

# ROBUST AND ACTIVE TRAJECTORY TRACKING FOR AN AUTONOMOUS HELICOPTER UNDER WIND GUST

Adnan Martini, François Léonard and Gabriel Abba

Industrial Engineering and Mechanical Production Lab (LGIPM), ENIM, Ile du Saulcy, 57045 Metz, cedex 1, France

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**Abstract:** The helicopter manoeuvres naturally in an environment where the execution of the task can easily be affected by atmospheric turbulences, which lead to variations of its model parameters. The originality of this work relies on the nature of the disturbances acting on the helicopter and the way to compensate them. Here, a nonlinear simple model with 3-DOF of a helicopter with unknown disturbances is used. Two approaches of robust control are compared via simulations: a nonlinear feedback and an active disturbance rejection control based on a nonlinear extended state observer (ADRC).

## 1 INTRODUCTION

The control of nonlinear systems under disturbance is an active sector of research in the last decades especially in aeronautics where several elegant approaches were presented. We consider here the problem of control of a Lagrangian model with 3-DOF of a helicopter assembled on a platform (VARIO 23cc). It is subjected to a wind gust and it carries out a vertical flight (takeoff, slope, flight, descent and landing). The mathematical model of the system is very simple but its dynamic is not trivial (nonlinear in state, and underactuated).

Basically, the methods of control which address the attenuation of the disturbance, can be classified according to the different kinds of disturbances. A possible approach is to model the disturbances by a stochastic process, which leads to the theory of nonlinear stochastic control (Gokçek et al., 2000). Another approach is the nonlinear control (Marten et al., 2005) where it is supposed that the energy of the disturbances is limited. A third approach is to treat the disturbances produced by a neutral stable exogenous system using the nonlinear theory of output regulation (Isidori, 1995) (Byrnes et al., 1997) and (Marconi and Isidori, 2000). (Wei, 2001) showed the control of the nonlinear systems with unknown disturbances, where an approach based on the disturbance observer

based control (DOBC) is carried out: a nonlinear observer of disturbance is presented to estimate the unknown disturbances. This is integrated with a conventional controller by using techniques based on the observation of the disturbance. (Hou et al., 2001) proposed a method of active disturbance rejection control (ADRC) which estimates the disturbance with an extended state observer. Many industrial applications use this method (Gao et al., 2001) (Zeller et al., 2001) (Jiang and Gao, 2001) and (Hamdan and Gao, 2000).

In this paper, an observer methodology is proposed to control a disturbed drone helicopter. It is based on the concept of active disturbance rejection control (ADRC). In this approach the disturbances are estimated by using an extended state observer (ESO) and are compensated for each sampling period.

In section 2, a model of a disturbed helicopter is presented. Details of the section of ADRC control are given in section 3. Section 4 presents an application of this method on our problem. Section 5 is dedicated to the zero-dynamics analysis. In section 6, several simulations of the helicopter under wind gust show the relevance of the two controls which are described in this work.

## 2 MODEL OF THE DISTURBED HELICOPTER

This section presents the nonlinear model of the disturbed helicopter (Martini et al., 2005) starting from a non disturbed model (Vilchis et al., 2003). The Lagrange equation, which describes the system of the helicopter-platform with the disturbance (see figure1), is given by:

$$M(q)\ddot{q} + C(q, \dot{q}) + G(q) = Q(q, \dot{q}, u, v_{raf}) \quad (1)$$

The input vector of the control and the state vector are respectively  $u = [u_1 \ u_2]^T$ ,  $x = [z \ \dot{z} \ \phi \ \dot{\phi} \ \gamma \ \dot{\gamma}]^T$ . The induced gust velocity is noted  $v_{raf}$ . Moreover,  $q = [z \ \phi \ \gamma]^T$ , where  $z$  represents the helicopter altitude,  $\phi$  is the yaw angle and  $\gamma$  represents the main rotor azimuth angle,  $M \in R^{3 \times 3}$  is the inertia matrix,  $C \in R^{3 \times 3}$  is the Coriolis and centrifugal forces matrix,  $G \in R^3$  represents the vector of conservative forces,  $Q(q, \dot{q}, u, v_{raf}) = [f_z \ \tau_z \ \tau_\gamma]^T$  is the vector of generalized forces. The variables  $f_z$ ,  $\tau_z$  and  $\tau_\gamma$  represent respectively, the vertical force, the yaw torque and the main rotor torque. Finally, the representation of the reduced system of the helicopter, which is subjected to a wind gust, can be written in the following state form (Martini et al., 2005):

$$\begin{aligned} \dot{x}_1 &= x_2 = \dot{z} \\ \dot{x}_2 &= \frac{1}{c_0} [c_8 \dot{\gamma}^2 u_1 + c_9 \dot{\gamma} + c_{10} - c_7] + \frac{1}{c_0} c_{16} \dot{\gamma} v_{raf} \\ \dot{x}_3 &= x_4 = \dot{\phi} \\ \dot{x}_4 &= \frac{1}{c_1 c_5 - c_4^2} [c_5 c_{11} \dot{\gamma}^2 u_2 - c_4 ((c_{12} \dot{\gamma} + c_{13} + c_8 \dot{\gamma} v_{raf}) u_1 + c_{14} \dot{\gamma}^2 + c_{15})] \\ &\quad - \frac{c_4}{c_1 c_5 - c_4^2} [2c_9 v_{raf} + c_{17} v_{raf}^2] = \ddot{\phi} \\ \dot{x}_5 &= x_6 = \dot{\gamma} \\ \dot{x}_6 &= \frac{1}{c_1 c_5 - c_4^2} [c_{11} c_4 \dot{\gamma}^2 u_2 + c_1 c_4 ((c_{12} \dot{\gamma} + c_{13} + c_8 \dot{\gamma} v_{raf}) u_1 + c_{14} \dot{\gamma}^2 + c_{15})] \\ &\quad + \frac{1}{c_1 c_5 - c_4^2} [2c_9 v_{raf} + c_{17} v_{raf}^2] = \ddot{\gamma} \end{aligned} \quad (2)$$

where  $c_i (i = 0, \dots, 17)$  are the physical constants of the model.

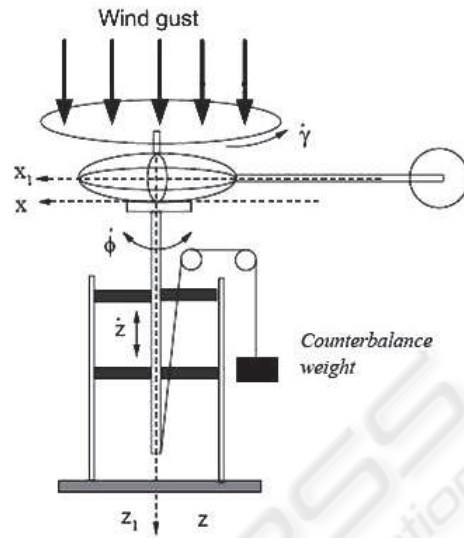


Figure 1: Helicopter-platform (Vilchis et al., 2003).

## 3 NONLINEAR EXTENDED STATE OBSERVER (NESO)

The primary reason to use the control in closed loop is that it can treat the variations and uncertainties of model dynamics and the outside unknown forces which exert influences on the behavior of the model. In this work, a methodology of generic design is proposed to treat the combination of two quantities, denoted as disturbance.

A second order system described by the following equation is considered (Gao et al., 2001)(Hou et al., 2001):

$$\ddot{y} = f(y, \dot{y}, w) + bu \quad (3)$$

where  $f(\cdot)$  represents the dynamics of the model and the disturbance,  $w$  is the input of unknown disturbance,  $u$  is the input of control, and  $y$  is the measured output. It is assumed that the value of the parameter  $b$  is given. Here  $f(\cdot)$  is a nonlinear function.

An alternative method is presented by (Han, 1999)(Han, 1995) as follows. The system in (3) is initially increased:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 + bu \\ \dot{x}_3 = \dot{f} \end{cases} \quad (4)$$

where  $x_1 = y$ ,  $x_2 = \dot{y}$ ,  $x_3 = f(y, \dot{y}, w)$ .  $f(\cdot)$  is treated as an increased state. Here  $f$  and  $\dot{f}$  are unknown. By considering  $f(y, \dot{y}, w)$  as a state, it can be estimated with a state estimator. Han in (Han, 1999) proposed a nonlinear observer for (4):

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + Bu + Lg(e, \alpha, \delta) \\ \hat{y} &= C\hat{x} \end{aligned} \quad (5)$$

where:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}, C = [1 \ 0 \ 0] \quad (6)$$

and  $L = [L_1 \ L_2 \ L_3]$ . The observer error is  $e = y - \hat{x}_1$  and:

$$g_i(e, \alpha_i, \delta)_{i=1,2,3} = \begin{cases} |e|^{\alpha_i} \text{sign}(e) & |e| > \delta \\ \frac{e}{\delta^{1-\alpha_i}} & |e| \leq \delta \end{cases} \quad \delta > 0$$

The observer is reduced to the following set of state equations, and is called extended state observer (ESO):

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + L_1 g_1(e, \alpha, \delta) \\ \dot{\hat{x}}_2 = \hat{x}_3 + L_2 g_2(e, \alpha, \delta) + bu \\ \dot{\hat{x}}_3 = L_3 g_3(e, \alpha, \delta) \end{cases} \quad (7)$$

The active disturbance rejection control (ADRC) is then defined as a method of control where the value of  $f(y, \dot{y}, w)$  is estimated in real time and is compensated by the control signal  $u$ . Since  $\hat{x}_3 \rightarrow f$ , it is used to cancel actively  $f$  by the application of:

$$u = (u_0 - \hat{x}_3)/b \quad (8)$$

This expression reduces the system to:

$$\ddot{y} = (f - \hat{x}_3) + u_0 \approx u_0 \quad (9)$$

The process is now a double integrator with a unity gain, which can be controlled with a PD controller:

$$u_0 = k_p(r - \hat{x}_1) - k_d \hat{x}_2 \quad (10)$$

where  $r$  is the reference input. The observer gains  $L_i$  and the controller gains  $k_p$  and  $k_d$  can be calculated by a pole placement. The configuration of ADRC is presented in figure2 :

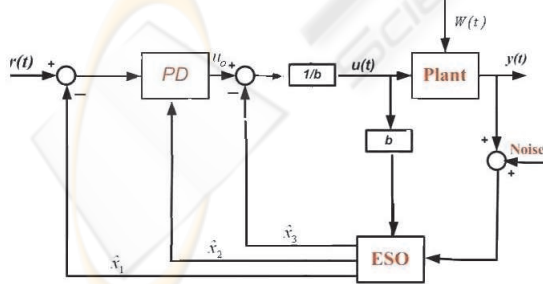


Figure 2: ADRC structure.

## 4 CONTROL OF DISTURBED HELICOPTER

### 4.1 Control by Nonlinear Feedback

Firstly, the nonlinear terms of the non disturbed model ( $v_{raf} = 0$ ) are compensated by introducing two new controls  $V_1$  and  $V_2$  such as(see Fig.3):

$$\begin{aligned} u_1 &= \frac{1}{c_8 \dot{\gamma}^2} [c_0 V_1 - c_9 \dot{\gamma} - c_{10} + c_7] \\ u_2 &= \frac{1}{c_5 c_{11} \dot{\gamma}^2} [(c_1 c_5 - c_4^2) V_2 + c_4 ((c_{12} \dot{\gamma} + c_{13}) u_1 + c_{14} \dot{\gamma}^2 + c_{15})]. \end{aligned} \quad (11)$$

By using the above controls  $V_1$  and  $V_2$ , for  $v_{raf} = 0$ , an uncoupled linear system is obtained which is represented by two equations:  $\ddot{z} = V_1$ ,  $\ddot{\phi} = V_2$ . Stabilization is carried out by a pole placement. To regulate altitude  $z$  and the yaw angle  $\phi$ , a PID controller is proposed:

$$\begin{aligned} V_1 &= -a_1 \ddot{z} - a_2 (\dot{z} - \dot{z}_d) - a_3 \int_0^t (z - z_d) dt \\ V_2 &= -a_4 \ddot{\phi} - a_5 (\dot{\phi} - \dot{\phi}_d) - a_6 \int_0^t (\phi - \phi_d) dt \end{aligned} \quad (12)$$

where  $z_d$  and  $\phi_d$  are the desired trajectories. The parameters of regulation were calculated using two dominant poles in closed loop such as:

$$z \begin{cases} \omega_z = 2 \text{rad/s} \\ \xi_z = 1 \end{cases} \quad \text{and} \quad \phi \begin{cases} \omega_\phi = 5 \text{rad/s} \\ \xi_\phi = 1 \end{cases} \quad (13)$$

where  $\omega_z$ ,  $\omega_\phi$  are the natural frequencies, and  $\xi_z, \xi_\phi$  are the damping ratios for the pole placement. These integral controllers are used to eliminate the effect of low frequency disturbance.

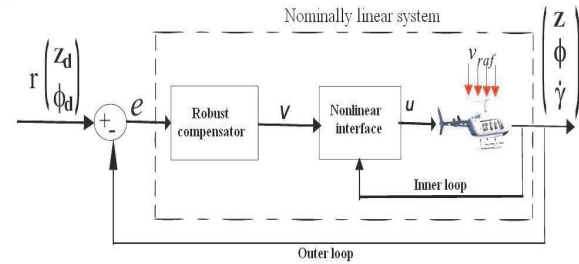


Figure 3: Architecture of nonlinear feedback control.

### 4.2 Active Disturbance Rejection Control (ADRC)

Since  $v_{raf} \neq 0$ , a nonlinear system of equations is obtained:

$$\begin{aligned} \ddot{z} &= V_1 + \frac{1}{c_0} c_{16} \dot{\gamma} v_{raf} \\ \ddot{\phi} &= V_2 - \frac{c_4 c_0 v_{raf}}{(c_1 c_5 - c_4^2) \dot{\gamma}} V_1 - \frac{c_4 v_{raf}}{c_1 c_5 - c_4^2} \left[ \frac{c_7 - c_{10}}{\dot{\gamma}} + c_9 + c_{17} v_{raf} \right] \end{aligned} \quad (14)$$

The stabilization is always done by pole placement. To regulate altitude  $z$  and the yaw angle  $\phi$ , we can notice that (14) represent two second order systems which can be written as in (3):

$$\ddot{y} = f(y, \dot{y}, w) + bu \quad (15)$$

with  $b = 1, u = V_1$  or  $V_2$  and:

$$\begin{aligned} f_z(y, \dot{y}, w) &= \frac{1}{c_0} c_{16} \dot{\gamma} v_{raf} \\ f_\phi(y, \dot{y}, w) &= -\frac{c_4 c_0 v_{raf}}{(c_1 c_5 - c_4^2) \dot{\gamma}} V_1 - \frac{c_4 v_{raf}}{c_1 c_5 - c_4^2} \left[ \frac{c_7 - c_{10}}{\dot{\gamma}} + \right. \\ &\quad \left. + c_9 + c_{17} v_{raf} \right] \end{aligned}$$

For each control, an observer is built using (7):

- for altitude  $z$ :

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + L_1 g_1(e_z, \alpha_1, \delta_1) \\ \dot{\hat{x}}_2 = \hat{x}_3 + L_2 g_2(e_z, \alpha_2, \delta_2) + bV_1 \\ \dot{\hat{x}}_3 = L_3 g_3(e_z, \alpha_3, \delta_3) \end{cases} \quad (16)$$

where  $e_z = z - \hat{x}_1$  is the observer error,  $g_i(e_i, \alpha_i, \delta_i)$  is defined as exponential function of modified gain.

$$g_i(e_z, \alpha_{iz}, \delta_i)_{i=1,2,3} = \begin{cases} |e_z|^{\alpha_{iz}} \text{sign}(e_z), & |e_z| > \delta_i \\ \frac{e_z}{\delta_i^{1-\alpha_{iz}}}, & |e_z| \leq \delta_i \end{cases}$$

with  $0 < \alpha_i < 1$  and  $0 < \delta_i$ , a *PID* controller is used in stead of *PD*(10) to attenuate the effects of disturbance:

$$V_1 = -k_1 \hat{x}_2 - k_2 (\hat{x}_1 - z_d) - k_3 \int_0^t (\hat{x}_1 - z_d) dt - \hat{x}_3 \quad (17)$$

The control signal  $V_1$  takes into account the terms which depend on the observer  $(\hat{x}_1, \hat{x}_2)$ . The fourth part, which also comes from the observer, is added to eliminate the effect of disturbance in this system.

- for the yaw angle  $\phi$ :

$$\begin{cases} \dot{\hat{x}}_4 = \hat{x}_5 + L_4 g_4(e_\phi, \alpha_4, \delta_4) \\ \dot{\hat{x}}_5 = \hat{x}_6 + L_5 g_5(e_\phi, \alpha_5, \delta_5) + bV_2 \\ \dot{\hat{x}}_6 = L_6 g_6(e_\phi, \alpha_6, \delta_6) \end{cases} \quad (18)$$

where  $e_\phi = \phi - \hat{x}_4$  is the observer error, with  $g_i(e_\phi, \alpha_{i\phi}, \delta_i)$  is defined as exponential function of modified gain:

$$g_i(e_\phi, \alpha_{i\phi}, \delta_i)_{i=4,5,6} = \begin{cases} |e_\phi|^{\alpha_{i\phi}} \text{sign}(e_\phi), & |e_\phi| > \delta_i \\ \frac{e_\phi}{\delta_i^{1-\alpha_{i\phi}}}, & |e_\phi| \leq \delta_i \end{cases}$$

$$V_2 = -k_5 \hat{x}_4 - k_4 (\hat{x}_5 - \phi_d) - k_6 \int_0^t (\hat{x}_4 - \phi_d) dt - \hat{x}_6 \quad (19)$$

$z_d$  and  $\phi_d$  are the desired trajectories. *PID* parameters are designed to obtain two dominant poles in closed-loop:

$$\text{for } z \begin{cases} \omega_{c1} = 2 \text{rad/s} \\ \xi_1 = 1 \end{cases} \quad \text{and for } \phi \begin{cases} \omega_{c2} = 5 \text{rad/s} \\ \xi_2 = 1 \end{cases} \quad (20)$$

## 5 ZERO DYNAMICS PROBLEM

The zero dynamics of a nonlinear system are its internal dynamics subject to the constraint that the outputs (and, therefore, all their derivatives) are set to zero for all times (Isidori, 1995)(Slotine and Li, 1991). Nonlinear systems with nonasymptotically stable zero-dynamics are called strictly (or weakly, if the zero dynamics are marginally stable) nonminimum phase system. The output of our system is  $q = [z \ \phi]^T$  and its control input  $u = [u_1 \ u_2]^T$ . The calculation of the relative degrees gives:  $r_1 = r_2 = 2$ . The dimension of our model  $n = 5$  so that:  $r_1 + r_2 < n$  what implies the existence of an internal dynamics. If a linearizable feedback is used, it is necessary to check the stability of this internal dynamics. In fact the  $\dot{\gamma}$  dynamics represents the zeros-dynamics of (2). Moreover the nonlinear terms of the non disturbed model ( $v_{raf} = 0$ ) can be compensated by introducing two new controls  $V_1$  and  $V_2$ . Since  $v_{raf} = 0$ , a nonlinear system of equations is then obtained:

$$\begin{aligned} \ddot{z} &= V_1; \quad \ddot{\phi} = V_2 \\ \dot{\gamma} &= \frac{1}{c_1 c_5 - c_4^2} [b_1 V_1 + b_2 V_2 + b_3] \end{aligned} \quad (21)$$

Where:

$$\begin{aligned} b_1 &= \frac{c_4 c_0}{c_5 c_8 \dot{\gamma}^2} (c_{12} \dot{\gamma} + c_{13})(c_1 c_5 + c_4) \\ b_2 &= \frac{c_4}{c_5} (c_1 c_5 - c_4^2) \\ b_3 &= \frac{c_4}{c_5} (c_1 c_5 + c_4) [(c_{12} \dot{\gamma} + c_{13})(-c_9 \dot{\gamma} - \\ &\quad c_{10} + c_7) \times \frac{1}{c_8 \dot{\gamma}^2} + c_{14} \dot{\gamma}^2 + c_{15}] \end{aligned} \quad (22)$$

Zero dynamics of nondisturbed model can then obviously be got by putting  $z = \phi = 0 \Rightarrow \dot{z} = \dot{\phi} = 0 \Rightarrow \ddot{z} = \ddot{\phi} = 0 \Rightarrow V_1 = V_2 = 0$ :

$$\ddot{\gamma} = \frac{1}{c_1 c_5 - c_4^2} b_3 \quad (23)$$

Simplified in:

$$\ddot{\gamma} = b_4 \dot{\gamma}^2 + \frac{b_5}{\dot{\gamma}^2} + \frac{b_6}{\dot{\gamma}} + b_7 \quad (24)$$

With:  $b_4 = 4.1425 \times 10^{-5}$ ,  $b_5 = -778300$ ,  $b_6 = -6142$  and  $b_7 = 0.1814$ . To get possible equilibrium points dynamics of (24), the following equation is solved:

$$b_4 \dot{\gamma}^4 + b_7 \dot{\gamma}^2 + b_6 \dot{\gamma} + b_5 = 0 \quad (25)$$

The four solutions of (25) are  $\dot{\gamma}^* = -219.5 \pm 468.2i$ ,  $563.71$  and  $-124.6 \text{rad/s}$ . Only the two last values of  $\dot{\gamma}^*$  have physical meaning for the system. On the other hand, the value  $\dot{\gamma}^* = 563.7 \text{rad/s}$  is too high regarding the blade fragility. As a result, the only

equilibrium point to consider is  $\dot{\gamma}^* = -124.6 \text{ rad/s}$ . The  $\dot{\gamma}$ -dynamics(24), linearized around the equilibrium point of interest  $\dot{\gamma}^* = -124.634 \text{ rad/s}$ , has a real eigenvalue equal to  $-0.419$ . As a consequence, all trajectories starting sufficiently near  $\dot{\gamma}^*$  converge to the latter. Then it follows that the zeros-dynamics of (21) has a stable behavior. Simulation results show that  $\dot{\gamma}$  remains bounded away from zero during the flight. For the chosen trajectories and gains  $\dot{\gamma}$  converges rapidly to a constant value(see Figure6). This is an interesting point to note since it shows that the dynamics and feedback control yield flight conditions close to the ones of real helicopters which fly with a constant  $\dot{\gamma}$  thanks to a local regulation feedback of the main rotor speed (which does not exist on the *VARIO* scale model helicopter).

## 6 RESULTS IN SIMULATION

To show the efficiency of active disturbance rejection control (*ADRC*), it is compared to the nonlinear control, which uses a *PID* controller. The various numerical values are the following:

### 6.1 Control by Nonlinear Feedback

We have  $a_1 = 24$ ,  $a_2 = 84$ ,  $a_3 = 80$ ,  $a_4 = 60$ ,  $a_5 = 525$  and  $a_6 = 1250$ , the numerical values are calculating by pole placement as defined in (13).

### 6.2 Active Disturbance Rejection Control (*ADRC*)

- For  $z$ :

$k_1 = 24$ ,  $k_2 = 84$ ,  $k_3 = 80$  (the numerical values are calculating by pole placement as defined in (20)). Choosing a triple pole located in  $\omega_{0z}$  such as  $\omega_{0z} = (3 \sim 5)\omega_{c_1}$ , one can choose  $\omega_{0z} = 10 \text{ rad/s}$ ,  $\alpha_1 = 0.5$ ,  $\delta_1 = 0.1$ . Using pole placement method, the gains of the observer for the case  $|e| \leq \delta$  (i.e linear observer) can be evaluated:

$$\begin{aligned} \frac{L_1}{\delta_1^{1-\alpha_1}} &= 3\omega_{0z} \\ \frac{L_2}{\delta_1^{1-\alpha_1}} &= 3\omega_{0z}^2 \\ \frac{L_3}{\delta_1^{1-\alpha_1}} &= \omega_{0z}^3 \end{aligned} \quad (26)$$

which leads to:  $L_i = \{9.5, 95, 316\}$ ,  $i \in [1, 2, 3]$

- For  $\phi$ :

$k_4 = 60$ ,  $k_5 = 525$ ,  $k_6 = 1250$  and  $\omega_{0\phi} = 25 \text{ rad/s}$ ,  $\alpha_2 = 0.5$ ,  $\delta_2 = 0.025$ . And by the same

method in (26) one can find the observer gains:  $L_i = \{11.9, 296.5, 2470\}$ ,  $i \in [4, 5, 6]$

For the (*ADRC*) control, one can show that if the gains of the observer  $L_{i=1,2,3,4,5,6}$  are too large, the convergence of  $\hat{x}_{i=1,2,3,4,5,6}$  to the following values  $(z, \dot{z}, \ddot{z}, \phi, \dot{\phi}, \ddot{\phi})$  is very fast but the robustness against the noises quickly deteriorated. By choosing  $L_{i+1} \gg L_i$  ( $i = 1, 2, \dots, 6$ ), higher order observer state  $\hat{x}_{i+1}$  will converge to the actual value more quickly than lower order state  $\hat{x}_i$ . Therefore, the stability of *ADRC* system will be guaranteed.  $\delta$  is the width of linear area in the nonlinear function *ADRC*. It plays an important role to the dynamic performance of *ADRC*. The larger  $\delta$  is, the wider the linear area. But if  $\delta$  is too large, the benefit of nonlinear characteristics would be lost. On the other hand, if  $\delta$  is too small, then high frequency chattering will happen just the same as in the sliding mode control. Generally, in *ADRC*,  $\delta$  is set to be approximately 10% of the variation range of its input signal.  $\alpha$  is the exponent of

tracking error. The smaller  $\alpha$  is, the faster the tracking speed is, but the more calculation time is needed. In addition, very small  $\alpha$  will cause chattering. In reality, selecting  $\alpha = 0.5$  will provide a satisfactory result. The induced gust velocity operating on the main rotor is chosen as (G.D.Padfield, 1996):

$$v_{raf} = v_{gm} \sin\left(\frac{2\pi V t}{L_u}\right) \quad (27)$$

where  $V$  in  $m/s$  is the rise speed of the helicopter and  $v_{gm} = 0.68 \text{ m/s}$  is the gust density. This density corresponds to an average wind gust, and  $L_u = 1.5 \text{ m}$  is its length (see Figure8). In simulation the gust is applied at  $t=80\text{s}$ .

A band limited white noise of covariance  $2 \times 10^{-8} \text{ m}^2$  for  $z$  and  $2 \times 10^{-9} \text{ rad}^2$  for  $\phi$ , has been added equally to the measurements of  $z$  and  $\phi$  for the two controls. The compensation of this noise is done by using a Butterworth second-order low-pass filter. Its crossover frequency for  $z$  is  $\omega_{cz} = 10 \text{ rad/s}$  and for  $\phi$  is  $\omega_{c\phi} = 25 \text{ rad/s}$ .

The parameters used for 3DOF standard helicopter model are based on a *VARIO 23cc* small helicopter(see figure 1).

Figure4 shows the desired trajectories. Figure7 illustrates the variations of control inputs, where from initial conditions when  $\|\dot{\gamma}\|$  increases quickly, the control output  $u_1$  and  $u_2$  saturates. Nevertheless the stability of the closed-loop system is not destroyed.

One can observe that  $\dot{\gamma} \rightarrow -124.6 \text{ rad/s}$  as expected from the previous zero dynamics analysis. One can also notice that the main rotor angular speed is similar for the two controls as illustrated in Figure6.

The difference between the two controls appears in Figure5 where the tracking errors are less significant by using the *PID (ADRC)* control than *PID* controller. One can see in Figure8 that the main rotor thrust converges to values that compensate the helicopter weight, the drag force and the effect of the disturbance on the helicopter.

The simulations show that the control by nonlinear feedback *PID (ADRC)* is more effective than nonlinear *PID* controller, i.e. the tracking errors are less significant by using the first control. But the *PID (ADRC)* control is a little more sensitive to noise than *PID* controller. Moreover, under the effect of noise, the second control allows the main rotor thrust  $T_m$  to be less away from its balance position than the first control (Figure8). Figure9 represent the effectiveness of the observer:  $\hat{x}_3$  and  $f_z(y, \dot{y}, w)$ , are very close and also  $\hat{x}_6$  and  $f_\phi(y, \dot{y}, w)$ . Observer errors are presented in the Figure10. By tacking a large disturbance ( $v_{raf} = 3m/s$ ), the *ADRC* control shows a robust behavior compared to the nonlinear *PID* control as illustrated in Figure11.

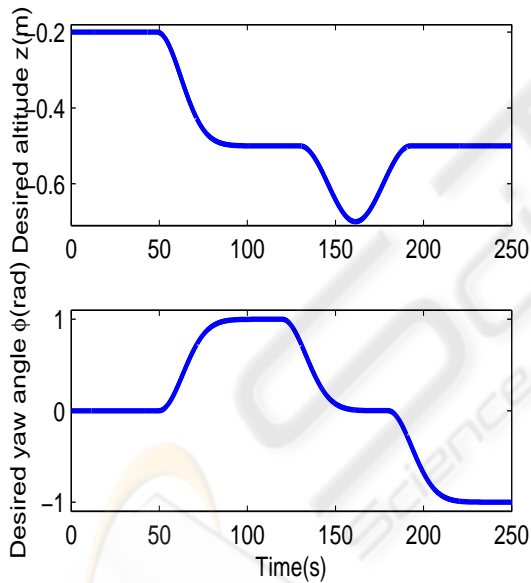


Figure 4: The desired trajectories in  $z$  and  $\phi$ .

## 7 CONCLUSION

In this paper, the active disturbance rejection control (*ADRC*) has been applied to the drone helicopter control. The basis of *ADRC* is the extended state observer. The state estimation and compensation of the change of helicopter parameters and disturbance variations are implemented by *ESO* and *NESO*. By using

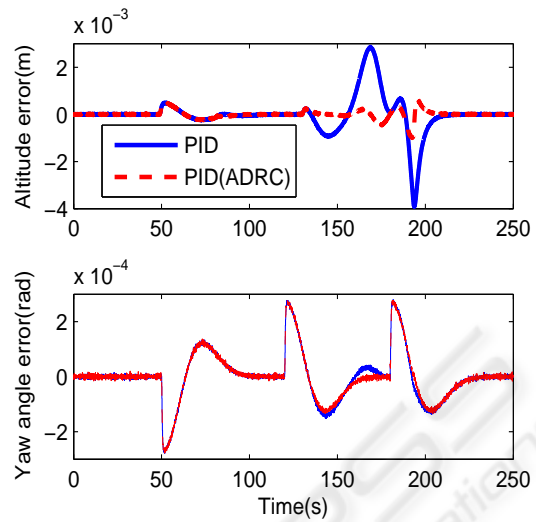


Figure 5: Tracking error in  $z$  and  $\phi$ .

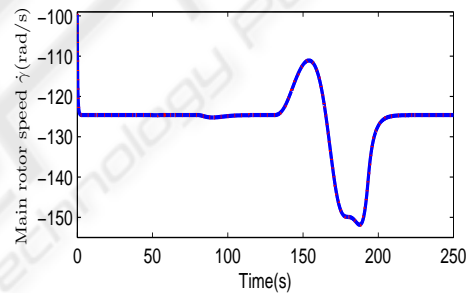


Figure 6: Variations of the main rotor angular speed  $\dot{\gamma}$ .

*ESO*, the complete decoupling of the helicopter is obtained. The major advantage of the proposed method is that the closed loop characteristics of the helicopter system do not depend on the exact mathematical model of the system. Comparisons were made in detail between *ADRC* and conventional nonlinear *PID* controller. It is concluded that the proposed control algorithm produces better dynamic performance than the nonlinear *PID* controller. Even for large disturbance  $v_{raf} = 3m/s$ (figure11), the proposed *ADRC* control system is robust against the modeling uncertainty and the external disturbance in various operating conditions. It is indicated that such scheme can be applicable to aeronautical applications where high dynamic performance is required. We note that the next step will be the validation of this study on the real helicopter model VARIO 23cc.

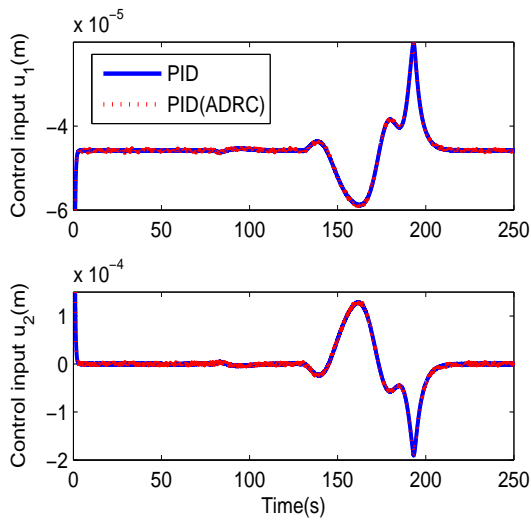


Figure 7: The control inputs  $u_1$  and  $u_2$ .

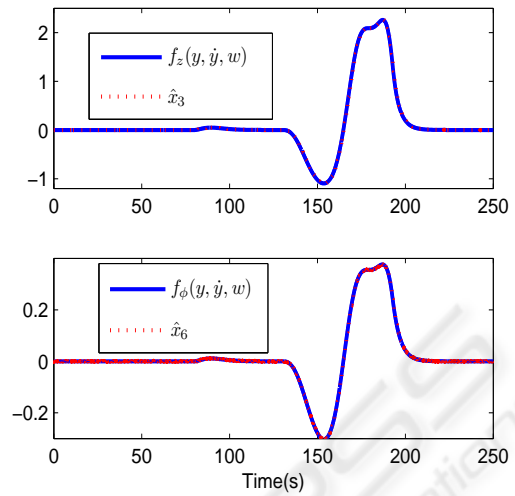


Figure 9: Estimation of  $f_z$  and  $f_\phi$ .

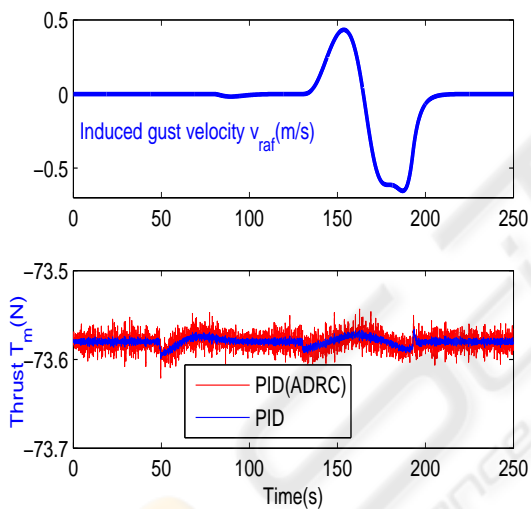


Figure 8: The induced gust velocity  $v_{raf}$  and the variations of the main rotor thrust  $T_M$ .

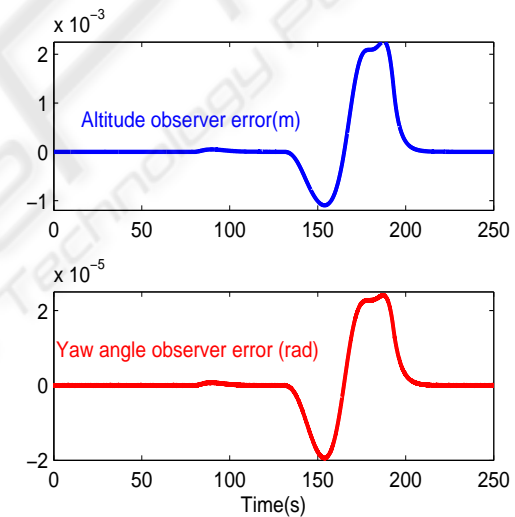


Figure 10: Observer error in  $z$  and  $\phi$ .

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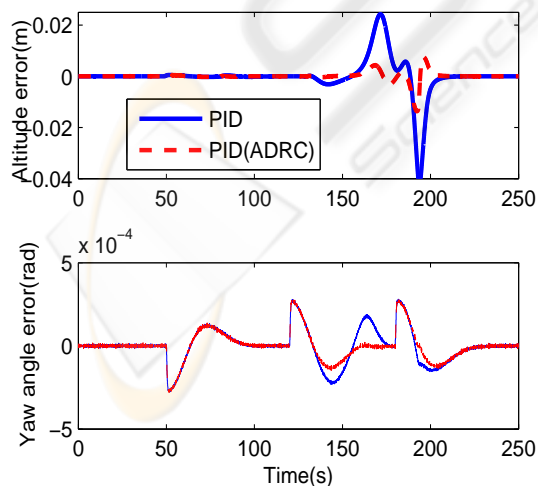


Figure 11: Test of a large disturbance  $v_{raf} = 3m/s$ .