

# FUZZY TIME MANIPULATION IN A RELATIONAL DB

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**Abstract:** Temporal Databases (TDB) have as a primary aim to offer a common framework to those DB applications that need to store or handle temporal data of different nature or source, since they allow to unify the concept of time from the point of view of its meaning, its representation and its manipulation. Up to now, most of the efforts have been devoted to extend the relational model in order to be adapted to the new way of inserting, deleting and updating the DB, together with the integrity constraints involved. In this paper we deal with these problems when the time is imprecisely expressed by means of a fuzzy interval of dates. Along the text, we will see how the delete and the insert operations can be carried out and how different types of queries to such a fuzzy temporal database are solved.

## 1 INTRODUCTION

Temporal Databases, in the widest sense, offer a common framework for all database applications that involve some temporal aspects when organizing data. These databases allow to unify the time concept from several points of view: the representation, the semantics and the manipulation.

Database applications involving temporal data are not a new subject and up to now, most of the work -both theoretical and implementations- made on this topic has been carried out using as starting point the relational DB model (since it is the most complete and consolidated one) and base the results in the extension of the table schemata (Tansel et al., 1993), the range of operators to be used (Elmarsy and Wu, 1990) and in the addition of specific integrity constraints related to the new data types (Snodgrass et al., 1995), (Etzion et al., 1998), (Bettini et al., 1998).

From the point of view of the real world, there exist two basic ways for associating temporal concepts to a fact:

1. *Punctual Facts.* a fact is related to an only time mark that depends on the granularity and informs about the time when it happened. As instances, birthdays, the order date, an academic year, ...
2. *Time Periods.* that are represented by a starting instant and an ending one, so the duration (or valid time) of the fact is implicit. Some examples are:

[admission date, discharge date], [start contract date, end contract date], ...

This way of time interpretation is called **valid time**.

For example, in the valid time relation (VTR) EMP each tuple represents a version for the available information about an employee, and this version is valid only when used in the time interval [VST, VET]. The up-to-date version, also called valid tuple, is *indefinite-valued* in the attribute VET. In the figure 1 an instance of EMP is shown.

Table 1: Instance example of the valid time relation EMP.

NAME	ID	SAL.	BOSS	EXP.	VST	VET
GRANT	1245	1500	9877	TRAINEE	15-06-97	31-05-98
GRANT	1245	1800	9877	JUNIOR	01-06-98	Undefined
REDFORD	9877	1200	4588	TRAINEE	20-08-94	31-01-96
REDFORD	9877	1500	4588	JUNIOR	01-02-96	31-03-97
REDFORD	9877	2200	9989	SENIOR	01-04-97	Undefined
BROWN	1278	2800	4588	JUNIOR	01-05-05	10-08-08
STREEP	6579	4000	9877	TRAINEE	15-06-97	Undefined

At first sight, time inclusion into a DB seems to be easy and direct, but extending schemata with these new attributes have many complex consequences, such as:

- In a VTR, the old primary key is not unique. The new primary key is the result of combining the previous value for the key and one of the valid time attributes, VST or VET. If the case of the employees table EMP is considered, the primary

key is not EMPID (employee’s code) but EMPID+VST or EMPID+VET.

- There can only exist a valid tuple for each entity in a concrete moment. Because of this, every operation has to be strictly controlled because valid time periods of distinct versions of the same entity must not overlap.
- Internal implementation of common operations is completely different to those implemented within a non temporal database. For instance, when an UPDATE is required, a new version of the updated tuple is created and the up-to-now valid version of the tuple is closed by fixing its VET value with the final time. In this case, the user is responsible for giving the valid time for an update operation; when closing the active version, the value of the attribute VET is updated with the previous grain to the value for VST attribute in the new version.

Together with these special considerations, sometimes it is not possible for the user to give an exact but an imprecise starting/ending point for the validity period of a fact. This is the case, for example, when a patient does not exactly know when a concrete ailment or symptom started. In this case, the use of fuzzy sets theory is necessary for not missing such important information since fuzzy time values can be used. Many authors working in the area of soft computing have opted to study temporal data affected by imprecision and/or uncertainty. That is the case of the paper (Chountas and Petrounias, 2005) what deals with this problem from the point of view of uncertainty exclusively, by attaching a probability distribution to every temporal data. Also in (Tre et al., 1997) the problem of fuzzy time is addressed in an object-oriented DB system and some interesting results have been obtained.

This paper is devoted to the fuzzy representation of time together with the UPDATE and the SELECT operation when the time is expressed in fuzzy terms. Note that the study of the UPDATE operation includes also the DELETE and the INSERT operations as particular cases so that the results obtained can be extrapolated to the whole data manipulation language.

### 1.1 Previous Concepts on Fuzzy Sets

A fuzzy value is a fuzzy representation about the real value of a property (attribute) when it is not precisely known.

In this paper, according to Goguen’s Fuzzification Principle (Goguen, 1967), we will call every fuzzy set of the real line *fuzzy quantity*. A *fuzzy number* is a particular case of a fuzzy quantity with the following properties:

**Definition 1.** The fuzzy quantity  $A$  with membership function  $\mu_A(x)$  is a **fuzzy number** (Dubois and Prade, 1985) iff:

1.  $\forall \alpha \in [0, 1], A_\alpha = \{x \in R \mid \mu_A(x) \geq \alpha\}$  ( $\alpha$ -cuts of  $A$ ) is a convex set.
2.  $\mu_A(x)$  is an upper-semicontinuous function.
3. The support set of  $A$ , defined as  $Supp(A) = \{x \in R \mid \mu_A(x) > 0\}$ , is a bounded set of  $R$ , where  $R$  is the set of real numbers.

We will use  $\tilde{R}$  to denote the set of fuzzy numbers, and  $h(A)$  to denote the height of the fuzzy number  $A$ . For the sake of simplicity, we will use capital letters at the beginning of the alphabet to represent fuzzy numbers.

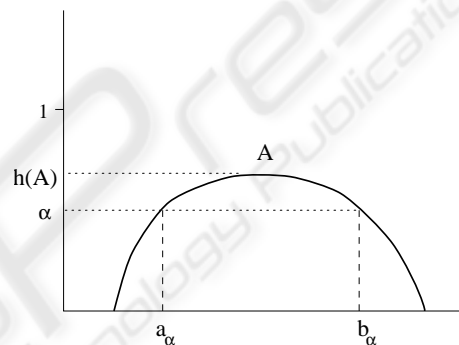


Figure 1: Fuzzy number.

The interval  $[a_\alpha, b_\alpha]$  (see figure 1) is called the  $\alpha$ -cut of  $A$ . So then, fuzzy numbers are fuzzy quantities whose  $\alpha$ -cuts are closed and bounded intervals:  $A_\alpha = [a_\alpha, b_\alpha]$  with  $\alpha \in (0, 1]$ .

If there is, at least, one point  $x$  verifying  $\mu_A(x) = 1$  we say that  $A$  is a *normalized* fuzzy number.

Sometimes, a trapezoidal shape is used to represent fuzzy values. This representation is very useful as the fuzzy number is completely characterized by four parameters  $(m_1, m_2, a, b)$  as shows figure 2 and the height  $h(A)$  when the fuzzy value is not normalized. We will call *modal set* all values in the interval  $[m_1, m_2]$ , i.e, the set  $\{x \in Supp(A) \mid \forall y \in R, \mu_A(x) \geq \mu_A(y)\}$ . The values  $a$  and  $b$  are called *left and right spreads*, respectively.

In our approach, we will use trapezoidal and normalized fuzzy values.

### 1.2 FSQL (Fuzzy SQL)

The FSQL language (Galindo et al., 1998),(Galindo et al., 2006) extends the SQL language in order to express flexible queries. Due to its complex format, we only show here an abstract with the main extensions to the `select` command.

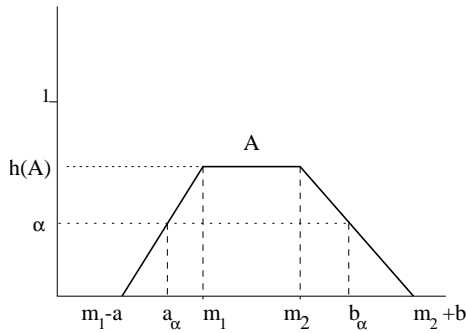


Figure 2: Trapezoidal fuzzy number.

Table 2: Fuzzy Comparators for FSQL (Fuzzy SQL).

Comparator for:		Significance
Possibility	Necessity	
FEQ	NFEQ	Fuzzy Equal (Possibly/Necessarily Equal)
FGT	NFGT	Fuzzy Greater Than
FGEQ	NFGEQ	Fuzzy Greater or Equal
FLT	NFLT	Fuzzy Less Than
FLEQ	NFLEQ	Fuzzy Less or Equal
MGT	NMGT	Much Greater Than
MLT	NMLT	Much Less Than

- **Linguistic Labels.** These labels will be preceded by the symbol \$ to distinguish them easily and have associated a trapezoidal possibility distribution. So, for example, we can define the labels \$Short, \$Normal, \$Tall, ... on the Height attribute.

- **Fuzzy Comparators.** Besides the typical comparators (=, >...), FSQL includes the fuzzy comparators shown in table 2. Like in SQL, fuzzy comparators compare one column with one constant or two columns of the same type.

Possibility comparators are less restrictive than necessity comparators are. For example, NFEQ uses the following equation:

$$\Theta^{NFEQ}(\tilde{p}, \tilde{p}') = \inf_{d \in U} \max(1 - \mu_{\tilde{p}}(d), \mu_{\tilde{p}'}(d)) \quad (1)$$

- **Fulfilment Thresholds.** ( $\gamma$ ) For each simple condition a fulfilment threshold may be established (default is 1) with the format:

$$\langle \text{condition} \rangle \text{ THOLD } \gamma$$

indicating that the condition must be satisfied with minimum degree  $\gamma \in [0, 1]$  to be considered. The reserved word THOLD is optional and may be substituted by a traditional crisp comparator (=,  $\leq$ ...). Find the people that are necessarily taller than label \$Tall (in minimum degree 0.8):

Table 3: Default computation for function CDEG with logic operators in FSQL.

<Condition>	CDEG(<Condition>)
<cond1> AND <cond2>	$\min(\text{CDEG}(\langle \text{cond1} \rangle), \text{CDEG}(\langle \text{cond2} \rangle))$
<cond1> OR <cond2>	$\max(\text{CDEG}(\langle \text{cond1} \rangle), \text{CDEG}(\langle \text{cond2} \rangle))$
NOT <cond1>	$1 - \text{CDEG}(\langle \text{cond1} \rangle)$

Table 4: Fuzzy constants that may be used in FSQL queries.

F. Constant	Significance
UNKNOWN	Unknown value but the attribute is applicable.
UNDEFINED	The attribute is not applicable or it is meaningless.
NULL	Total ignorance: We know nothing about it.
\$(a,b,c,d)\$	Fuzzy trapezoid ( $a \leq b \leq c \leq d$ ).
\$Label	Linguistic Label: It may be a trapezoid or a scalar.
[n,m]	Interval "Between n and m" ( $a=b=n$ and $c=d=m$ ).
#n	Fuzzy value "Approximately n" ( $b=c=n$ and $n-a=d-n=\text{margin}$ ).

```
SELECT * FROM Person WHERE Height NFGT $Tall
      THOLD 0.8
```

- **Function CDEG(<attribute>).** It shows a column with the fulfilment degree of the condition imposed on a specific attribute. If logic operators appear, the calculation of this compatibility degree is carried out as table 3 shows. We use the minimum T-norm and the maximum T-conorm, but the user may change these values by modifying a view (FSQL\_NOTANDOR). In this view the user can set the function to be used for every logic operator (NOT, AND, OR). Obviously, that function must be implemented in the FSQL Server or by the user himself.
- **Fuzzy Constants.** In FSQL we can use the fuzzy constants detailed in table 4.

## 2 FUZZY TIME REPRESENTATION

In the introduction we have seen that, in classical TDB, the valid time is managed thanks to the extension of the tables schemata by adding two new attributes, the valid start time -VST- and the valid end time -VET- to determine the period of validity of the fact expressed by a tuple.

In this paper we are going to consider that the information provided by the VST and VET for the classical TDB is fuzzy, in the sense that we are not completely sure about when the current values of the tuple began to be valid.

The more immediate solution to this problem is to soften the VST and the VET in such a way that they may contain fuzzy dates represented by means of a fuzzy number. This means that, if we use the para-

metrical representation for fuzzy numbers, we need to store four values for the VST and four values for the VET, as shown in figure 3. Since the meaning of the attributes VST and VET is the period of time during which the values of a tuple are valid, it is more convenient to summarize the information given by the two fuzzy attributes in an only but fuzzy interval. This situation can be represented by the trapezoidal fuzzy set shown in figure 4 which incorporates the semantics of our problem. As can be seen in such figure, the right side of the interval (VET) is set to 1 indefinitely while the tuple remains valid. On the contrary, the left side of the interval is the part that reflects the imprecision about the starting time point of the interval or VST. In this figure, the fuzzy value represents an interval where the VST corresponds to *middle february* and the VET corresponds to *now* or *indefinitely*.

EMPNAM	EMPID	SALARY	BOSS	EXPERTISE	VST	VET
GRANT	1245	1500	9877	TRAINEE	~15-06-1997	~ 31-05-1998
GRANT	1245	1500	9877	JUNIOR	-01-06-1998	~undefined

(31-12-2050,31-12-2050,0,0)  
 ↓  
 (01-06-1998,01-06-1998,2,2)

Figure 3: Internal representation of a fuzzy date.

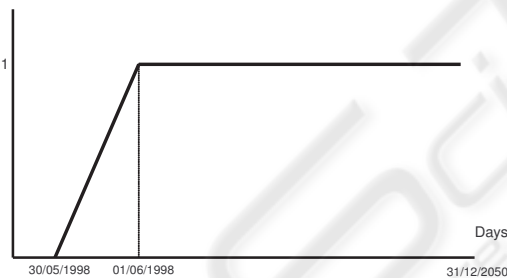


Figure 4: Fuzzy Period of Time for a Valid Tuple.

This representation has the advantage that, not only periods of time, but fuzzy dates can also be represented in a unified way. Think that a parametrical representation as (m,m,a,b) represents a central time point with some imprecision at both sides, what is interpreted as a fuzzy date.

As it was explained in section 1.2, it is quite easy to represent a fuzzy interval with this characteristics since only four parameters need to be stored in order to specify it. In our example, the parametrical representation should be (17/02/1996,31/12/2050,5,0)<sup>1</sup>. This representation is more flexible, since admits to represent both precise

<sup>1</sup>We have assumed that 31/12/2050 is the ending point of the time domain

and imprecise time intervals. For example, a precise interval like [18/03/1997,25/06/1998] is represented as (18/03/1997,25/06/1998,0,0).

In (Medina et al., 1994), (Medina et al., 1995) is presented a generalized model of fuzzy DB that supports this representation for fuzzy data and the corresponding implementation in a classical relational DB system (Oracle).

### 3 INSERTING AND UPDATING IN THE FUZZY TDB

As we explained in the introduction, in a TDB information is never deleted when an update operation is being carried out. The process now is to leave the old version of data in the DB and to add a new version with the suitable modifications achieved, but closing the old one by adding a valid end time value which is the immediately previous granule to the valid start time of the tuple inserted. Note that closing the old version of a tuple is a *deletion* operation in the TDB environment whereas adding a new version corresponds to an *insert* operation.

One the most important problems that arise when the time period considered is fuzzy is that we can not say which is the time point immediately previous to a given one for a concrete granularity, since many values with different possibility degree are possible. To compute this value is very important for the update operation since, as explained below, we need to close the old version for the new one is valid. So now, the given solution is not precise but also imprecise.

**Definition 2.** Let us note by  $\mu_O(x)$  the membership function associated to the fuzzy interval of the *old* version of the tuple to be updated and  $\mu_N(x)$  the membership function associated to the new fuzzy interval. Then, the membership function of the fuzzy interval ( $\mu'_O(x)$ ) that serves to *close* the validity time of the old one is:

$$\mu'_O(x) = \begin{cases} \mu_O(x) & \forall x \mid \mu_N(x) = 0 \\ 1 - \mu_N(x) & \forall x \mid \mu_N(x) > 0 \end{cases}$$

This result can be graphically seen in figure 5. It is obvious, that the non-overlapping condition required for the crisp TDB is not valid now, but the overlapping degree will never reach value 1 for the sake of consistency.

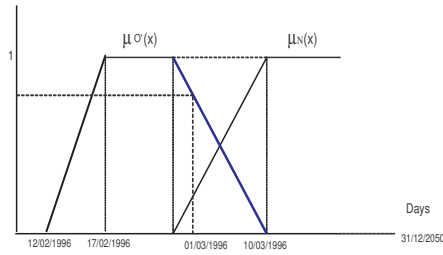


Figure 5: Membership function of the fuzzy value  $O'$  that closes a version of a tuple with new valid time  $N$ .

**Definition 3.** We will call *closing interval* all the values  $x \in D$  such that  $0 < \mu_N(x) < 1$  and  $0 < \mu_{O'}(x) < 1$ .

As can be observed, in the closing interval of the domain  $D$ , the more is the possibility to belong to the new interval, the less is the possibility to belong to the old one, what makes sense because, for the old valid time the interval is finishing whereas for the new valid time it is starting.

It is also important to note that we have used  $1 - \mu(x)$  as the complement of a fuzzy set (it is not the only possible function in the literature) because it is the simplest one from the computational point of view and easy to be understood for a user since it directly reflects the semantics of the problem.

#### 4 QUERYING THE FUZZY TDB

Once we are able to represent fuzzy periods of valid time, queries about a crisp/fuzzy date or period to the valid time relations can be solved computing the corresponding fulfillment degree between the time we are querying about (QT) and the database valid time (FVP). In the next section we explain the most representative types of queries.

In order to illustrate all the cases, let us consider the following fuzzy TDB instance.

Table 5: Fuzzy Temporal Database Instance.

NAME	ID	...	EXP.	FVP
GRANT	1245		TRAINEE	(15/06/1997, 31/05/1998, 2,2)
GRANT	1245		JUNIOR	(02/06/1998, 31/12/2050,2,0)
REDFORD	9877		TRAINEE	(20/08/1994, 31/01/1996, 2,3)
REDFORD	9877		JUNIOR	(03/02/1996, 31/03/1997,3,4)
REDFORD	9877		SENIOR	(04/04/1997, 31/12/2050,4,0)
BROWN	1278		JUNIOR	(01/05/1996,10/08/97,0,0)
STREEP	6579		TRAINEE	(15/06/1997,31/12/2050,0,0)
NEWMAN	5546		SENIOR	(18/06/1997,29/04/1998,8,10)

##### 4.1 Queries about a Precise Date

In this case, QT is a date  $d$  and the system must find the tuples whose FVP includes  $d$  in the support set, that is, those tuples for which the membership of this

date to the fuzzy period is greater than 0. Once found, the fulfillment degree of the resulting tuple will be computed as:

$$\mu_{FVP}(d)$$

It should be noted that this degree will be 1 when the date  $d$  belongs to the modal set of the fuzzy interval.

As explained before, it may happen that not only one but two versions of the same tuple have a validity period that includes  $d$  in the support set. This is the case when the mentioned date belongs to the closing interval of two consecutive periods. In this case, the answer will be the tuple whose fulfillment degree is:

$$\oplus(\mu_{FVP1}(d), \mu_{FVP2}(d))$$

i.e. the one whose validity time best fits the query date. The fuzzy set theory provides many different operators that may be chosen depending on the desired effect. For example, in this case we could use the *maximum*.

One example of this type of query is *Find the expertise level of employee number 9877 on 1st April 1997*. This situation can be graphically seen in figure 6 and the formal expression of this query using FSQL syntax is:

```

fsql> SELECT empnam,expertise,CDEG(fvp)
FROM employees
WHERE empid=9877 AND
fvp FEQ TO_DATE('01/04/1997') THOLD 0.0;
    
```

NAME	EXP.	FVP	CDEG(FVP)
REDFORD	JUNIOR	(03-02-1996, 31-03-1997, 3, 4)	0.75
REDFORD	SENIOR	(04-04-1997, 31/12/2050, 4, 0)	0.25

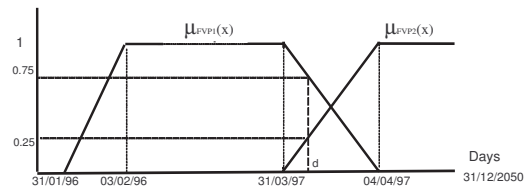


Figure 6: Membership degrees when date  $d$  belongs to the closing interval of two tuples.

##### 4.2 Queries about a Fuzzy Period of Time

In this case, QT is another fuzzy interval of dates and the system should find the tuples whose FVP includes

QT and compute to what degree this inclusion is fulfilled. This situation can be modelled by means of the implication:

$$QT \longrightarrow FVP$$

If we assume that the implication function  $I(QT(d), FVP(d))$  used is the material implication, then the fulfillment degree of this fuzzy inclusion will be:

$$N(FVP|QT) = \min_{x \in D} I(\mu_{QT}(x), \mu_{FVP}(x))$$

$$N(FVP|QT) = \min_{x \in D} \{(1 - \mu_{QT}(x)) \oplus \mu_{FVP}(x)\}$$

If the t-conorm considered is the *maximum*, the resulting measure is a *necessity*. Note that this measure includes the classical sets inclusion as a particular case.

If more than one version of a tuple give a positive result for the inclusion degree, then the best answer will be the one with the highest degree.

One example of this type of query is *Find the boss of employee number 9877 by the beginning of April 1997 (01/04/97,05/04/97,0,2)*. The formal expression of this query using FSQL syntax is:

```
fsql> SELECT e.empnam, e.expertise, e.boss,
        CDEG(e.fvp)
FROM employees WHERE e.empid=9877 AND
$['01/04/1997','05/04/1997',0,2] NFEQ fvp
THOLD 0.0;
```

NAME	EXP.	BOSS	CDEG(FVP)
REDFORD	SENIOR	9989	0,25

where NFEQ (necessity-based fuzzy equal) computes to what degree the left side fuzzy set is included in the right side one using the expression given above.

### 4.3 Simultaneous Events

The problem now is to compute to what degree two events are simultaneous. This operation is very useful since *joins* are based on it. The simultaneity degree or temporal equality between two periods of time can be carried out computing the degree to which both fuzzy/crisp sets  $-FVP_1$  and  $FVP_2-$  are mutually included one in the other, that is, we should compute:

$$\otimes\{N(FVP_1/FVP_2), N(FVP_2/FVP_1)\}$$

as the final value for the fuzzy equality degree between the two fuzzy sets where  $\otimes$  stands for a T-norm (*minimum* is our case).

A query of this type could be *Find employees with the boss 9877 during the same period of time*. The formal expression of this query using FSQL syntax is:

```
fsql> SELECT e.empnam, f.empnam, CDEG(*)
FROM employees e, employees f
WHERE e.boss=9877 AND f.boss=9877
AND e.empid<>f.empid
AND (f.fvp NFEQ e.fvp) >0
AND (e.fvp NFEQ f.fvp) >0
```

E.EMPNAM	F.EMPNAM	CDEG(*)
GRANT	NEWMAN	0,4

where CDEG(\*) computes the minimum of the fulfillment degrees obtained.

## 5 CONCLUSIONS AND FUTURE WORK

In this paper we have shown the advantages of representing fuzzy temporal data with a parametrical representation. On this temporal data, we have explained how an update operation can be carried out by taking into account that no deletion is possible when temporal information is stored. As a result of this operation, a modification to the old version of the tuple is needed by changing some of the parameters that define it. As a consequence of our approach, the queries return a set of tuples together with a fulfillment degree when a query is made on these fuzzy temporal data. The paper also analyzes the different types of queries that can be made on these data. We are currently analyzing the behavior of other operators (after, before, short time, long time ago, etc.) and considering a wider range of temporal data. We are also studying the problem of primary keys in the presence of fuzzy intervals instead of VST and VET attributes and trying to find out new indexing techniques that take the new primary keys into account.

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