IDENTIFICATION OF A CAR-LIKE VEHICLE *via* MODULATING FUNCTIONS

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Abstract: This paper describes an interesting application of the modulating functions technique to model identification of a car-like vehicle that has to face various types of surface. Several models have been obtained in different operating conditions. The construction of a 'mean model' will make possible the design of a robust control for unmanned guidance purposes. An alternative control strategy based on adaptive methods is also suggested by means of an online implementation of the technique.

1 INTRODUCTION

In model identification literature, the modulating functions technique plays a very important role. Several published works have shown excellent results as for the characterization of linear models, see (Shinbrot, 1957) or (Balestrino et al., 2003). Recently, an appealing application has involved the characterization of various classes of nonlinear models, see for example (Pearson, 1992). In (Balestrino et al., 2000) a very useful adaptation of the method is provided for the identification of systems with an unknown delay. In general, the most frequently investigated fields are model identification of electrical engines (Daniel-Berhet and Unbehauen, 1996), switching converters (Balestrino et al., 2003) and robot components (Daniel-Berhet and Unbehauen, 1997).

The aim of this work is the application of such technique to model identification of a car-like vehicle, that has to face several types of surface (dry and wet asphalt, dry and wet grass) and different operating conditions (presence or absence of a person on board). Various linear models are then derived in the above-mentioned conditions and a satisfying fitting of real data is consequently obtained. We are also going to provide a comparison between the estimation errors of the original model supplied by the manufacturer and the model deduced from the identification process.

The further step will be the design of a control law for the unmanned guidance of the vehicle. The computation of a "mean model" will allow us to set the problem of the controller's design in terms of robust control problem.

Last, the possibility of an online identification is presented. In this way an adaptive control strategy can be implemented.

This paper is organized as follows: in Section 2 the vehicle and the manufacturer's model; in section 3 the modulating functions method is reviewed and the identified models in different conditions are computed; in section 4 the mean model is derived and the control problem is formulated; in Section 5 online identification is presented and, in Section 6 some conclusions are reported.

2 THE CAR-LIKE VEHICLE

The vehicle[1] has been originally conceived as a caddy vehicle for golf courses (see Figure 1). It is a three wheels vehicle equipped with a DC motor supplied with two 12 volt batteries; the two back wheels are for the traction, the wheel in the front is for the direction. Our final goal is the realization of an unmanned guidance; for this purpose the caddy has been equiped with a linear actuator[2] for the steering, and

[1]By courtesy of Bestkart S.N.C., Pionca di Vigonza (Padova), Italy.

[2]By courtesy of Ognibene Elettromeccanica Srl, Bologna, Italy.

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with a couple of sensors[3] for the speed and position of the wheels. We have also designed control and data acquisition electronics interfacing the motor, the linear actuator and the sensors. A microcontroller Microchip PIC 16F874 plays the main role. The chip has 40 PIN: 4 are for the supply, 1 is for the asynchronous reset, 3 are for synchronous serial communication, 2 are for the asynchronous serial communication, the remnant is for input and output data.

Figure 1: The caddy vehicle.

To realize the caddy control and the setting of the interface card we will have to figure out an estimation of the behavior of the different components that compose the vehicle.

The caddy can be split into two independent subsystems:

- Driver + traction motor
- Linear actuator driver + steering

Concerning the subsystem driver + traction motor, the manufacturer endows the caddy with an input-output characteristic derived in no load condition (i.e. without a passenger). To verify the coherence with real data we have designed a program test implemented on the interface card. In this test we have furnished various torque steps as inputs to the driver of the motor and we have measured the velocity with a 100 Hz sampling frequency. We have distributed in a quite uniform way the voltage interval 0-5 Volt into the digital range 0-255, with a little more dense fitting in proximity of the bends. As a result we have obtained a fairly good approximation of the ideal characteristic (see Figure 2), but this however fails if we consider, for example, a passenger.

As regards the transfer function, the manufacturer provides the following first order model:

$$G(s) = \frac{1}{1.8s + 22.7} \tag{1}$$

By comparing the output of this model with the real outputs measured in different operating conditions (with or without passenger) with respect to the

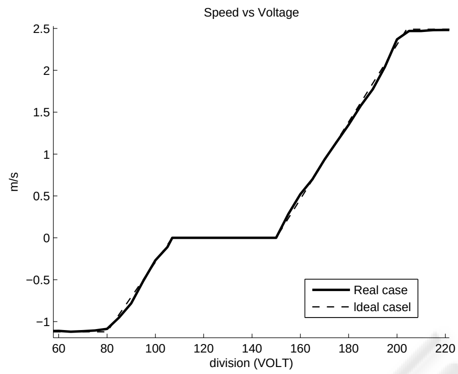

Figure 2: Ideal and real input\output characteristic in no load condition.

same inputs (random sequence of steps), it turns out from the analysis of data that the description of the dynamic behavior of the system is not satisfying. The gap increases in presence of a passenger and in the case of motion on grass (see Figures 3 and 4).

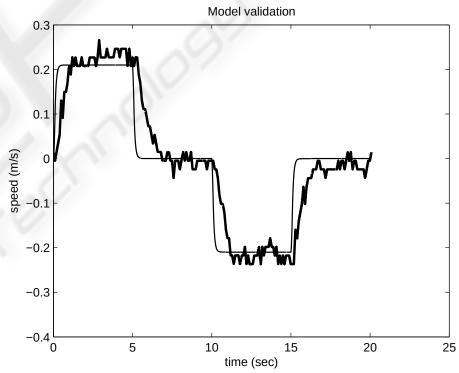

Figure 3: Asphalt test with passenger (thin: first order model response, thick: real response).

In order to achieve a better description of the dynamics of the caddy, an identification procedure is needed.

3 THE MODULATING FUNCTIONS IDENTIFICATION

By means of input-output data analysis we are looking for a computation of a linear model for the caddy. The structure of this model must be as simple as possible (i.e. with a minimum order).

The measured output signal presents a non negligible noise component and there are also unwelcome variations due to the unevennesses of the ground. For

[3]By courtesy of Tekel Instruments Srl, Torino, Italy

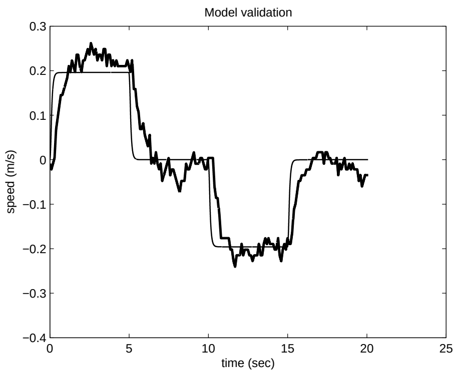

Figure 4: Grass test without passenger (thin: first order model response, thick: real response)

this reason, an identification procedure capable to extract reliable models even in these conditions is necessary.

3.1 The modulating functions

Consider continuous n-th order linear processes modeled by the following differential equation:

$$\sum_{i=0}^{n} b_i \frac{d^i y(t)}{dt^i} = \sum_{i=0}^{m} a_i \frac{d^i u(t)}{dt^i} \qquad (2)$$

where u(t) is the input signal and y(t) the output one. The coefficients a_i and b_i are the unknows of the system.

Now define a set of smooth functions $\Phi(t)$ having the following properties:

$$\Phi(t) = \begin{cases} \Phi(t) & t \in [0,T] \\ 0 & t \notin [0,T] \end{cases} \qquad (3)$$

$$\exists \, \frac{d^i \Phi}{dt^i}, \, \forall i = 1, \ldots, p, \, p \geq n \qquad (4)$$

$$\Phi^{(i)}(0) = \Phi^{(i)}(T) = 0, i \leq n. \qquad (5)$$

The time interval $I = [0,T]$ is commonly named integration window. We thus denote modulating functions $\Phi(t)$ function along with with its derivatives.

Multiplying both members of (2) by $\Phi(t)$ and integrating over I we have:

$$\sum_{i=0}^{n} b_i \int_0^T \Phi(t) y^{(i)}(t) dt = $$
$$= \sum_{i=0}^{m} a_i \int_0^T \Phi(t) u^{(i)}(t) dt \qquad (6)$$

Applying the integration by parts rule and keeping the property 5 in mind, we get:

$$\sum_{i=0}^{n} (-1)^i b_i \int_0^T \Phi^{(i)}(t) y(t) dt =$$
$$= \sum_{i=0}^{m} (-1)^i a_i \int_0^T \Phi^{(i)}(t) u(t) dt \qquad (7)$$

The previous statement is an algebraic equation. The unknows are the coefficients of (2) and the general solution is:

$$\alpha_i = (-1)^i \int_0^T \Phi^{(i)}(t) y(t) dt \qquad (8)$$

$$\beta_i = (-1)^i \int_0^T \Phi^{(i)}(t) u(t) dt \qquad (9)$$

Without loss of generality we assume:

$$b_0 = 1 \qquad (10)$$

Therefore, in order to determine all the unknown parameters a_i and b_i, at least $n+m+1$ linearly independent equations are required. These equations can be generated discretely by shifting the modulating function along the measured input-output data. In this manner we obtain the following linear algebraic system:

$$Az = c \qquad (11)$$

where z is the vector of unknown parameters. The matrix A and the vector c consist of terms depending on the coefficients α_i and β_i:

$$A = \begin{bmatrix} \alpha_{0,1} \ldots \alpha_{m,1} & -\beta_{1,1} \cdots - \beta_{n,1} \\ \ldots & \ldots \\ \alpha_{0,k} \ldots \alpha_{m,k} & -\beta_{1,k} \cdots - \beta_{n,k} \end{bmatrix} \qquad (12)$$

$$c = [\beta_{0,1} \ldots \beta_{0,k}]^T \qquad (13)$$

with $k \geq n+m+1$.

If the equation's number is greater than the number of unknown parameters, a pseudoinverse solution can be determined.

Summarizing, the most important features of this technique are:

- the identification process occurs directly in the time domain;
- the knowledge of time derivatives of the input and the output is not necessary;
- the integral nature of the procedure cuts the high frequency dynamics: the bandwidth is $1/T$.

Due to good filtering characteristics and easiness of implementation, the most popular class of modular functions is the class of spline functions introduced by Maletinsky (see (Maletinsky, 1978), (Maletinsky, 1979)). In Figure 5 we are showing some examples of spline functions.

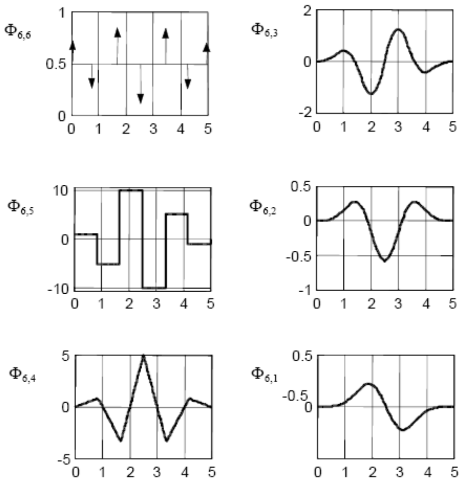

Figure 5: Spline functions of order 6.

Set now as operating point the condition in which the caddy receives the constant input $U = U_0$. At the output terminal a constant signal $Y = Y_0$ is measured. In order to apply the modulating function algorithm the following steps are required:

1. excitation of the system with the input $U = U_0 + u$. The u term represents the variation with respect to the fixed operating point. The u input must excite all the dynamics of the system;
2. storage of the corresponding output signal $Y = Y_0 + y$, where y is the variation component;
3. computation of u and y from signals U and Y;
4. selection of the system structure (number of poles and zeros) and selection of the integration window;
5. identification procedure applied to u and y.

An appropriate set of data is needed for derive a number of independent equations sufficient for the parameter's calculation.

3.2 The Identification Procedure

The likely operating conditions that we have considered are are the following:

1. Dry asphalt;
2. Dry asphalt with load (passenger);
3. Grass;
4. Grass with load.

The reference input is $U_0 = 170$ (digital scale). For each operative condition several data-sets have been acquired for different input signals u and a validation data-set have been selected.

The input signal has been built with rectangular pulses of variable amplitude. The duration of the pulses changes along the data sets.

Different models have been obtained for each data set and have then been compared to the validation data-set of the corresponding operating condition. For this purpose the measured output has been compared to the predicted output of the model and the mean square error has been computed.

The models have been obtained by different model structure (number of poles and zeros) and different amplitude of the integration window. Some examples are shown in Table 1.

Table 1: Examples of different identifications. Only the best models with respect to the integrator window T are listed.

Operating condition 1		
Model Structure	T (s)	MSE (m/s)
1 pole	4	0.04
2 poles	2.5	0.02
2 poles, 1 zero	2	0.03
manufacturer	-	0.12
Operating condition 2		
Model Structure	T (s)	MSE (m/s)
1 pole	5	0.036
2 poles	3.5	0.024
2 poles, 1 zero	3.5	0.038
manufacturer	-	0.122
Operating condition 3		
Model Structure	T (s)	MSE (m/s)
1 pole	4.5	0.054
2 poles	3.5	0.048
2 poles, 1 zero	2.5	0.054
manufacturer	-	0.121
Operating condition 4		
Model Structure	T (s)	MSE (m/s)
1 pole	4	0.054
2 poles	4.5	0.049
2 poles, 1 zero	2	0.049
manufacturer	-	0.119

The examples are related to the data sets showing the features depicted in Table 2. In figures 6-9 it is shown a comparison with real data.

In terms of mean square error, good results are obtained even with simple models such as second order transfer function with or without zero. In our case the presence of the zero does not change significantly the mean square error; therefore, for the sake of simplicity, we can select the second order model without zero.

The application of the delay version of modulating function algorithm (Balestrino et al., 2000) does not have shown any remarkable delay.

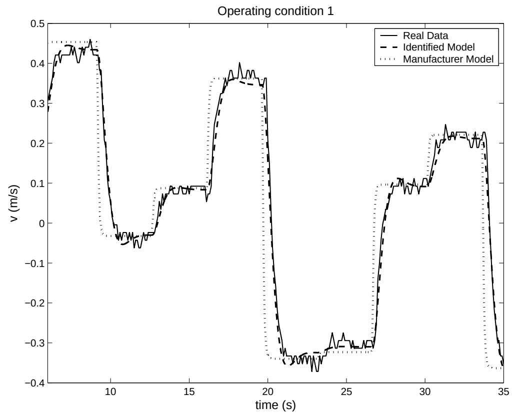

Figure 6: Operating Condition 1.

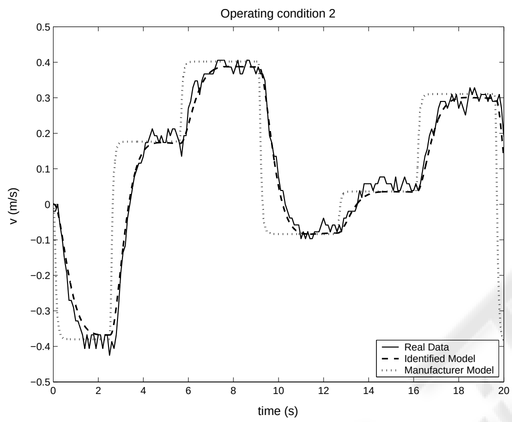

Figure 7: Operating Condition 2.

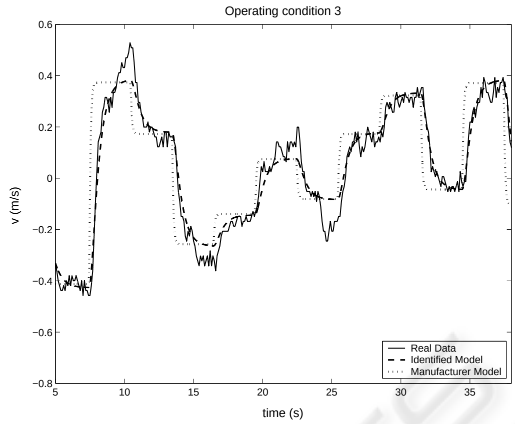

Figure 8: Operating Condition 3.

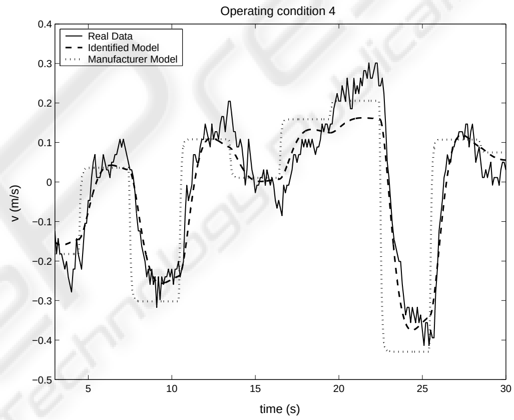

Figure 9: Operating Condition 4.

4 BASIS FOR ROBUST CONTROL

Thanks to previous analysis, we currently own a set of maps describing the behavior of the vehicle in different conditions. The basic idea is the construction of a mean model, that, for our purposes, plays the role of nominal plant. For this aim a very simple approach is followed: a mean model is derived for every operating condition and after the final mean model is computed as a mean of these results:

$$z_0 = \sum_{j=1}^{N_{op}} \frac{\sum_{i=1}^{N_{test}} \frac{z_{ij}}{N_{test}}}{N_{op}} \tag{14}$$

where N_{op} is the number of operating conditions (dry asphalt with or without a person on board, dry grass with or without a person on board, etc.), N_{test} is the number of models derived in different tests in a given operating condition.

The term ”uncertainty” refers to the differences or errors between models and reality. For this reason we are defining uncertainties the deviations with respect to the mean model (14) of all models computed. The structured representation of uncertainty in the so-called multiplicative form is (Zhou and Doyle, 1998):

$$P_\Delta = (I + W_m(s)\Delta_m(s))P_0(s) \tag{15}$$

where $W_m(s)$ is a stable transfer matrix describing the spatial and frequency structure of uncertainty. In this manner P_Δ is confined in a normalized neighborhood of the nominal plant $P_0(s)$. For every model derived in Section 2 the multiplicative uncertainty is defined as:

$$\Delta_{im}(s) = \frac{G_i(s) - G_0(s)}{G_0(s)} \tag{16}$$

We then look for a weight transfer function W_m such that:

$$|W_m(j\omega)| \geq |\Delta_{im}(j\omega)|$$

In our case the function $W_m(s)$ is also proper since $G_0(s)$ and $G_i(s)$ have the same relative degrees. Applying this procedure to the data set, we can find the

Table 2: Data set features.

Operating Mode	1	2	3	4
surface	Asphalt	Asphalt	Grass	Grass
Passenger	No	Yes	No	Yes
Min/Max step	-10/+10			
Step duration (s)	1.5	1.5	2	2

following result:

$$W_m(s) = \frac{1.3(s+0.6)}{(s+3.8)}$$

In Figure 10 the bounded region including all the models is shown, whereas in Figure 11 the frequency response of the $W_m(j\omega)$ and $\Delta_{im}(j\omega)$ is reported.

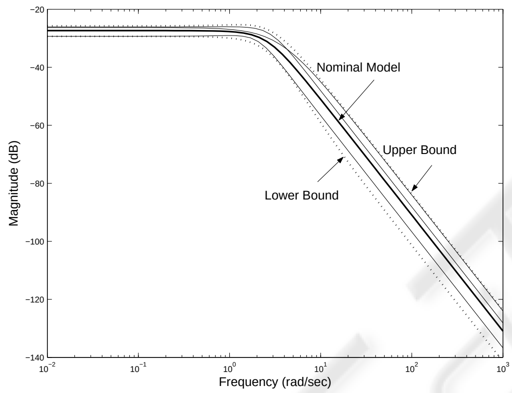

Figure 10: Bounding region of $P_\Delta = [1 + W_m(s)\Delta(s)]P_0(s)$

5 ONLINE IDENTIFICATION

The offline version of the modulating function technique has been used to estimate uncertainties with respect to the chosen nominal model. In order to obtain a likely uncertain model several tests are necessary.

An online version of this technique can be easily implemented (Balestrino et al., 2004). In this way an alternative approach for the control design, based on adaptive method, seems to be possible. The design of the controller will require an analysis of the convergence of the method.

To perform the online version of modulating functions algorithm we will have to choose:

- The model structure;
- The number of rows of the A matrix defined in 12;

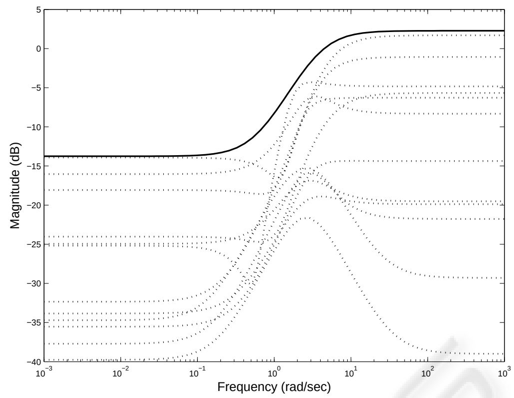

Figure 11: Δ_{im} (dashed lines) and the bound W_m (solid line).

- The integration window duration(T);
- An initial estimation of z (given, for example, by a offline procedure);

Input-output data are stored continuously. Every T seconds a new equation is generated using (7) and the following steps are performed (see Figure 12):

1. Starting from the previous matrix A_{k-1} and the vector c_{k-1}, the A_k matrix and the vector c_k are updated with a First In First Out strategy taking in account the new equation;
2. The following index is calculated:
$$\Delta = \|A_k z_{k-1} - c_k\| \tag{17}$$
were z_{k-1} is the previous parameter's vector;
3. If $\Delta > \epsilon$, where ϵ is a fixed positive threshold, then the new row brings meaningful new information and the algorithm can proceed to the next step. Otherwise, the A_{k-1} and c_{k-1} are restored and the algorithm returns to step 1;
4. computation of the new z_k. Return to step 1.

It is worth to notice that, in order to increase the frequency of the parameter's computation, the integration windows can overlap. In Figure 13 we are showing the application of the online algorithm in dry asphalt operating mode.

6 CONCLUSION

The modulating functions technique guarantees the identification of a second order linear model, which thoroughly describes the dynamics of a caddy vehicle. The offline classic version of the procedure allows to set the control problem for unmanned guidance from the point of view of robustness. The online version of the technique enables an adaptive control strategy.

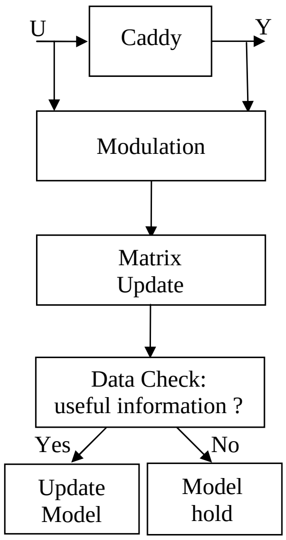

Figure 12: Block diagram of online algorithm.

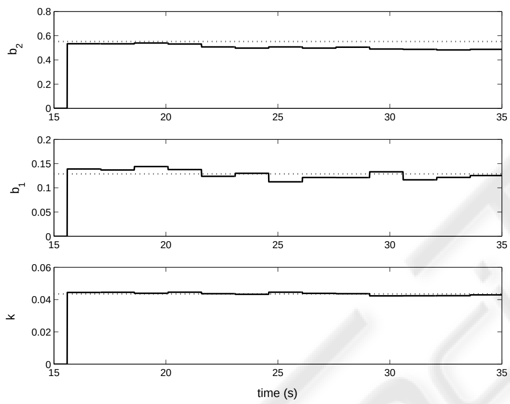

Figure 13: Second order model in dry asphalt: Offline parameters (dashed line), Online parameters (solid line).

Our intent in a near future is the design and the implementation of controllers following the guidelines showed in this work.

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