MULTIMODELLING STEPS FOR FREE-SURFACE HYDRAULIC SYSTEMS CONTROL

Eric Duviella, Philippe Charbonnaud and Pascale Chiron
Laboratoire Génie de Production
Ecole Nationale d'Ingénieurs de Tarbes
47, avenue d'Azereix, BP 1629
65016 Tarbes Cedex, France

Keywords: Multimodelling, operating modes, on-line selection, hydrographic network.

Abstract: The paper presents multimodelling steps for the design of free-surface hydraulic system control strategies. This method is proposed to represent simply and accurately the non-linear hydraulic system dynamics under large operating conditions. It is an interesting alternative to the use of Saint Venant partial differential equations because it allows the design, the tuning and the validation of control strategies. The multimodelling steps of the proposed method are performed in order to lead to the determination of a finite number of models. The models are selected on-line by the minimization of a quadratic criterion. The evaluation of the multimodelling method is carried out by simulation within the framework of a canal with trapezoidal profile.

1 INTRODUCTION

The hydrographic networks are systems geographically distributed conveying gravitationally water quantities. They are composed of open-surface hydraulic systems (canals, rivers, etc.) which are used to satisfy the requests related to human activities. The efficient management of these systems is essential today, according to the recognized importance of water resource. This management requires the proposal, the design and the tuning of control strategies through simulation, before their implementation on real systems. The free-surface hydraulic system dynamics is characterized by nonlinearity and important transfer delays. Although the Saint Venant Partial Differential Equations (PDE) accurately represent hydraulic systems dynamics (Chow et al., 1988; Malaterre and Baume, 1998), their resolution involves numerical approaches according to discretization scheme which are rather complex to handle in the control strategy design and tuning steps. The PDE simplification and linearization around an operating point led to simplify models of the hydraulic system dynamics (Litrico and Georges, 1999a). In the literature, most authors have proposed control strategies based on the PDE linearization (Malaterre et al., 1998). However, the accuracy of these models is only acceptable on restricted interval around the operating point, and their use on large operating conditions requires robust controller design, as proposed in (Litrico and Georges, 1999b).

The representation of the non linear systems with variable transfer delays involves the identification problem of a model with variable parameters or multiple models. In the literature, multimodelling approaches for the predictive control are described in (Palma and Magni, 2004) and in (Özkan and Kothare, 2005). These methods are based on switching technics amongst several simple models. In the case of nonlinear systems with variable transfer delays, an algorithm for estimation of the models most representative of the system dynamics is proposed in (Petridis and Kehagias, 1998). In these approaches, the models number and their operating range are known *a priori*. The nonparametric modelling of the nonlinear system dynamics can also be carried out by Gaussian approaches (Gregorcic and Lightbody, 2002). The identification and control of open-channel systems using Linear Parameter Varying (LPV) models is proposed in (Bolea et al., 2004; Puig et al., 2005). The dynamics of hydraulic systems is modelled by a first order differential equation with time delay. The parameters of the LPV model have been identified using a parameter estimation algorithm. These approaches require an important quantity of adapted data.

In this article, the multimodelling steps which lead to the determination of a finite number of models, are proposed to design and tune control strategies for hydraulic systems subject to large operating conditions.

Duviella E., Charbonnaud P. and Chiron P. (2006).
MULTIMODELLING STEPS FOR FREE-SURFACE HYDRAULIC SYSTEMS CONTROL.
In *Proceedings of the Third International Conference on Informatics in Control, Automation and Robotics*, pages 32-38
DOI: 10.5220/0001203400320038

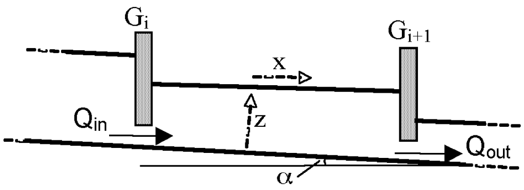

Figure 1: Canal reach.

In section 2, the multimodelling steps according to the celerity coefficient is described. An on-line multimodels selection method is presented in section 3. In section 4, the method is used to identify the dynamic of a canal with trapezoidal profile. The effectiveness of the multimodelling steps compared to the Saint Venant PDE is presented in section 5.

2 MODELLING OF FREE-SURFACE HYDRAULIC SYSTEMS

Open-channel systems are characterized by large sized, nonlinear dynamics and important transfer delays. They are generally broken down into several reaches which are located between two measurement points or gates G_i and G_{i+1} (*see* Figure 1). The diffusive wave equation (Chow et al., 1988), expressed by relation (1) can be used to represent accurately the canal reach dynamics.

$$\frac{\partial Q(x,t)}{\partial t}+C(Q,z,x)\frac{\partial Q(x,t)}{\partial x}-D(Q,z,x)\frac{\partial^2 Q(x,t)}{\partial x^2}=0, \quad (1)$$

where $Q(x,t)$ is the reach flow discharge $[m^3/s]$, $C(Q,z,x)$ the celerity coefficient $[m/s]$ and $D(Q,z,x)$ the diffusion coefficient $[m^2/s]$ expressed by:

$$\begin{cases} C(Q,z,x)=\dfrac{1}{L^2\frac{\partial J}{\partial Q}}\left[\dfrac{\partial L}{\partial x}-\dfrac{\partial(LJ)}{\partial z}\right], \\ D(Q,z,x)=\dfrac{1}{L\frac{\partial J}{\partial Q}}, \end{cases} \quad (2)$$

where L is the water surface width and J is the friction slope. Several empirical formulas can be used to express the friction slope J (Kovacs, 1988). Generally, the Manning-Strickler formula (3) is used. The friction slope J is considered equal to the canal slope α when the flow depth is normal:

$$J=\frac{Q^2 n_M^2 P^{\frac{4}{3}}}{S^{\frac{10}{3}}}, \quad (3)$$

where n_M is the Manning coefficient associated with the hydraulic system considered (river, channel) and with his bed type (n_M lies between 0.02 and 0.01 for a concrete canal). The Manning coefficient determination can be carried out from physical knowledge of the hydraulic system or by identification (Ooi et al., 2003).

The diffusive wave equation (1) can be linearized around an operating discharge Q_e (Litrico and Georges, 1999a), and the identified celerity and diffusion parameters are denoted C_e and D_e.

$$\frac{dq(x,t)}{dt}+C_e\frac{dq(x,t)}{dx}-D_e\frac{d^2q(x,t)}{dx^2}=0. \quad (4)$$

where $Q=Q_e+q$. The discharge variation q from the reference discharge Q_e is flowed out with a mean speed of constant celerity C_e and is diffused with a constant diffusion D_e.

The linearization of the diffusive wave equation leads to a finite order transfer function:

$$F(s)=\frac{e^{-\tau s}}{1+a_1 s+a_2 s^2}, \quad (5)$$

where the transfer function parameters a_1, a_2 and τ are calculated by the moment matching method as described in (Georges and Litrico, 2002):

$$\begin{cases} a_1=\left(\dfrac{6XD_e^2}{C_e^5}+\sqrt{\dfrac{4X^2D_e^3}{C_e^9}\left(\dfrac{9D_e}{C_e}-2X\right)}\right)^{\frac{1}{3}} \\ \quad +\left(\dfrac{6XD_e^2}{C_e^5}-\sqrt{\dfrac{4X^2D_e^3}{C_e^9}\left(\dfrac{9D_e}{C_e}-2X\right)}\right)^{\frac{1}{3}}, \\ a_2=\dfrac{2XD_e}{C_e^3}\left(1-\dfrac{3D_e}{a_1C_e^2}\right), \\ \tau=\dfrac{X}{C_e}-a_1. \end{cases}$$

The model order is determined according to the adimensional coefficient C_M (Malaterre and Baume, 1998):

$$C_M=\frac{2C_eX}{9D_e} \quad (6)$$

where X is the reach length. The canal reach dynamic is modelled by:

- a second order plus delay transfer function (5) when $C_M>1$,
- a first order plus delay transfer function when $\frac{4}{9}<C_M\le 1$, with $a_2=0$,
-a first order transfer function when $C_M\le\frac{4}{9}$, with $a_2=0$ and $\tau=0$.

The linearization of the diffusive wave model leads to the identification of dynamics of free-surface hydraulic systems. Validity of the model decreases as the operating point of system moves away from the identification point. Accurate representation of open-surface hydraulic system dynamic on large operating conditions requires a multimodelling method. It involves to determine the necessary number of models, *i.e.* Operating Modes (OM), and their validity boundaries.

3 MULTIMODELLING STEPS OF FREE-SURFACE HYDRAULIC SYSTEMS

The multimodelling method consists in defining the models number n necessary to represent the system dynamics under large operating conditions. The process model M of the hydraulic system is decomposed into a finite class of linear models $M = \{M_1, M_2, ...M_n\}$, where the i^{th} linear model of the hydraulic system is denoted M_i, and $n = \text{card}(M)$. The celerity coefficient values are used in order to fix the number n of OM.

The open-surface hydraulic system dynamics under large operating conditions are represented by the following relation:

$$\left\{ \begin{array}{l} \dot{x} = \mathbf{A}_i x(t) + \mathbf{B}_i u(t-\tau), \\ y = \mathbf{C}_i x(t), \end{array} \right. \tag{7}$$

where u and y are respectively the input and output variables, x the state. Identification matrices $\mathbf{A}_i$, $\mathbf{B}_i$ and $\mathbf{C}_i$ are computed for the model M_i which corresponds to the i^{th} OM. They are expressed, according to the transfer functions (5), by relation: $\mathbf{A}_i = \begin{bmatrix} -\frac{a_{1i}}{a_{2i}} & 1 \\ -\frac{1}{a_{2i}} & 0 \end{bmatrix}$, $\mathbf{B}_i = \begin{bmatrix} 0 \\ \frac{1}{a_{2i}} \end{bmatrix}$ and $\mathbf{C}_i = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

The celerity coefficient can be considered as the most representative parameters of the open-channel system dynamics. Therefore, a model is considered as available as soon as the error on the celerity coefficient is inferior to a fixed percentage Π_c. The validity boundaries are defined for each OM. The value of parameter Π_c is choosen according to the system dynamics. The multimodelling method is described by an algorithm (*see* Table 1), where the OM are determined starting with C_{med}. This one is computed with the parameters $C_{\min}$ and $C_{\max}$ corresponding respectively to the minimum and maximum discharges of the system $Q_{\min}$ and $Q_{\max}$.

This algorithm leads to the determination of the celerity coefficient C_{id_r} used to identify the r^{th} linear model and the OM validity boundaries C_{inf_r} and C_{sup_r}. According to C_{id_r}, the water elevation z_{id_r} of each r^{th} OM are determined, with one millimeter accuracy, by the digital resolution of the relation (8) with Newton method.

$$C_{id} = \frac{\sqrt{J}S^{\frac{5}{3}}}{nP^{\frac{2}{3}}L^2}\left[-\frac{1}{2}\frac{\partial L}{\partial z} - \frac{L}{3P}\left(2\frac{\partial P}{\partial z} - 5\frac{P}{S}\frac{\partial S}{\partial z}\right)\right], \tag{8}$$

Table 1: Multimodelling Algorithm.

Input : $C_{\max}, C_{\min}, \Pi_C$
Output : $C_{id_r}, C_{\text{sup}_r}, C_{\text{inf}_r}$,
$C_{med} = \dfrac{C_{\max} + C_{\min}}{2}$,
$r = 1$,
For i : $\left\lceil \dfrac{\ln \frac{C_{\min}}{C_{med}}}{\ln \frac{(1+\Pi_C)}{(1-\Pi_C)}} \right\rceil$ to $\left\lfloor \dfrac{\ln \frac{C_{\max}}{C_{med}}}{\ln \frac{(1+\Pi_C)}{(1-\Pi_C)}} \right\rfloor$,
| $C_{id_r} = \left(\dfrac{1+\Pi_C}{1-\Pi_C}\right)^i C_{med}$,
| $C_{\text{sup}_r} = (1+\Pi_C)\left(\dfrac{1+\Pi_C}{1-\Pi_C}\right)^i C_{med}$,
| $C_{\text{inf}_r} = (1-\Pi_C)\left(\dfrac{1+\Pi_C}{1-\Pi_C}\right)^i C_{med}$,
| $r++$,
EndFor.

where L, P and S parameters are expressed, according to the geometrical characteristics, interms of the water elevation z_{id}. The computation of water elevation z_{id_r} allows for the determination of the diffusion coefficient D_{id_r} (2). Finally, the matrices A_i, B_i and C_i are computed using C_{id_r} and D_{id_r} values.

The multimodelling method is used to identify the free-surface hydraulic system dynamics with several OM. The discharge boundary conditions Q_{inf_i} and Q_{sup_i} are computed according to the relation:

$$Q = \frac{\sqrt{J}S^{\frac{5}{3}}}{n_M P^{\frac{2}{3}}}, \tag{9}$$

where P and S parameters depend on the water elevation boundary conditions z_{inf_r} and z_{sup_r}.

The presented algorithm can be used for various hydraulic system profiles, *i.e.* rectangular, trapezoidal and circular profiles. The multimodelling steps were used in (Duviella et al., 2006) within the framework of a dam gallery with a circular profile. In order to simulate and implement control strategies for hydraulic systems under large operating conditions, it is necessary to propose an on-line selection method of the multi model.

4 ON-LINE SELECTION CRITERION OF OPERATING MODE

An analytical expression of the hydraulic system output y can be approached as:

$$y \simeq \hat{y} = \sum_{j=1}^{n} \delta_m^j . y_j, \quad (10)$$

where n is the number of OM, m denotes the actual OM, y_j is the answer of the j^{th} model M_j, and δ_m^j is equal to 1 if $m = j$ and 0 otherwise. The main problem of OM detection lies in the real-time estimation of m at the boundary between two OM, *i.e.*, the current behavior of the physical process. The selection method has to figure out the OM actual value. For that, a criterion J_j (11), $0 < j \leq n$, is defined for each OM and computed at each sample period kT_s.

$$J_j(k) = \frac{1}{N-1} \sum_{i=0}^{N-1} \varepsilon_{j,i}(k), \quad (11)$$

where N is the size of a sliding window, $\varepsilon_{j,i}(k)$ is the j^{th} identification error and:

$$\varepsilon_{j,i}(k) = (y(k-i) - y_j(k-i))^2. \quad (12)$$

The multi-model output recursive square error criterion $\mathbf{J}(k) = [J_1(k)\ J_2(k)\ ...\ J_j(k)...]$ is computed with the recursive formula:

$$J_j(k) = J_j(k-1) + \frac{1}{N-1}(\varepsilon_{j,0}(k) - \varepsilon_{j,0}(k-N)). \quad (13)$$

At each sampling period, a minimization of the criterion given by (11) is carried out to determine the adequat transfer function. The correspondant OM number is denoted d(k) and satisfies:

$$d(k) = \underset{1 \leq j \leq n}{\arg\min} J_j(k)\}. \quad (14)$$

At the starting time, it is assumed that $d(0) = 1$. The detection time is defined by:

$$t_d(k) = \{kT_s,\ d(k) \neq d(k-1)\}, \quad (15)$$

The multimodelling steps which lead to determine the different OM, associated to on-line selection criterion, makes possible to represent the hydraulic systems dynamics on the totality of their operating range. In the following section, the multimodelling steps and on-line selection criterion are illustrated within the framework of a canal with trapezoidal profile. The multimodelling approach is then compared to a method based on Saint Venant equations.

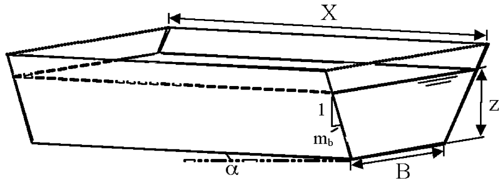

Figure 2: Canal reach with trapezoidal profile.

Table 2: Canal reach downstream limiting conditions.

$Q\ [m^3/s]$	0.5	1.4	2.6	4.2	6.2	8.7	10
$z\ [m]$	0.3	0.5	0.7	0.9	1.1	1.3	1.4

5 APPLICATION TO A CANAL WITH TRAPEZOIDAL PROFILE

The application of the multimodelling steps is carried out on a canal reach with a trapezoidal profile (*see* Figure 2). Its geometrical characteristics are given below:
- bottom width $B = 2.85\ m$,
- average fruit of the banks $m_b = 0.99$,
- profile length $X = 1732\ m$,
- Manning coefficient $n_M = 0.02$,
- reach slope $\alpha = 0.13\ \%$,
- minimal discharge $Q_{\min} = 1\ m^3/s$,
- maximal discharge $Q_{\max} = 10\ m^3/s$.

This canal reach is firstly modelled by the Saint Venant equations, and secondly by the multimodelling approach. The resolution of Saint Venant equations is realized with the downstream limiting conditions (*see* Table 2) according to the software SIC[1]. This one allows the dynamic simulation of rivers and canals according to the Preissmann scheme. Among the resolution algorithms proposed, the Newton algorithm which offers the best performances in spite of a longer simulation time, is chosen. To avoid the instability periods during simulation, the time and space steps must be tuned so that the Courant number (16) is equal to one.

$$Cr = \frac{dt}{dx}(V + C), \quad (16)$$

where V is the mean velocity of the flow expressed by $V = \frac{Q}{S}$.

The multimodelling step applied according to the algorithm (*see* Table 1) with an tolerated error Π_c of 10% leads to the identification of three OM. In the

[1]SIC user's guide and theorical concepts. CEMAGREF, Montpellier, 1992. http://canari.montpellier.cemagref.fr/

Table 3: Identification discharge Q_{id_r}, operating range Ω_r, and characteristics z_{id_r}, C_{id_r} and D_{id_r} of each OM.

Q_{id_r} $[m^3/s]$	Ω_r $[m^3/s]$	z_{id_r}	C_{id_r}	D_{id_r}
1.5	[1 ; 2.2[	0.469	1.3	148
3.4	[2.2 ; 5.3[	0.775	1.6	300
8.7	[5.3 ; 10]	1.320	1.9	617

Table 4: Identification discharge Q_{id_r}, operating range Ω_r, and characteristics a_{1i}, a_{2i} and τ_i of each OM.

Q_{id_r} $[m^3/s]$	Ω_r $[m^3/s]$	a_{1i}	a_{2i}	τ_i
1.5	[1 ; 2.2[	742	154520	608
3.4	[2.2 ; 5.3[	736	135470	369
8.7	[5.3 ; 10]	678	77690	226

case of a hydraulic system with a trapezoidal profile, celerity C_e and diffusion D_e are expressed by:

$$\begin{aligned} C_e &= \frac{Q_e}{L^2}\left[-m_b + \frac{L}{3}\left(\frac{2B}{Pz} + \frac{5L}{S} - \frac{2}{z}\right)\right], \\ D_e &= \frac{Q_e}{2LJ}, \end{aligned} \tag{17}$$

with :
- $L = B + 2m_b z$,
- $S = (B + m_b z)z$,
- $P = B + 2z\sqrt{1 + m_b{}^2}$.

The three OM, the correspondant identification discharge Q_{id_r}, operating range Ω_r, and characteristics z_{id_r}, C_{id_r} and D_{id_r} are given in Table 3. Characteristics a_{1i}, a_{2i} and τ_i are given in Table 4.

In order to visualize the identified OM, parameters z_{id_r}, C_{id_r} and D_{id_r} are represented in Figure 3, and parameters a_1, a_2 and τ in Figure 4 according to the discharge Q. The values of each parameter were calculated beforehand for each discharge of the operating range, *i.e.* $[1; 10]$, with a step of 1 m^3/s. These values are represented by points in Figures 3 and 4.

The evaluation of the multimodelling approach is carried out by comparison with the Saint Venant approach. Figure 5 shows the responses to steps around the discharges used for the parameters identification of the three OM. The setpoints are in bold continuous line, the outputs resulting from SIC are also in bold continuous line, those resulting from the first model M_1 are in dot line, those resulting from the second M_2 are in dashed line, and finally those resulting from the third M_3 are in dash-dot line.

Figure 5.a shows the M_1 and Saint Venant answers of a step of 1.2 m^3/s starting from a discharge of 1 m^3/s, Figure 5.b, the answers of a step of 3.1 m^3/s starting from a discharge of 2.2 m^3/s, and Figure 5.c, the answers of a step of 3.7 m^3/s starting from a discharge of 5.3 m^3/s. According to the simulation results, the dynamics modelled by transfer functions are close to those from SIC for each OM.

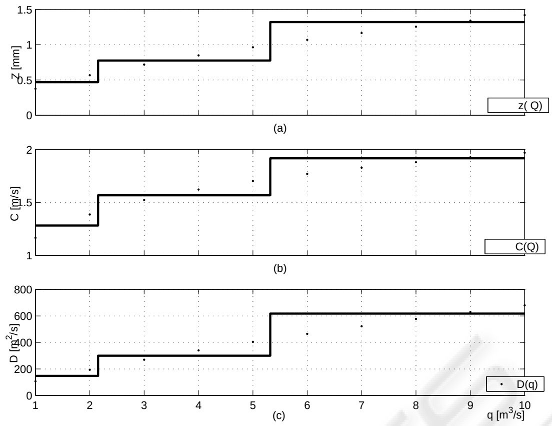

Figure 3: Variation of the parameters z, C and D according to the considered OM.

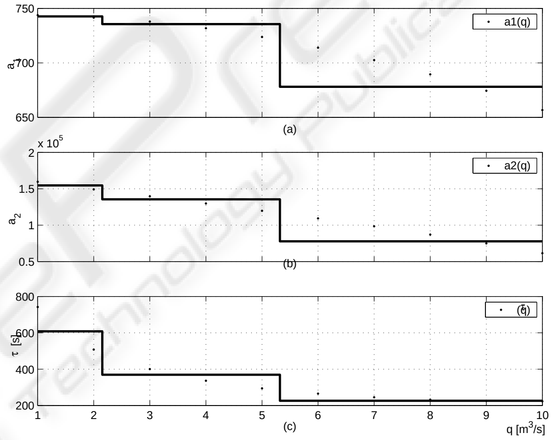

Figure 4: Variation of the parameters a_1, a_2 and τ according to the considered OM.

Then, the comparison between the two modelling approaches is carried out by simulation on the whole operating range of the canal reach. The setpoint input is represented in bold continuous line in Figure 6.a. It corresponds to setpoints with discharge amplitude from 1 to 9 m^3/s.

The simulation results obtained by SIC are represented in continuous line, and those from multimodelling in dashed line in Figure 6.a. The on-line selection of the transfer function is represented in Figure 6.b. This selection is realized according to the rules presented in section 4.

The outputs resulting from the two approaches are very similar on the totality of the canal reach operating range. Light differences between these outputs appear around 8 m^3/s when the discharge increase.

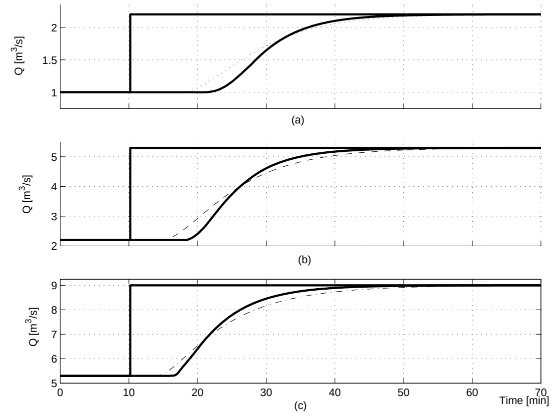

Figure 5: Input step setpoints (bold continuous line) arround (a) $1.5\ m^3/s$, (b) $3.5\ m^3/s$ and (c) $8.5\ m^3/s$, and corresponding outputs resulting from SIC (bold continuous line), from M_1 (dot line), M_2 (dashed line) and M_3 (dash-dot line).

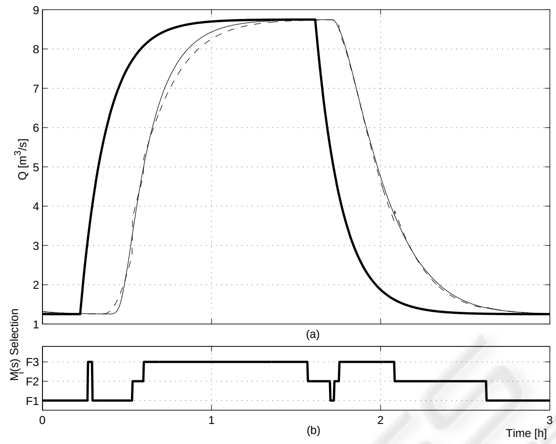

Figure 6: (a) Setpoint input (bold continuous line), SIC output (continuous line) and multimodelling output (dashed line) and (b) β the selected transfer function.

The maximum output error between these two approaches is reached for $7.5\ m^3/s$ and corresponds to an error percentage of $4.2\ \%$. These differences are sufficiently weak to conclude on the effectiveness of the multimodelling approach. Moreover for this simulation case, the excecution time for the SIC method was ten times longer than for the multimodelling method.

The multimodelling approach interest lies in the faithful representation of the hydraulic systems dynamics on the totality of their operation range, and in the facility of its implementation. This approach requires only the knowledge of the physical characteristics of the hydraulic system, the OM determination and the on-line selection method implementation. Moreover, the multimodelling approach constitutes an effective tool for the design and the tuning of regulation and reactive control strategies.

6 CONCLUSION

The efficient management of hydraulic systems required the proposal and the design of regulation and reactive control strategies. The development of these techniques is facilitated using an operational and faithful simulation tool. A multimodelling approach is proposed to design and tune the control strategies of hydraulic systems subject to large operating conditions. It leads to obtain a finite number of operating modes accurately reproducing the hydraulic systems dynamics around operating discharges. This multimodelling method is carried out by considering an acceptable percentage of error on the celerity. The on-line operating modes selection method based on the minimization of a quadratic criterion leads to the accurate identification of the adequat model.

The multimodelling steps and the on-line operating modes selection criterion are illustrated within the framework of a canal reach with trapezoidal profile. The dynamic identified by multimodel are compared to those resulting from a discretization scheme using for the Saint Venant equations resolution. Simulation results lead to conclude to the multimodelling approach effectiveness.

REFERENCES

Bolea, Y., Puig, V., Blesa, J., Gomez, M., and Rodellar, J. (2004). An LPV model for canal control. *10th IEEE International Conférence on Methods and Models in Automation and Robotics, MMAR, 30 August - 2 September, Miedzyzdroje, Poland*, 1:659–664.

Chow, V. T., Maidment, D. R., and Mays, L. W. (1988). *Applied Hydrology*. McGraw-Hill, New York, Paris.

Duviella, E., Chiron, P., and Charbonnaud, P. (2006). Multimodélisation des systèmes hydrauliques à surface libre. *Conférence Francophone de Modélisation et Simulation, MOSIM'06, Rabat (Maroc), 3-5 avril*.

Georges, D. and Litrico, X., editors (2002). *Automatique Pour la Gestion Des Ressources En Eau*. Hermès Science Publications, Lavoisier, 11 rue Lavoisier, 75008 Paris.

Gregorcic, G. and Lightbody, G. (2002). Gaussian processes for modelling of dynamic non-linear systems. *In Proc. of the Irish Signals and Systems Conference, ISSC'02, Cork, Ireland, 25-26 Juin 2002*.

Kovacs, Y. (1988). *Modèles de Simulation D'écoulement Transitoire En Reseau D'assainissement*. PhD thesis, ENPC - CERGRENE.

Litrico, X. and Georges, D. (1999a). Robust continuous-time and discrete-time flow control of a dam-river system. (i) modelling. *Applied Mathematical Modelling 23*, pages 809–827.

Litrico, X. and Georges, D. (1999b). Robust continuous-time and discrete-time flow control of a dam-river system. (II) controller design. *Applied Mathematical Modelling 23*, pages 829–846.

Malaterre, P., Rogers, D., and Schuurmans, J. (1998). Classification of canal control algorithms. *Journal of irrigation and drainage engineering*, 124(1):3–10.

Malaterre, P.-O. and Baume, J.-P. (1998). Modeling and regulation of irrigation canals: Existing applications and ongoing researches. *IEEE International Conference on Systems, Man, and Cybernetics*, 4:3850–3855.

Ooi, S. K., Krutzen, M., and Weyer, E. (2003). On physical and data driven modelling of irrigation channels. *Control Engineering Practice*, 13:461–471.

Özkan, L. and Kothare, M. V. (2005). Stability analysis of a multi-model predictive control algorithm with application to control of chemical reactors. *Journal of Process Control, In Press, Corrected Proof.*

Palma, F. D. and Magni, L. (2004). A multi-model structure for model predictive control. *Annual Reviews in Control, , Issue 1*, 28:Pages 47–52.

Petridis, V. and Kehagias, A. (1998). A multi-model algorithm for parameters estimation of time-varying non-linear systems. *Automatica, Elsevier Science Ltd*, 34(4):469–475.

Puig, V., Quevedo, J., Escobet, T., Charbonnaud, P., and Duviella, E. (2005). Identification and control of an open-flow canal using LPV models. *CDC-ECC'05, 44 th IEEE conference on decision and control and european control conference, Seville, Spain, 12-15 december 2005*, pages 1893–1898.