{pmatrix}^\top, \tag{3}$$

$$A = \begin{pmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & e^{-\Delta T/\Theta} & 0 \\ 0 & 0 & 0 & e^{-\Delta T/\Theta} \end{pmatrix}, \tag{4}$$

$$Q = \rho^2 e^{-2\Delta T/\Theta} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{5}$$

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad (6)$$

and

$$R = \delta \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (7)$$

Thereby x_1 and x_2 are the Cartesian coordinates of the target and ρ and Θ are parameters that are modeling the acceleration of the target. A small Θ and a large ρ correspond to a large acceleration. ΔT is the temporal interval between two consecutive measurements. In the calculations $\Theta = 3$, $\rho = 40$ and $\delta = 1000$ are used.

3.2 The Gate

The gate is realised with the help of the Kalman filter (Kalman, 1960) (Maybeck, 1979) (Shumway and Stoffer, 2000), which makes a prediction $y(k+1|k)$ for the measurement of the object based on the last estimate $x(k|k)$ for its position via the formulae

$$x(k+1|k) = Ax(k|k) \quad (8)$$

and

$$y(k+1|k) = Bx(k+1|k). \quad (9)$$

Thereby $x(k|l)$ is the estimate for x at time k based on the observations up to time l. Then the Mahalanobis distance

$$\lambda = (z_i(k+1) - y(k+1|k))^\top \cdot [S(k+1)]^{-1} \cdot \cdot (z_i(k+1) - y(k+1|k)) \quad (10)$$

with the innovations covariance $S(k+1)$ from the Kalman filter is computed for every sensor reading $z_i(k+1)$ at time $k+1$. The Mahalanobis distance has been introduced in (Mahalanobis, 1936). In contrast to the Euclidian distance it incorporates the different uncertainnesses in the varying directions. To illustrate this one can think of $S(k+1)$ as a diagonal matrix. A greater uncertainty in a direction then results in a greater associated diagonal entry in $S(k+1)$ and thus in a smaller associated diagonal entry in $[S(k+1)]^{-1}$. Consequently deviations from $y(k+1|k)$ in this direction will be multiplicated with a smaller factor and thus will be weighted less in the calculation of the Mahalanobis distance. More precisely, the points with the same Mahalanobis distance are lying on the surface of an ellipsoid with the semi–major axis in the direction of the greater uncertainty (see also the next paragraph). In our experiments $\lambda = 4$ has been used.

Deviant from the Kalman filter the Matrix $S(k+1)$ is computed as

$$S(k+1) = BP(k+1|k)B^\top + R + E(k) \quad (11)$$

with the matrix $E(k)$, that describes the shape and the extension of the object in order to account for these attributes. A detailed description of the matrix $E(k)$ will be given in the next paragraph. The recursion formula for $P(k+1|k)$ is

$$P(k+1|k) = AP(k|k)A^\top + Q. \quad (12)$$

λ is χ^2 distributed with two degrees of freedom, so a χ^2 test is used. All sensor readings with a λ lower than a given threshold are passing the gate.

3.3 The Extension of the Object

To describe the extension of the tracked object a positive definite matrix can be used, because for a positive definite matrix P and a given $\epsilon > 0$ all points y with $y^\top Py = \epsilon$ are lying on the surface of an ellipsoid with the origin as the centre (Torrieri, 1984). So shape, orientation, and extension of the objects are approximated by an ellipsoid. The appropriate positive definite matrix is computed as follows. At first it is assumed that all points that have passed the gate successfully are originated from the target, so that it is reasonable to use all these points for the computation of the matrix $E(k)$. Hence the covariance of these points which is positive definite is computed and used as the matrix $E(k)$.

3.4 The Viterbi Algorithm

The Viterbi algorithm (Viterbi, 1967) is a recursive algorithm. Thus as an initialisation $x(0|0)$, $P(0|0)$ and $E(0)$, that describe the object at the beginning, are used. Now a description of a single step of the recursion is given. The Viterbi algorithm uses a directed graph to determine the optimal sequence of measurements for a given set of measurements Z^T. Thereby, it is $Z^T = \{Z_k\}_{k=1}^T$, and $Z_k = \{z_{k,j}\}$ is the set of selected measurements $z_{k,j}$ at time k. The selected measurements correspond to the nodes in the graph. Given the selected measurements Z_k at time k, the set of selected measurements at time $k+1$ is computed as follows.

For every measurement $z_{k,j}$ the gate is applied to the measurements at time $k+1$. That results in the sets $Z_{k+1,j}$ of measurements which have passed the particular gate for the measurement $z_{k,j}$ successfully. The set $Z_{k+1} = \{z_{k+1,i}\}_i$ of selected measurements $z_{k+1,i}$ at time $k+1$ is then just the union of these sets, i.e. it is

$$Z_{k+1} = \cup_j Z_{k+1,j}. \quad (13)$$

In the next step for every selected measurement $z_{k+1,i}$ its predecessor is identified. For this purpose only those selected measurements $z_{k,j}$ whose gates have been passed by $z_{k+1,i}$ successfully are considered (this technique is due to (Pulford and Scala, 1995)). For each of these measurements $z_{k,j}$ the length $d_{k+1,j,i}$ of the path from $x(0|0)$ to $z_{k+1,i}$

through $z_{k,j}$ is calculated.

$$d_{k+1,j,i} = d_{k,j} + a_{k+1,j,i} \tag{14}$$

is used. Thereby, $d_{k,j}$ is the length of the path that ends in the node $z_{k,j}$. $a_{k+1,j,i}$ is the distance between the nodes $z_{k,j}$ and $z_{k+1,i}$. For $a_{k+1,j,i}$

$$a_{k+1,j,i} = \frac{1}{2}\nu_{k+1,j,i}^{\top}[S_{k+1,j}]^{-1}\nu_{k+1,j,i} + \\ + \ln\left(\sqrt{|2\pi S_{k+1,j}|}\right) \tag{15}$$

is used. $\nu_{k+1,j,i}$ is the innovation defined as

$$\nu_{k+1,j,i} = z_{k+1,i} - y_j(k). \tag{16}$$

In doing so $S_{k+1,j}$ and $y_j(k)$ are the innovations covariance respectively the prediction evaluated by the Kalman filter based on the nodes or the measurements $\{z_{l,i(l,j)}\}_{l=1}^{k}$ belonging to the path ending in $z_{k,j}$. The sequence

$$Z_{k,j} = \{z_{l,i(l,j)}\}_{l=1}^{k} \tag{17}$$

of these nodes is called the tracking history belonging to the node $z_{k,j}$ and the tracking histories define an association τ which assignes to every j and $l \leq k$ a measurement $z_{l,i(l,j)}$. Thereby the same measurement can be associated to more than one tracking history and not all measurements have to be associated. This proceeding causes

$$p(z_{k+1,i}|Z_{k,j}) = \exp\left(-a_{k+1,j,i}\right) \tag{18}$$

and therefore with Bayes' rule as well

$$p(Z_{k,j}|Z_0) = \exp\left(-d_{k,j}\right) \tag{19}$$

with $Z_0 = x(0|0), P(0|0), E(0)$. Finding the tracking histories with the minimal distance corresponds therefrom strictly to determining the tracking history with the maximal likelihood. Furthermore it is with Bayes' law for a association τ of the measurements

$$p(\tau|Z^T) = \frac{p(Z^T|\tau)p(\tau)}{p(Z^T)}. \tag{20}$$

Thereby, $p(Z^T)$ is a normalization constant. $p(\tau)$ is the prior probability of the association τ. In the absence of prior information one usually assumes the priors to be equal for all associations. In this case searching for the association τ with the most likely posterior probability $p(\tau|Z^T)$ coincides with maximizing the likelihood function $p(Z^T|\tau)$ of the measurements over all associations.

The predecessor of $z_{k+1,i}$ is now just the node $z_{k,j}$, by which $d_{k+1,j,i}$ is kept at a minimum. For this node it is written $z_{k,i(k,j)}$. With the appendant predictions $x(k+1|k)_i$, $P(k+1|k)_i$ and $y(k+1|k)_i$ then the Kalman filter is used to get the estimates $x(k+1|k+1)_i$ and $P(k+1|k+1)_i$ using the well known equations from Kalman filtering

$$x(k+1|k+1)_i = x(k+1|k)_i + W_{k+1,i} \cdot \\ \cdot (z_{k+1,i} - y(k+1|k)_i) \tag{21}$$

and

$$P(k+1|k+1)_i = [I - W_{k+1,i}B]P(k+1|k)_i \tag{22}$$

with the Kalman gain

$$W_{k+1,i} = P(k+1|k)_i B^{\top} S_i^{-1}(k+1). \tag{23}$$

In doing so the subscript i at the matrices $P(k+1|k)$, $S(k+1)$, $P(k+1|k+1)$ and W_{k+1} traces back to the different matrices $E(k)$ used in equation 11. According to this these matrices depend on the measurements Z^T aberrant from the customary Kalman filter.

Thus the description of one step of the recursion is finished.

4 CROSSING EXTENDED OBJECTS

4.1 Mathematical background

First a mathematical description of the problem of tracking two expanded objects is given. An expanded object is defined as an object that can be the origin of more than one measurement. A crossing of two objects is defined by the fact that the validation gates of the two objects are intersecting. Each measurement of the intersection can then be associated with both targets. The full association θ of the n measurements in this intersection can be represented by a binary $n \times 2$ indicator matrix $\Omega(\theta)$, i.e. a matrix with entries 1 or 0. If $\Omega(\theta)_{ij} = 1$, measurement i is associated with target j by the association θ, otherwise not. Since there is no clutter in our application each measurement in the intersection has to be associated with exactly one target. Therefore, the sum of the entries of each row of Ω has to be one and the sum of all entries has to be equal to n. Since we are dealing with expanded objects, the sum of the entries of a column of Ω can vary from 0 to n. This is different from the case of punctiform targets, where this sum can only be 1 or 0. In the case of an expanded target a sum of n means that all measurements in the intersection are caused by the corresponding target, and a sum of 0 means that all measurements come from the other target. θ_k refers to the association θ at time k. The association history $(\theta_1, \ldots, \theta_k)$ is referred to as θ^k.

As discussed in the previous section, the Viterbi algorithm developes the probability $p(Z_{k,j})$ to the maximum. For two crossing targets the MAP estimate is calculated as follows. For each association θ_k the

Viterbi algorithm is run for each measurement $z_{k,j}$ to find the tracking history that minimizes the corresponding distance. Thereafter the sum over all distances is calculated. The MAP estimate then corresponds to that association, for which this sum reaches its minimum. This approach implies that the likelihoods $p(\theta_k)$ for all associations are equal. Thus, this procedure can obviously be improved by considering different likelihoods $p(\theta_k)$ for these associations. Since the associations θ_k are mutually exclusive and exhaustive, the law of total probability leads to

$$p(Z_{k,j}) = \sum_{\theta_k} p(Z_{k,j}, \theta_k) \tag{24}$$

and with Bayes' rule it is

$$\sum_{\theta_k} p(Z_{k,j}, \theta_k) = \sum_{\theta_k} p(Z_{k,j}|\theta_k)p(\theta_k). \tag{25}$$

In order to find the improved MAP solution we have to calculate $p(\theta_k)$ for every θ_k, and to maximise $p(Z^k|\theta_k)$ using the Viterbi algorithm as proposed in the previous section.

In practice this approach is infeasible for several reasons. First, if the number of measurements in the intersection of the validation gates is n then there are 2^n possible associations θ_k. A typical example for n in our application is 10. So, since 2^n is 1024, there might be some hundreds of associations to deal with. Therefore, the calculation of the exact Bayesian solution in form of the Gaussian sum $\sum_{\theta_k} p(Z^k|\theta_k)p(\theta_k)$ often cannot be obtained efficiently. Second, the calculation of the likelihood $p(\theta_k)$ is difficult, because it depends on the shape, the extension and the orientation of the two crossing objects. Furthermore, there is no guarantee that the two objects will be separated after the crossing by this Bayesian approach. Consequently, we tried to develop a heuristic solution for this problem.

As in the case of a single target in clutter there exists an optimal Bayesian approach. With the stacked state vector $x(k) = \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix}$ it is

$$E[x(k)|Z^k] = \sum_{\theta^k} E[x(k), \theta^k|Z^k] =$$

$$= \sum_{\theta^k} E[x(k)|\theta^k, Z^k]p(\theta^k|Z^k). \tag{26}$$

The expected value $E[x(k)|\theta^k, Z^k]$ could be approximated by known techniques (see the next paragraph for details), but the probabilities $p(\theta^k|Z^k)$ again cannot be efficiently calculated. Moreover, because a crossing lasts for about ten time steps there would be about $1000^{10} = 10^{30}$ association histories to consider. A method which runs a Viterbi algorithm for each association history θ^k and looks for the association history with the minimal sum of distances produces unattainable computational costs.

An algorithm similar to the PDAF algorithm for punctiform targets in clutter can be constructed as follows: it is

$$E[x(k)|Z^k] = \sum_{\theta_k} E[x(k), \theta_k|Z^k] =$$

$$= \sum_{\theta_k} E[x(k)|\theta_k, Z^k]p(\theta_k|Z^k). \tag{27}$$

Thereby, $E[x(k)|\theta_k, Z^k]$ can again be approximated by known techniques based on the estimate $E[x(k-1)|Z^{k-1}]$. More precisely, the Kalman filter can be run for each of the two targets with a weighted average of the measurements $z_{k,j}$. The weights can be calculated using the normal distribution as in (Koch and Stannus, 2003) via the formula

$$\alpha_l^t = \frac{N(\nu_{k,l}^t; 0, R)}{\sum_{l,\Omega(\theta_k)_{l,t}=1} N(\nu_{k,l}^t; 0, R)} \tag{28}$$

or similar as in (Bar-Shalom and Tse, 1975) via the formula

$$\alpha_l^t = \frac{N(\nu_{k,l}^t; 0, S^t(k))}{\sum_{l,\Omega(\theta_k)_{l,t}=1} N(\nu_{k,l}^t; 0, S^t(k))}. \tag{29}$$

These algorithms are equivalent to running a Kalman filter for each target $t \in \{1, 2\}$ with the combined innovation

$$\nu_k^t = \sum_{\theta_k} \sum_{l=1,\Omega(\theta_k)_{l,t}=1}^{m} p(\theta_k|Z^k)\alpha_l^t \Omega(\theta_k)_{l,t}\nu_{k,l}^t. \tag{30}$$

Finally, an algorithm similar to the JPDAF algorithm for the crossing of two punctiform targets in clutter can be constructed as follows: for each target t a Kalman filter is run with the innovations

$$\nu_k^t = \sum_{j=1}^{m} \beta_j^t \nu_{k,j}^t. \tag{31}$$

Thereby, m is the number of measurements $z_{k,j}$ in the intersection, β_j^t is the posterior probability that measurement $z_{k,j}$ originated from target t and $\nu_{k,j}^t$ is the innovation $z_{k,j} - y(k|k-1)^t$. The predicted measurement $y(k|k-1)^t$ is calculated by the equations of Kalman filtering based on the last estimated state vector $x(k-1|k-1)^t$ of target t and, furthermore, it is

$$\beta_j^t = \sum_{\theta_k} p(\theta_k|Z^k)\Omega(\theta_k)_{jt}. \tag{32}$$

$p(\theta_k|Z^k)$ can be aproximated via the formula

$$p(\theta_k|Z^k) = \frac{1}{c}\prod_{t=1}^{2}$$

$$\prod_{j|\Omega(\theta_k)_{j,t}=1} \exp[(z_{k,j} - y(k|k-1)^t)^\top \cdot$$

$$\cdot\{S^t(k)\}^{-1}(z_{k,j} - y(k|k-1)^t)] \frac{1}{|2\pi S^t(k)|^{1/2}}. \quad (33)$$

Thereby c is a normalisation constant. The last equation can be derived similar to the corresponding equation of the JPDA filter. This algorithm is then equivalent to running a kalman filter with the combined innovation

$$\nu_k^t = \sum_{\theta_k} \sum_{j=1}^{m} p(\theta_k|Z_k)\Omega(\theta_k)_{lt}\nu_{k,j}^t. \quad (34)$$

4.2 A heuristic approach

Our approach to deal with two crossing expanded targets relies on a specific feature of the Viterbi algorithm. The Viterbi algorithm is able to cope with bimodal probability densities to some degree. It has this feature in common with Schulz' SJPDAF algorithm proposed in (Schulz et al., 2001). But while Schulz' algorithm uses particle filtering and thus has to deal with several hundreds of points, the Viterbi algorithm in our application only handles a few (typically between 3 and 12) points. Additionally, these points contain some information about the surface of the tracked objects as proposed in (Kräußling et al., 2004). If a crossing between two targets occurs the Viterbi algorithm shows the following behaviour. As soon as the crossing occurs the algorithm tracks all points originated from both objects simultaneously. When the crossing is over, these points are again separated into two distinct clouds. The two clouds of points are still tracked simultaneously. Our heuristical approach now is based one some observations from our experiments and consists of three different steps:

1. At every time step k a binary matrix in form of an upper triangular matrix is calculated. The elements a_{ij} of this matrix are set to 1, if the gates of the objects i and j intersect, otherwise they are set to 0. Once a crossing has been detected the element a_{ij} remains 1 until the end of the crossing is recognized in step 2.
2. For each pair of targets i and j with $a_{ij} = 1$ it is examined whether the crossing has finished. This is done by testing if the points have dispersed into two distinct clouds with a minimum Euclidian distance. For people tracking this is the case when there are at least two points with a minimal Euclidian distance of 300 cm. This is due to the fact, that the maximal distance of the points on the two different legs of a person in human walking can be assumed to not exceed 150 cm. In our experiments the value of 300 cm worked well also with regard to the problem that the gates of the two targets should not intersect again once the end of the crossing has been detected.
3. As soon as the end of the crossing has been observed the two corresponding clouds have to be separated and assigned to the two objects. In order to do so an arbitrarily chosen point of the combined cloud is assigned to the object with the lower index. Then, for every other point it is determined wether the Euclidian to the first point is larger or lower than 150 cm. In the first case it is assigned to the object with the higher index and otherwise to that with the lower one. Of course, since the first point is arbitrarily chosen the two objects might be interchanged after the crossing by this procedure. But in our opinion there is no general solution to this problem which is based only on laser distance information. An improvement over this approach can easily be achieved in the case of additional knowledge about the objects. For example, different reflecting power of the laser beams could be due to the fact that two persons are wearing differently coloured trousers. Or, in the case of one human and one circular robot, the different shapes could be used in order to correctly identify the separated objects.

Finally, at the end of the tracking process for each object the path with the minimum length of the path is determined and the Kalman smoother (Shumway and Stoffer, 2000) is applied for the estimated state vector via the formula

$$x(t-1|n) = x(t-1|t-1) +$$
$$+ J_{t-1}(x(t|n) - x(t|t-1)). \quad (35)$$

Thereby it is

$$J_{t-1} = P(t-1|t-1)A^\top [P(t|t-1)]^{-1}. \quad (36)$$

The backward recursion for the estimates $x(t-1|n)$ is initialised with the estimate $x(n|n)$ of the Kalman filter for the final time step n.

5 EXPERIMENTS AND SIMULATIONS

The new approach has been tested on simulated data and on data provided by a real experiment. In both cases the new algorithm was able to separate the two targets after the crossing successfully whereas the original algorithm as proposed in (Kräußling et al., 2004) failed to achieve this goal (see figure 1). In simulation the motion of two robots of radius 26 cm is explored. The lower robot is moving from left to right and the upper robot is moving in the opposite direction. In the middle of the two paths the crossing occurs. During the crossing the estimate for the extension of the targets is increasing. The isolated estimate in the path of the upper target when using the improved algorithm (see figure 2) is due to the fact that the Kalman smoother is used. Otherwise there would be a gap between the region where the crossing takes place and the region where the upper target is tracked again after the crossing. In this example the two targets are assigned correctly after the crossing.

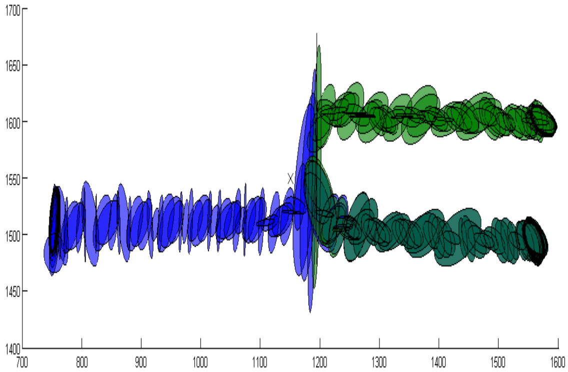

Figure 1: Simulated Data, tracked with the original algorithm.

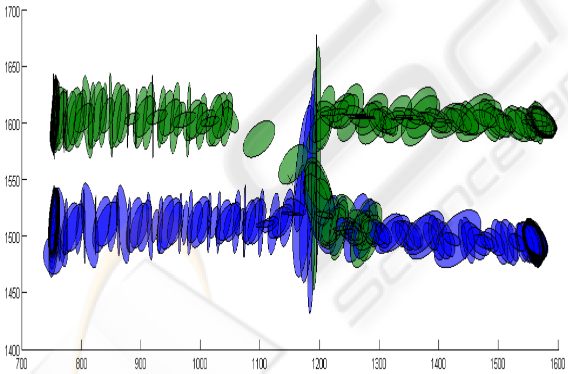

Figure 2: Simulated Data, tracked with the improved algorithm.

In the second example, which uses real data of two walking persons, the assignment after the crossing is wrong (see figure 3). But again, the two objects are separated well after the crossing has finished.

Finally, to illustrate the special features of the improved algorithm, we investigate the processing of four consecutive crossings with this algorithm (see figure 4). As the first two diagrams in figure 4 show, the number of measurements associated with each target grows rapidly when a crossing occurs. This is due to the fact, that the measurements of both of the targets are associated with each target. When the improved algorithm separates the targets again after the crossing, the number of measurements associated with each of the targets consequently declines rapidly. The third diagram in figure 4 shows, during a crossing all of the measurements associated with one of the targets are associated with each of the two targets, so that the difference of the number of measurements associated with each of the targets equals zero.

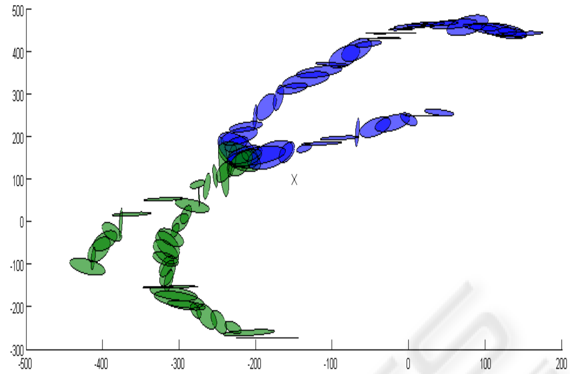

Figure 3: Real data, tracked with the improved algorithm.

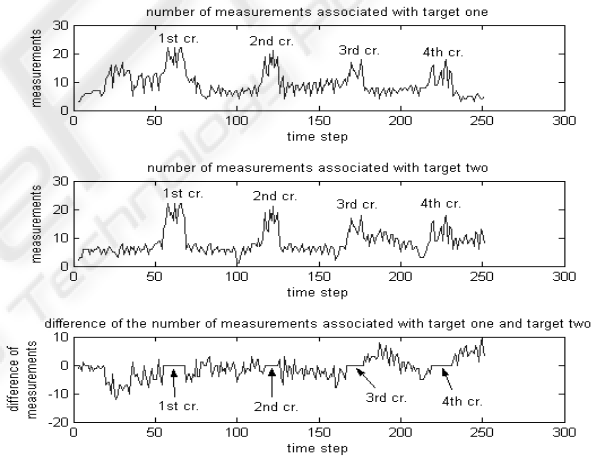

Figure 4: Real data, tracked with the improved algorithm.

6 CONCLUSION AND FUTURE WORK

In this paper a new method for tracking expanded objects as an application of the Viterbi algorithm has been introduced. The special problem of tracking two crossing targets has been investigated qualitatively. The mathematical background of this topic has been analysed briefly. A solution to the problem in form of a heuristic approach has been suggested and evaluated on simulated and real data. It has been demonstrated by means of these examples that the new algorithm performs well under general conditions.

A more detailed and advanced analysis of the mathematical description of crossing targets together with further experiments might be the content of a consecutive journal paper.

ACKNOWLEDGEMENTS

We want to thank Dr Martin Ulmke from the department SDF at the FGAN for drawing our attention to the fact, that particle filters can represent bimodal probability distributions. From this we recognized that our implementation of the Viterbi algorithm has a comparable property and that this characteristic can be used to develop a robust algorithm for the problem of expanded crossing targets. Furthermore we want to thank Dr Koch, the department head of SDF, for a lot of helpful suggestions about the philosophy of a tracking algorithm.

REFERENCES

Bar-Shalom, Y. and Tse, E. (1975). Tracking in a cluttered environment with probabilistic data association. *Automatica*, Volume 11, pp. 451–460.

Elfes, A., Prassler, E., and Scholz, J. (1999). Tracking people in a railway station during rush hour. In *Proc. of the International Conference on Computer Vision (ICVS)*, pages 13–15, Las Palmas, Spain. Springer Verlag.

Fod, A., Howard, A., and Mataric, M. J. (2001). Laser–based people tracking. Technical report, Computer Science Department, University of Southern California, Los Angeles, USA.

Forney, Jr., G. D. (1973). The viterbi algorithm. In *Proceedings of the IEEE*, volume Volume 61, pages 268–278.

Fortmann, T. E., Bar-Shalom, Y., and Scheffe, M. (1983). Sonar tracking of multiple targets using joint probabilistic data association. *IEEE Journal Of Oceanic Engineering*, Volume OE–8(Number 3).

Gordon, N., Salmond, D., and Smith, A. (1993). A novel approach to nonlinear/non–gaussian bayesian state estimation. *IEEE Proceedings F*, Volume 140(Number 2):107–113.

Kalman, R. E. (1960). A new approach to linear filtering and prediction problems. *Transactions of the ASME, Journal Basic Engineering*, Volume 82:34–45.

Koch, W. and Stannus, W. (2003). A new application of the em algorithm: Robot–borne extended object tracking. Technical Report Number 66, Research Institute for Communication, Information Processing and Ergonomics of the Research Establishment of Applied Sciences (FGAN), Wachtberg, Germany.

Kräußling, A., Schneider, F. E., and Wildermuth, D. (2004). Tracking expanded objects using the viterbi algorithm. to be published in the Proceedings of the IEEE Conference on Intelligent Systems, Varna, Bulgaria.

Mahalanobis, P. C. (1936). On the generalized distance in statistics. In *Proc. Natl. Inst. Science*, volume Volume 12, pages 49–55, Calcutta, India.

Maybeck, P. S. (1979). *Stochastic Models, Estimation, and Control*. Academic Press, New York, San Francisco, London.

Pitt, M. and Shephard, N. (1999). Filtering via simulation: auxiliary particle filters. *Journal of the American Statistical Association*, Volume 994(Number 446).

Pulford, G. W. and Scala, B. L. (1995). Over–the–horizon radar tracking tracking algorithm using the viterbi algorithm – third report to high frequency radar division. Technical report, Cooperative Research Centre for Sensor Signal and Information Processing, University of Melbourne, Australia.

Quach, T. and Farrooq, M. (1994). Maximum likelihood track formation with the viterbi algorithm. In *Proceedings of the 33rd Conference on Decision and Control*, Lake Buena Vista, FL, USA.

Schulz, D., Burgard, W., Fox, D., and Cremers, A. B. (2001). Tracking multiple moving objects with a mobile robot. In *Proceedings of the 2001 IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR 2001)*, Kauai, Hawaii, USA.

Sclaroff, S. and Rosales, R. (1998). Improved tracking of multiple humans with trajectory prediction and occlusion modeling. In *IEEE Conference on Computer Vision and Pattern Recognition, Workshop on the interpretation of Visual Motion*, Santa Barbara, CA, USA.

Shumway, R. H. and Stoffer, D. S. (2000). *Time Series Analysis and Its Applications*. Springer.

Thrun, S. (1999). Learning metric–topological maps for indoor mobile robot navigation. *Artificial Intelligence*, (Number 1):21–71.

Thrun, S., Fox, D., and Burgard, W. (1999). Markov localization for mobile robots in dynamic environments. *Artificial Intelligence*, (Number 11):391–427.

Torrieri, D. J. (1984). Statistical theory of passive location systems. *IEEE Transactions on Aerospace and Electronic Systems*, Volume AES–20(Number 2).

Viterbi, A. J. (1967). Error bounds for convolutional codes and an asymptotically optimum decoding algorithm. *IEEE Transactions On Information Theory*, Volume IT–13(Number 2).