MPC FOR SYSTEMS WITH VARIABLE TIME-DELAY
Robust Positive Invariant Set Approximations

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Abstract: This paper deals with the control design for systems subject to constraints and affected by variable time-delay. The starting point is the construction of a predictive control law which guarantees the existence of a nonempty robust positive invariant (RPI) set with respect to the closed loop dynamics. In a second stage, an iterative algorithm is proposed in order to obtain an approximation of the maximal robust positive invariant set. The problem can be treated in the framework of piecewise affine systems due to the explicit formulations of the control law obtained via multiparametric programming.

1 INTRODUCTION

The delays (constant or time-varying, distributed or not) describe coupling between the dynamics, propagation and transport phenomena, heredity and competition in population dynamics. Various motivating examples and related discussions can be found in (Niculescu, 2001), (Michiels and Niculescu, 2007). There is an consensus in defining *delay* as a *critical parameter* in understanding dynamics behavior and/or improving (overall) system's performances. Independently of the mathematical problems related to the appropriate representation of such dynamics, the delay systems are known to rise challenging control problems due to the instabilities introduced by the deferred input actions. One of the natural ways to counteracting the effects of dead-time is to predict the system evolution but particular care has to be shown to the sensitivity of predictions for unstable models.

MPC - "Model Predictive Control" is a popular control technique based on the resolution of a finite-time optimal control problem over a receding horizon. Several strategies were proposed in order to reinforce the MPC stability (Maciejowski, 2002; Goodwin et al., 2004) having as main ingredients the terminal cost functions and the positive invariant terminal constraints (Mayne et al., 2000). Unfortunately, considering similar uncertainty interpretation in the context of time-varying delays lead to complex min-max optimization problems, difficult to handle on-line.

The present paper proposes an alternative issue for handling such a control problem. More precisely, we propose to use a simple MPC design constructed upon the nominal prediction. The resulting piecewise affine control law will transform the closed-loop dynamic in a piecewise affine system, the variable delay inducing in fact a model uncertainty. The existence of a non-empty positive invariant set can be guaranteed under mild conditions. Two problems related to the robustness of the designed control law will be dealt in detail:

- tuning the nominal MPC using inverse optimality;
- characterisation of the maximal robust positive invariant (MRPI) set.

It should be noted that the MRPI set may not be finitely determined and the second point will iteratively construct a dual expansive/contractive procedure for providing an inner approximation.

The paper is organized as follows: section 2 formulates the control problem and defines the models to be further used in the MPC design; section 3 deals with the construction of the explicit piecewise affine control law and section 4 details the approximation of the maximal robustly positive invariant set for the closed-loop system. Finally section 5 presents an example whereas section 6 draws the conclusions.

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2 PROBLEM FORMULATION

Consider a linear continuous time system:

$$\dot{x} = A_c x(t) + B_c u(t-\tau) \tag{1}$$

affected by a variable time delay $\tau \in [0, \tau_{max}]$.

Note the discrete time instants $x_k = x(t_k) = x(kT_e)$ where T_e is the sampling time. Consider:

$$\begin{aligned} d &= \lceil \bar{\tau}/T_e \rceil \\ \varepsilon &= dT_e - \bar{\tau} \end{aligned} \tag{2}$$

where $\bar{\tau}$ is the "probable" value of the delay. The nominal discrete time LTI model is:

$$x_{k+1} = Ax_k + Bu_{k-d} - \bar{\Delta}(u_{k-d} - u_{k-d+1}) \tag{3}$$

The matrices $A, B, \bar{\Delta}$ are given by:

$$A = e^{A_c T_e} \tag{4}$$

$$B = \int_0^{T_e} e^{A_c(T_e-\theta)} B_c d\theta \tag{5}$$

$$\bar{\Delta} = \int_{-\varepsilon}^{0} e^{-A_c\theta} B_c d\theta \tag{6}$$

obtained by assuming that the control action u is maintained constant $u(t) = u_k, \forall t \in [t_k, t_{k+1})$.

Despite this nominal model 3, in the general case, the variable time-delay implies a variable limit for the integration for ε. By considering Δ as a matrix affected by polytopic uncertainty (corresponding to the variation $0 \leq \varepsilon \leq T_e$) one can obtain the embedding:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_{k-v} - \Delta(u_{k-v} - u_{k-v+1}) \\ \Delta &\in Co\{\Delta_0, \Delta_1, \ldots, \Delta_n\} \\ & v \in \{0, 1, 2, \ldots, h\} \end{aligned} \tag{7}$$

where the maximum value of discrete delay is:

$$h = \left\lceil \frac{\tau_{max}}{T_e} \right\rceil \tag{8}$$

a set of $n+1$ vertices Δ_i are obtained using the Jordan form of A_c (Olaru and Niculescu, 2008).

Using an extended state space representation based on the equation (3), one can obtain the nominal prediction model:

$$\xi_{k+1} = \bar{F}\xi_k + \bar{G}u_k \tag{9}$$

.

For the same state vector ξ_k, by using A, B and the polytopic embedding for Δ with the extreme realizations $\Delta_i, i = \{0, \ldots, n\}$ in the extended state space, the polytopic model can be described:

$$\begin{aligned} \xi_{k+1} &= F\xi_k + Gu_k \\ (F, G) &\in Co\{(F_1, G_1), \ldots, (F_s, G_s)\} \end{aligned} \tag{10}$$

with $s = nh + 1$.

The system evolution has to satisfy physical limitations leading to a set of linear inequalities:

$$C\xi_k \leq W \tag{11}$$

The control objective is the regulation of the state ξ_k to origin while satisfying the constraints using a receding horizon optimal control approach.

3 EXPLICIT CONTROL DESIGN: ROBUSTNESS ISSUE

3.1 Predictive Control

A standard MPC strategy, for the delay system considered here, will construct at each sampling instant k the optimal control sequence:

$$\mathbf{k}_u^* = \{u_{k|k}, \ldots, u_{k+N-d-1|k}\} \tag{12}$$

with respect to a *performance index* which evaluates the system dynamics over a finite horizon $k+1, \ldots, k+N$. As a basic remark, the prediction horizon has to be larger than the delay $N \geq d$ in order to have an effective measure of its effect at the system output. Knowing that the prediction is constructed upon the nominal model but the real system may be affected by delays up to h samples, it will be considered that $N \geq h$ in order to cope with all the possible variations.

The first component of $\mathbf{k}_u^*$ is effectively applied as control action to the system:

$$u_k = \mathbf{k}_u^*(1) = u_{k|k} \tag{13}$$

while the tail is discarded. Using the new measurements the optimisation procedure is restarted, thus obtaining a closed-loop control scheme.

The most popular performance index has a quadratic form and commensurate the state (tracking error) trajectory and the associated control effort. If the admissible trajectories are described by constraints as in (11), the MPC implementation passes by the resolution of the optimisation problem of the form:

$$\mathbf{k}_u^* = \arg\min_{\{u_{k|k}, \ldots, u_{k+N-d-1|k}\}} \left\{ \xi_{k+N|k}{}^T \bar{P} \xi_{k+N|k} + \sum_{j=1}^{N} \xi_{k+j|k}{}^T \bar{Q} \xi_{k+j|k} + \sum_{j=0}^{N-d-1} u_{k+j|k}^T \bar{R} u_{k+j|k} \right\} \tag{14}$$

subject to:

$$\begin{cases} \xi_{k+j+1|k} = \bar{F}\xi_{k+j|k} + \bar{G}u_{k+j|k} \\ C\xi_{k+j|k} \leq W; \ j = 1, \ldots, N-1 \\ u_{k+i|k} = 0, \quad i = N-d, \ldots, N-1 \\ \xi_{k+N} \in X_N; \end{cases}$$

The construction of the predictive control law will be influenced by the choice of the prediction horizon N, the weighting factors on the state trajectory $\bar{Q} = \bar{Q}^T \succ 0$ and the control effort, $\bar{R} = \bar{R}^T \succeq 0$. For the penalty on the terminal state the matrix $\bar{P}$ is usually constructed such that the prediction horizon to be extended to infinity by the introduction of the term

${\xi_{k+N|k}}^T \bar{P} \xi_{k+N|k}$ in (14). However $\xi_{k+N|k}$ has to satisfy some mild conditions materialized by the terminal constraint which force this prediction to reach the predefined invariant set X_N. The usual choice in this sense ((Gilbert and Tan, 1991)) is the maximal output admissible set $X_N = O_\infty$ constructed for the system (9) with the optimal control satisfying the discrete algebraic Riccati equation:

$$\begin{aligned} \bar{P} &= \bar{Q} + \bar{F}^T \bar{P} \bar{F} - \bar{K}^T \left(\bar{R} + \bar{G}^T \bar{P} G\right) \bar{K} \\ \bar{K} &= -\left(\bar{R} + \bar{G}^T \bar{P} G\right)^{-1} \bar{G}^T F \end{aligned} \tag{15}$$

This is the classical design for the MPC law. In the subsection 3.3, the choice of the performance index will be discussed (in particular the matrices $\bar{Q}, \bar{R}$ and indirectly $\bar{P}$) such that the resulting control law to present a certain degree of robustness with respect to the variable delay.

3.2 Multiparametric Programming

After expressing the predictions as functions of the current state and the future control action, the optimisation problem in (14) can be reformulated as a multiparametric quadratic problem ((Bemporad et al., 2002),(Goodwin et al., 2004),(Dua et al., 2007), (Olaru and Dumur, 2005))

$$\begin{aligned} &\mathbf{k}_u^*(\xi_k) = \arg\min_{\mathbf{k}_u} 0.5 \mathbf{k}_u^T H \mathbf{k}_u + \mathbf{k}_u^T G \xi_k \\ &\text{subject to}: \quad A_{in} \mathbf{k}_u \leq b_{in} + B_{in} \xi_k \end{aligned} \tag{16}$$

where the vector ξ_k plays the role of parameter.

Further, explicit solutions for the MPC law can be obtained by retaining the first component of $\mathbf{k}_u^*(\xi_k)$, thus expressing the predictive control in terms of a piecewise affine feedback law:

$$u_k = K_i^{MPC} \xi + \kappa_i^{MPC}, \quad \text{with} \quad i \text{ s.t. } x \in D_i, \tag{17}$$

for D_i, polyhedral regions in $\Re^{n+hm}$.

Remark 1. *The prediction model is linear, the origin is a feasible point (in the most cases placed on the interior of the feasible domain) and thus represents an equilibrium point for the system (9). The problems (14), and further (16), are feasible and more then that, the associated optimum will be unconstrained.*

The consequence is that the affine control law corresponding to the region D_{i_0} containing the origin ($0 \in D_{i_0}$) is in fact a linear feedback ($\kappa_{i_0}^{MPC} = 0$) and it corresponds to the unconstrained optimal control law ($K_{i_0}^{MPC} = K_{LG}$ if $\bar{P}$ is build upon (15)). If the constraints are symmetric, the region containing the origin will be the central region of the partition (the symmetry is inherited in the polyhedral decomposition of the state space).

3.3 Tuning MPC for Robustness

Consider an infinite-horizon min-max control problem for the polytopic system (10):

$$\min_K \quad \max_{F \in \Omega_\xi} \sum_{i=0}^{\infty} \xi_{k+i}^T Q \xi_{k+i} + u_{k+i}^T R u_{k+i} \tag{18}$$

$$u_k = K \xi_k \tag{19}$$

where $Q > 0$, $R > 0$ are suitable weighting matrices fixed *a priori* and K, the feedback gain playing in fact the role of the optimization argument.

Consider a quadratic function of the state

$$V(\xi) = \xi^T P \xi, \quad P > 0 \tag{20}$$

which represents an upper bound for J_∞ if the following inequality is satisfied $\forall F \in \Omega_\xi$:

$$V(\xi_{k+i+1}) - V(\xi_{k+i}) \leq -[\xi_{k+i}^T Q \xi_{k+i} + u_{k+i}^T R u_{k+i}] \tag{21}$$

Rewriting this equation using (19) the following inequality is obtained:

$$\begin{aligned} \xi_{k+i}^T [(F+GK)^T P (F+GK) \\ -P + K^T R K + Q] \xi_{k+i} \leq 0 \end{aligned} \tag{22}$$

or equivalently:

$$(F+GK)^T P (F+GK) - P + K^T R K + Q \leq 0 \tag{23}$$

Using the ideas in (Boyd et al., 1994), by noting $P = GS^{-1}$ and $Y = KS$, for $S \geq I$, the following LMI can be constructed:

$$\begin{bmatrix} S & SF^T + Y^T G^T & SQ^{1/2} & Y^T R^{1/2} \\ FS + GY & S & 0 & 0 \\ Q^{1/2} S & 0 & GI & 0 \\ R^{1/2} Y & 0 & 0 & GI \end{bmatrix} \succ 0, \tag{24}$$

Using now the fact that $F \in \Omega_\xi$, a stabilizing control law is given by $K = YS^{-1}$ where Y, S and the scalar G are the solutions of the LMI problem (similar with the construction in (Kothare et al., 1996)):

$$\begin{gathered} \min_{G,S,Y} G \\ \begin{bmatrix} S & SF_i^T + Y^T G_i^T & SQ^{1/2} & Y^T R^{1/2} \\ F_i S + G_i Y & S & 0 & 0 \\ Q^{1/2} S & 0 & GI & 0 \\ R^{1/2} Y & 0 & 0 & GI \end{bmatrix} \succ 0, \\ \text{for all} \quad i = 0, \ldots, s \\ S \geq I \end{gathered} \tag{25}$$

Remark 2. *This LMI based procedure is used in (Kothare et al., 1996) to design a robust MPC law. The LMI in (25) is not depending on the measured state and thus the resulting control law is represented by a fixed feedback control gain.*

The resulting law $u_k = Kx_k$ represents a robust stabilizing control in the unconstrained case. In the sequel, the idea is to use this information when tuning the nominal MPC parameters in (14), namely Q, R and P. We start with the remark that the MPC law is a piecewise affine function of the state and the central region (or the region containing the origin, if the constraints are not symmetric) is characterized by the unconstrained optimum for the chosen performance index in (14). Constructing this performance index such that the optimal solution corresponds to the LQ solution ($K = YS^{-1} \leftrightarrow K_{LQ}$) can be seen as an inverse optimality problem (Kalman, 1964).

Roughly speaking the tuning procedure is the following: given the matrices $\bar{F}, \bar{G}$ and Y, S from (25), the matrices $\bar{Q} \geq 0$ and $\bar{R} > 0$ (and indirectly $\bar{P} \geq 0$) will be constructed such that the optimal solution to the unconstrained problem (14) to be:

$$\mathbf{k}_u^* = \begin{bmatrix} YS^{-1} \\ YS^{-1}(\bar{F}+\bar{G}YS^{-1}) \\ \vdots \\ YS^{-1}(\bar{F}+\bar{G}YS^{-1})^{N-1} \end{bmatrix} \xi_k \qquad (26)$$

The (not unique) pair $(\bar{Q}, \bar{R})$ has to satisfy:

$$\bar{Q} = \bar{P} - \bar{F}^T\bar{P}\bar{F} + \left\{YS^{-1}\right\}^T(\bar{R}+\bar{G}^T\bar{P}\bar{G})YS^{-1} \qquad (27)$$

$$\bar{R}YS^{-1} + \bar{G}^T\bar{P}\bar{G}YS^{-1} + \bar{G}^T\bar{P}\bar{F} = 0 \qquad (28)$$

This problem can be solved in the general case by employing an LMI formulation (Larin, 2003):

$$\begin{gathered} \min \alpha \\ \bar{P} - \bar{F}^T\bar{P}\bar{F} + \left\{YS^{-1}\right\}^T(\bar{R}+\bar{G}^T\bar{P}\bar{G})YS^{-1} \succ 0 \\ \begin{bmatrix} Z & \bar{R}YS^{-1} + B^T\bar{P}BYS^{-1} + B^T\bar{P}A \\ * & I \end{bmatrix} \succ 0 \\ Z \prec \alpha I, \quad \bar{P} \succ 0 \end{gathered} \qquad (29)$$

Theorem 1. *The nominal MPC control law, designed upon a performance index obtained by inverse optimality with respect to an unconstrained robust linear feedback, is robustly stabilizing the system (10) despite of constraints on a nondegenerate neighborhood of the origin V.*

Proof: The proof is constructive and follows the arguments described in this section. Using the LMI formulation (25), a robustly stabilizing control law is obtained for the unconstrained system (10) affected by uncertainty. The corresponding gain $K = YS^{-1}$ will be used together with the nominal model for the resolution of the LMI problem (29) which provides by inverse optimality the matrix $\bar{R}$. The matrix $\bar{Q}$ is obtained with a simple evaluation of (27) and the structure of the performance index in (14) is completed. The prediction horizon of the same performance index can be chosen according with the desired performances and complexity of the explicit solution. Independently of this choice, if the matrix $\bar{P}$ satisfies (15), then the nominal MPC leads to a piecewise affine control law and for the region D_{i_0} with $0 \in Int(D_{i_0})$ the explicit control law will be

$$u_k = K_{i_0}^{MPC}\xi_k + \kappa_{i_0}^{MPC} = YS^{-1}\xi_k \qquad (30)$$

This region is polyhedral and the robust stabilizing properties are verified for an invariant subset with respect to the closed loop dynamics (10). If we consider the general form of the invariant set given by the level set:

$$E(\sigma) = \{\xi | \xi^T\bar{P}\xi \leq \sigma\} \qquad (31)$$

then one can find $\sigma > 0$ satisfying $V = E(\sigma) \subset D_{i_0}$. ■

4 RPI SET

The synthesis problem being solved, we dispose of a control law supposed to stabilize a time-varying delay system. The question is: *which is the maximal invariant set for the closed loop system?* An approximation can be obtained by constructing the maximal robust positive invariant set (MRPI) for a piecewise affine system (PWA) affected by uncertainty.

A PWA system is obtained from the embedding of the time-varying system in a linear model affected by polytopic uncertainty in closed loop with the piecewise affine control law:

$$\begin{aligned} \xi_{k+1} &= f_{PWA}(\xi_k) = (F+GK_i^{MPC})\xi_k + \kappa_i^{MPC} \\ &\qquad \text{for} \quad \xi_k \in D_i \\ (F,G) &\in Co\{(F_1,G_1),\ldots,(F_s,G_s)\} \end{aligned} \qquad (32)$$

where D_i are the polytopic partition $D = \cup_i D_i$.

The dynamics related to an extreme realization of the PWA polytopic uncertainty will be described by:

$$\xi_{k+1} = f_{PWA_i}^j(\xi_k) = (F_j + G_jK_i^{MPC})\xi_k + \kappa_i^{MPC} \quad \text{for} \quad \xi_k \in D_i,\ j \in \{0,1,\ldots,s\} \qquad (33)$$

The description of the MRPI set for such a PWA system is not immediate, even for simple cases the finite determiness can not be guaranteed. Nevertheless, the fact that the partition of the state space is given by polyhedral regions will be used in the following section to build appropriate approximations.

In order to describe these geometrical constructions, the image and preimage operators over the sets $\Psi \in \Re^{n+hm}$ will be defined as:

$$\begin{aligned} Im_{f_{PWA}}(\Psi) = \bigcup_j \{\zeta \in \Re^{n+hm} | \exists \xi \in \Psi, \quad \text{s.t.} \\ \zeta = (F_j + G_jK_i^{MPC})\xi + \kappa_i^{MPC} \quad \text{for} \quad \xi \in D_i \cap \Psi\} \end{aligned} \qquad (34)$$

$$PreIm_{f_{PWA}}(\Psi) = \bigcap_j \{\xi \in D | \exists \zeta \in \Psi, \quad \text{s.t.} \\ \zeta = (F_j + G_j K_i^{MPC})\xi + \kappa_i^{MPC} \text{ for } \quad \xi \in D_i\} \tag{35}$$

Contractive Procedure: The idea is to substract from the state partition $D = \cup_i D_i$ defining the PWA system, those regions for which one of the extreme dynamics will evolve outside D. This is an iterative procedure as long as after each iteration, the set D is modified and thus the possible evolutions are to be rechecked.

The complexity of the procedure is given by the fact that the subtraction of convex set is not a closed operation. In short, if D is convex, there is no guarantee that it will remain convex after an iteration of the contractive procedure. Indirectly this is acknowledging the fact that the MRPI set may not be convex.

Algorithm 1: Contractive Scheme

$V_0 = D$
$k = 0$
`while` *(precision condition)*
$V_{k+1} = PreIm_{f_{PWA}}(Im_{f_{PWA}}(V_k) \cap V_k)$
$k = k + 1$

Expansive Procedure: In this case instead of excluding gradually those regions outside the MRPI set, we start with an RPI set and add those regions which evolve in one step inside the RPI set. Again the resulting set is RPI and is monotonically increasing (in the sense of inclusion) and is limited by MRPI.

An important advantage of the expansive procedure is that the intermediate results are robust positive invariant and thus can be considered as candidate approximations for the MRPI set.

Algorithm 2: Expansive Scheme

`find` $\sigma > 0$ s.t. $E(\sigma) \subset D_{i_0}$
$V_0 = E(\sigma)$
$k = 0$
`while` *(precision condition)*
$V_{k+1} = PreIm_{f_{PWA}}(Im_{f_{PWA}}(D) \cap V_k)$
$k = k + 1$

Note the maximal robust positive invariant set Ψ and the iterates obtained with the expansive and contractive procedure by Ψ_i^e and Ψ_i^c respectively.

Neither the expansive procedure $\Psi_i^e \subset \Psi$, nor the contractive procedure $\Psi_i^c \supset \Psi$ do not dispose of a measure of the convergence toward the MRPI set. However, by mixing the two relations we obtain an inner approximation for the MRPI set:

$$\Psi_i^e \subset \Psi \subset \Psi_i^c \tag{36}$$

Considering the Hausdorff metric over the class of polyhedra. The distance $d_H(\Psi_i^c, \Psi_i^e)$ can provide a measure of the MRPI approximation offered by Ψ_i^e and thus a *precision condition*:

$$\Psi_i^e \subset \Psi \subset \Psi_i^c \subset \Psi_i^e \oplus \mathbf{B}_0(d_H(\Psi_i^c, \Psi_i^e)) \tag{37}$$

5 EXAMPLE

Consider the level control system as the one reported in (Furtmueller and del Re, 2006) with the bloc representation presented in figure 1. Beside the sen-

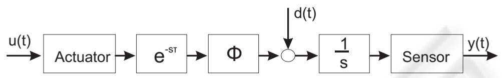

Figure 1: General scheme for the plant to be controlled.

sor and the actuator transfer functions we retrieve in this schema-block the variable time-delay; a nonlinear function Φ known and invertible and an integrator. The paper (Furtmueller and del Re, 2006) presented a method for the disturbance suppression, such that in the following we will consider the level control and replace the classical PI controller with a predictive controller and characterize the safety functioning region by the construction of the robust positive invariant region following the procedure presented in the previous sections.

The continuous time system to be controlled is a double integrator with variable-time delay and the discrete-time model is given by:

$$x_{k+1} = \begin{bmatrix} 1 & 0 \\ 0.1 & 1 \end{bmatrix} x_k + \begin{bmatrix} 0.1 \\ 0.05 \end{bmatrix} u_{k-i} - \\ \Delta(u_{k-v} - u_{k-v+1}), \text{ with v} \in \{0,1,2\} \tag{38}$$

In the first instance the embedding of the uncertainty matrix Δ have to be obtained. Due to the fact that in the original representation, we deal with a 2-dimensional state vector x_k, the poytopic uncertainty will be:

$$\Delta \in Co\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0.05 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0.013 \end{bmatrix} \right\} \tag{39}$$

In the extended state representation, a robustly stabilizing feedback gain is obtained for the unconstrained case by solving the LMI problem (25):

$$K = [\ -1.3188 \quad -0.5408 \quad -0.1292 \quad -0.0157 \quad -0.1511\ ] \tag{40}$$

The inverse optimality problem leads after solving (29) to the tuning of the nominal MPC law with the weighting matrices P, Q, R. By imposing a set of constraints on the input and the state:

$$-0.1 \le u_k \le 0.1 \\ \begin{bmatrix} -2 \\ -2 \end{bmatrix} \le x_k \le \begin{bmatrix} 2 \\ 2 \end{bmatrix} \tag{41}$$

and taking into account that the maximal delay is 3 sampling instants we choose a prediction horizon $N =$

5 in order to maintain a low complexity of the explicit solution (47 regions in the state space partition, see figure 2).

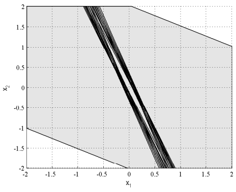

Figure 2: Projection of the explicit solution's partition on the first two components of the extended state space.

The polytopic model in the extended state representation which embeds (using 7 extreme realizations) the time-varying delay system will allow the use of the contractive procedure for the approximation of the maximal invariant set. In figure (3) cuttings through the approximation obtained after 5 iterations is presented.

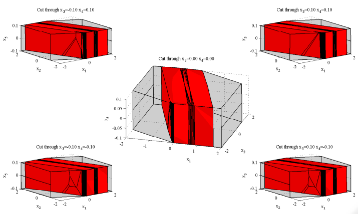

Figure 3: The explicit solution's partition and the approximation of the MRPI set.

Finally in figure (4-5) a time domain simulation with varying delay is presented (starting from the state $(0;-2)$), proving the versatility of the proposed control technique.

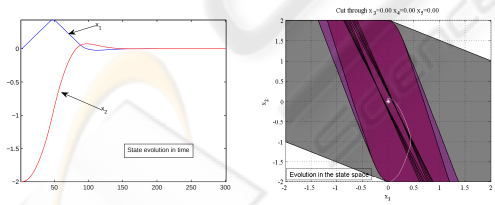

Figure 4: The time evolution of the state components.

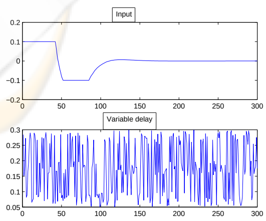

Figure 5: The control signal and the variation of the delay in time.

6 CONCLUSIONS

A model predictive control law was designed to deal with time-varying delay systems. The constraints are handled from the design stage and the iterative approximation of the maximal positive invariant set offers information about the region of the state space where the control policy is viable.

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