

AN OPTIMIZATION METHOD FOR REDEMPTION AND DUE DATE MATCHING IN ASSIGNMENT OF ELECTRONIC RECEIVABLES BY USING INTEGER LINEAR PROGRAMMING

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Abstract: This paper shows an optimization method for redemption and due date matching which assigns the receivables to the payments date under the pre-defined constraints which related companies specified. Our proposed method determines the pairs of the receivables and the payment with proper new settlement date which closes the fluid assets of companies to their target amounts by extending/shortening the redemption/due date. This paper shows that this matching program is categorized in integer linear programming. By applying this matching mechanism, transferors could utilize credit of issuers and also reduce fluid assets for payments. Effectiveness of our optimization method is shown by executing simulation which emulates the issuing and receiving status of receivables in Japanese companies.

1 INTRODUCTION

In recent years, various countries have made progress in the development of systems for handling credit transactions electronically, including EBPP/EIPP (Electronic Bill/Invoice Presentation and Payment) in the United States (Fairchild, 2003) and e-billing systems in South Korea (KFTC, 2002). Japan is now also expecting an increase in this type of business, and in June 2007 the Japanese parliament approved the Electronically Receivables Legislation (FSA, 2005) which is due to go into force at the end of 2008. Electronically receivables are a new form of credit whereby electronic registrations at organizations that register electronic receivables become only requirement for the generation and transfer of credit, and are expected to be utilized as a means for the exchange of nominative claims and credits to payment on which contract documents and printed bonds are based (Ikeda, 2006).

The assignment of claims allocated to the payment of electronic receivables owned by a business has the merit of allowing the transferor to utilize the remitter's credit rating (Oogaki, 2006). Specifically, when the transferor (company B) owns electronic receivables issued by an issuer (company A), even if com-

pany B has a poor credit rating and the electronic receivables issued by company B have not been received by the transferee (company C), then if we suppose that company A has a high credit rating then company B can pay by transferring credit issued by company A. This supplementation of reliability by a third party is particularly useful as a way of utilizing the credit rating of a parent company in the financing of subcontractor corporations in business affiliations that account for approximately 60% of such arrangements in Japan (JSBRI, 2007).

However, it has been pointed out that previous nominative claims and credits to payment are limited in terms of the opportunities for utilizing one's credit in the assignment of claims. One possible reason for this is that the payment conditions (amount payable and due date) and credit redemption conditions are not exactly the same. Since electronic receivables make it easy to rewrite the conditions electronically, it is relatively simple to accommodate differences in sums by splitting the electronic receivables. This has actually been implemented in the book entry transfer system for short term corporate bonds in Japan. On the other hand, in cases where the due dates do not match, it is necessary to adjust the gains and losses of the inter-

ested parties (issuer, transferor, transferee) before altering the conditions. This due date modification has the side benefits of allowing businesses to make effective use of surplus floating assets (referred to simply as “assets” in the following) and providing a means for supplementing shortfalls in assets. It is thus necessary to reach an agreement on items such as what the interest rate should be and by how much the term should be extended or contracted.

It is difficult to make an agreement between the interested parties regarding the change in due date by executing work flow between them because it can take much time to reach an agreement if consideration is given to factors such as changes of circumstances resulting from conflicting conditions.

Therefore in this study it is envisaged that making alterations to the due date of electronic receivables in an effective manner requires a mechanism whereby the participants first register assets fluctuation targets and variation conditions whereby changes in due date can be tolerated, and then combinations of payments and electronic receivables are determined to satisfy these constraints. For this study we therefore propose a matching scheme that optimizes combinations of payments and electronic receivables so as to achieve the best possible assets fluctuation targets for all participants. In particular, in this matching of electronic receivables and payments, the level of assets target achievement of the participants depends not only on which electronic receivables should be combined with which payments, but also on how the new due date for the redemption/payment of electronic receivables and payments is set and matched. We therefore formalize the issue of combining electronic receivables and payments by including the selection of a new due date whose selection range is determined by these combinations, and the results can be treated as a problem of integer linear programming. Also, by simulating the transaction of electronic receivables under conditions simulating the issue of corporate bills and accounts receivable in Japan, we first verify the extent to which the assignment of claim is promoted by the due date matching function. We also verify the optimization effects by comparing the optimized matching of electronic receivables and payments with some other combination methods.

2 METHOD FOR MATCHING THE DUE DATES OF ELECTRONIC RECEIVABLES

In this section we discuss a method for matching electronic receivables and payments based on the prior registration of due date alteration conditions which is a premise of this study. The processing flow is illustrated in Fig. 1. First, the participants register their respective asset fluctuation targets and due date adjustment criteria (1). The payment registrations are then accepted (2), the combination of electronic receivables and payments is determined based on these information (3), and approval is obtained for the matching results. The due date adjustment criteria are assumed to consist of a range of possible due dates for redemption payments, and interest conditions. In the following, we will discuss the asset fluctuation targets specified by the participants, the possible range of redemption/payment due dates, and the interest conditions.

- **Asset Fluctuation Targets.**

Each participant specifies when and how much they would like the current assets to fluctuate. For example, the estimation of future changes in current assets is prepared from predictions of the payment and receipt of electronic receivables (and other settlements), and by setting the target holdings of current assets at each future timing, the fluctuation targets are set according to the difference between the target holdings and estimated transitions.

- **Possible Range of Redemption/Payment due Dates.**

If necessary, the participants specify a range for the extension or contraction of payment due dates and/or redemption due dates.

- **Interest Conditions.**

The participants specify a lower limit of interest to be accepted in cases where the redemption due date is extended or the payment period is reduced, and conversely an upper limit of interest to be paid in cases where the redemption due date is brought forward or the payment due date is extended. Note that the interest is set according to the debtor's credit rating.

3 FORMULARIZATION OF THE PROBLEM OF MATCHING ELECTRONIC RECEIVABLES AND PAYMENTS

In this section we will devise a formula for the problem of matching electronic receivables and payments, and we will show that this problem can be expressed as a problem in integer linear programming (Schrijver, 1986)(Aardal et al., 2005).

The problem of matching electronic receivables and payments is expressed using the following notation.

Constants.

R_{ijm} . The m -th electronic receivable issued by issuer i and held by transferor (current holder) j .

$D(R_{ijm})$. Redemption date of electronic receivable R_{ijm}

$DE(R_{ijm})$. Upper limit of the extension of the redemption date of electronic receivables R_{ijm} set by issuer i

$DS(R_{ijm})$. Upper limit of the reduction of the redemption date of electronic receivables R_{ijm} set by issuer i

$V(R_{ijm})$. Redemption sum of electronic receivable R_{ijm}

P_{jkn} . n -th payment made to transferee k by transferor j

$D(P_{jkn})$. Payment date of payment P_{jkn}

$DE(P_{jkn})$. Upper limit of the extension of the payment date of payment P_{jkn} set by transferee k

$DS(P_{jkn})$. Upper limit of the reduction of the payment date of payment P_{jkn} set by transferee k

$V(P_{jkn})$. Payment sum of electronic receivable R_{ijm}

$IP(i, R_{ijm}, f) / IR(i, R_{ijm}, f)$. Upper/lower limit of interest to be paid/received by issuer i in cases where the redemption date of electronic receivable R_{ijm} is extended/contracted to date f . Issuer i decide the upper/lower limit of interest according floating assets needs.

$IP(j, R_{ijm}, f) / IR(j, R_{ijm}, f)$. Upper/lower limit of interest to be paid/received by transferor j in cases where the redemption date of electronic receivable R_{ijm} is contracted/extended to date f . Transferor j decides the

upper limit of interest according her/his floating assets needs and the lower limit according to credit rating of issuer of R_{ijm} and transferors (who transfer R_{ijm} before transferor j).

$IP(j, P_{jkn}, f) / IR(j, P_{jkn}, f)$. Upper/lower limit of interest to be paid/received by transferor j in cases where the payment date of payment P_{jkn} is contracted/extended to date f . Transferor j decides the upper/lower limit same as in case of $IP(j, R_{ijm}, f) / IR(j, R_{ijm}, f)$.

$IP(k, P_{jkn}, f) / IR(k, P_{jkn}, f)$. Upper limit of interest to be paid/received by transferee k in cases where the payment date of payment P_{jkn} is extended/contracted to date f . Transferee k decides the upper/lower limit same as in case of $IP(j, R_{ijm}, f) / IR(j, R_{ijm}, f)$.

C_{xe} . Fluctuation target sum of current assets of participant x on date e

Variables.

$v(R_{ijm}, P_{jkn}, f)$. Transferred sum whereby electronic receivable R_{ijm} is allocated to payment P_{jkn} at a new due date f

The Objective function of this matching problem can be expressed as shown in Formula 1.

Objective Function.

$$Max(\sum_{ijkmn} f |D(R_{ijm}) - D(P_{jkn})| \cdot v(R_{ijm}, P_{jkn}, f) \quad (1)$$

The target of this problem is to make the best possible effort to ensure that the fluctuation targets of all participants are satisfied by the fluctuation of current assets resulting from changes to the due dates of electronic receivables and payments. To achieve this goal, it is preferable to match the combinations of payments and electronic receivables having longer discrepancies of due dates with the greatest transferred sums, within the range of the following constraints. As a result, the target function can be expressed as the sum total of the values obtained by multiplying the transferred sums by the discrepancies between the redemption due dates and payment due dates, and can be expressed by formula 1.

Meanwhile, the constraint formulae can be expressed by Formulae 2 through 15 as follows:

Constraint formulae.

$$v(R_{ijm}, P_{jkn}, f) \geq 0 \quad (2)$$

$$V(R_{ijm}) \geq \sum_{kn} f v(R_{ijm}, P_{jkn}, f) \quad (3)$$

$$V(P_{jkn}) \geq \sum_{im} f v(R_{ijm}, P_{jkn}, f) \quad (4)$$

$$C_{xe} \geq \sum_{jkmn} f v(R_{xjm}, P_{jkn}, f) + \sum_{ijmn} f v(R_{ijm}, P_{jkn}, f) \quad (5)$$

$$\text{where } C_{xe} \geq 0 \ \&\& \ D(R_{xjm}) \leq e < f \ \&\& \ f < e \leq D(P_{jkn})$$

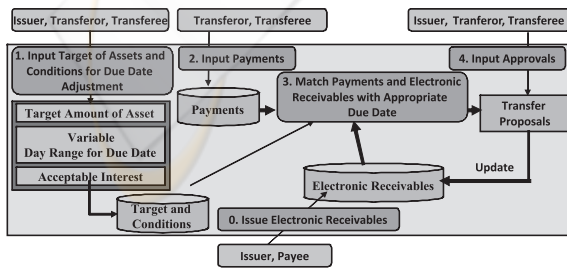


Figure 1: Process of Due Date Adjustment between Electronic Receivables and Payments.

$$C_{xe} \leq -\sum_{jkmn} v(R_{xjm}, P_{jkn}, f) - \sum_{ijm} v(R_{ijm}, P_{jxn}, f) \quad (6)$$

$$\text{where } C_{xe} < 0 \ \&\& \ f < e \leq D(R_{xjm}) \ \&\& \ D(P_{jxn}) \leq e < f$$

$$v(R_{ijm}, P_{jkn}, f) = 0 \quad (7)$$

$$\text{where } (f \leq D(R_{ijm}) \ \parallel \ D(P_{jkn}) \leq f) \ \&\& \ D(R_{ijm}) < D(P_{jkn}) \quad (8)$$

$$\text{where } (f \leq D(P_{jkn}) \ \parallel \ D(R_{ijm}) \leq f) \ \&\& \ D(P_{jkn}) < D(R_{xjm}) \quad (9)$$

$$\text{where } f < DS(R_{ijm}) \ \parallel \ f > DE(R_{ijm}) \ \parallel \ f < DS(P_{jkn}) \ \parallel \ f > DE(P_{jkn}) \quad (10)$$

$$\text{where } (IR(j, R_{ijm}, f) < IP(i, R_{ijm}, f) \ \parallel \ IR(j, P_{jkn}, f) < IP(k, P_{jkn}, f)) \ \&\& \ D(R_{ijm}) < D(P_{jkn}) \quad (11)$$

$$\text{where } (IR(i, R_{ijm}, f) < IP(j, R_{ijm}, f) \ \parallel \ IR(k, P_{jkn}, f) < IP(j, P_{jkn}, f)) \ \&\& \ D(P_{jkn}) < D(R_{xjm}) \quad (12)$$

$$\text{where } C_{ie} > 0 \ \&\& \ f < e \leq D(R_{ijm}) \quad (13)$$

$$\text{where } C_{ie} < 0 \ \&\& \ D(R_{ijm}) \leq e < f \quad (14)$$

$$\text{where } C_{ke} > 0 \ \&\& \ D(P_{jkn}) \leq e < f \quad (15)$$

$$\text{where } C_{ke} < 0 \ \&\& \ f < e \leq D(R_{jkn}) \quad (16)$$

The constraint formulae consist of the following three types:

1. Constraints in which the possible range of transferred sums is predetermined (formulae 2–4)
2. Constraints in which fluctuations of current assets arising from transfers associated with changes of due dates are kept within the fluctuation target (formulae 5–6)
3. Constraints in which the electronic receivables and payments that can be combined are predetermined (formulae 7–16)

Constraint formula 2 imposes the condition that the transferred sum must be positive.

Constraint formula 3 imposes the condition that the sum total of sums transferred when electronic receivables are transferred in separate parts does not exceed the sum of the original electronic receivables (but not necessarily equal since the splitting of electronic receivables may leave parts that are not suitable for transfer).

Constraint formula 4 imposes the condition that the sum total of payments made by the transfer of electronic receivables does not exceed the total amount of be paid (but not necessarily equal since new issues and combinations are made in cases where

the payable sum cannot be fully allocated by the transfer of electronic receivables alone).

Constraint formula 5 shows that when the fluctuation target of the current assets of participant x on day e is increased, the increase target of participant x on day e is greater than the sum of the total amount of electronic receivables drawn by participant x for an extension astride day e and the total amount of payments in which participant x is the transferee for a reduction astride day e (a participant's current assets are increased by the extension of redemption due dates on electronic receivables drawn by the same participant, or by the reduction in payment due date of payments received by the participant).

Constraint formula 6 is the converse of constraint formula 5, and shows that when the fluctuation target of the current assets of participant x on day e is decreased, the reduction target of participant x on day e is less than the sum of the total amount of electronic receivables drawn by participant x for a reduction astride day e multiplied by the negative value of the total amount of payments in which participant x is the transferee for an extension astride day e (a participant's current assets are reduced by the reduction of redemption due dates on electronic receivables drawn by the same participant, or by the extension in payment due date of payments received by the participant).

Constraint formulae 7–15 define the possible range of combinations of electronic receivables and payments. Specifically, the range for which reverse combinations are not possible is defined as a transferred amount of 0 in formula 7, and the applicable ranges are specified by formulae 8–16.

Range 8 and range 9 specify that electronic receivables and payments cannot be matched unless the new due date after modification is between the redemption date of the electronic receivables and the date on which the payment is due.

Range 10 specifies that electronic receivables and payments cannot be matched unless the new due date after modification is within the redemption dates of the electronic receivables and the possible range of payment due dates of the payment.

Range 11 specifies that in cases where the redemption due date of electronic receivables is before the date on which payment is due, matching of the electronic receivables and payments is not possible unless the interest to be paid by the receiver due to extension of the redemption due date on the electronic receivables is less than the interest required by the transferor due to this extension, and the interest paid by the assignee due to a reduction in the redemption due date is smaller than the interest required by the transferor

due to this reduction.

Range 12 relates to the case where the redemption due date comes after the date on which the payment is due, and is the converse of range 11 where the payer/payee relationships of interest payments by the debtor, transferor and transferee are reversed.

Range 13 specifies that when there is an increase in the fluctuation target for current assets on day e , it is not possible to perform matching with electronic receivables issued by the issuer involving a change in the redemption due date so as to reduce the redemption due date astride day e (a change that depletes current assets). Since this means it is possible to exceed the target if only one of the combinations is agreed upon, in this formularization the fact that fluctuation targets cannot be exceeded is added as a premise to this constraint.

Range 14 relates to the converse of range 13 in cases where the current formula fluctuation target of the issuer is reduced.

Ranges 15 and 16 are the same as ranges 13 and 14 where the issuer imposes limits on the redemption due data by means of restrictions on changes to the payment due date in transferee k .

With regard to the definition contents of the above target functions and constant formulae, first, target function 1 is the maximization of a primary function with $v(R_{ijm}, P_{jkn}, f)$ as a variable, and it takes an integer value. Also, the constraint formulae are all first-order inequalities that take $v(R_{ijm}, P_{jkn}, f)$ as a variable. This problem can therefore be classified as an integer linear programming problem.

4 EVALUATION

The effects of changes in due date and the effects of optimizing the combinations of electronic receivables and payments were verified by simulation. This section discusses the preconditions under which the simulation was conducted, and then presents the measurement results.

4.1 Preconditions

There is currently no statistical information relating to the issue of electronic receivables. We therefore performed the simulation by assuming conditions for the issue of electronic receivables based on financial information from Japanese businesses. Table 1 shows the financial information and the conditions for the issue of electronic receivables assumed in this simulation. The financial information was sourced from corporate statistics published by the National

Tax Agency and from settlement trends for 2003 published by the Bank of Japan, including the average sales figures for Japanese corporations, payable liabilities (accounts payable, bills payable), received credit (accounts receivable, bills receivable), average sum of bills cleared, and average sum of accounts receivable. The conditions for the issue of electronic receivables were assumed based on this financial information. Specifically, we made assumptions regarding the average redemption period of electronic receivables, the average frequency of issue and the average sum. The respective calculation formulae are shown below.

- Average redemption period of electronic receivables = receivable credit / sales \times 365
- Average number of electronic receivables issued = (bills receivable / average sum of bills cleared + accounts receivable / average sum of accounts receivable) / 365
- Average sum of credit = receivable credit / (bills receivable / average sum of bills cleared + accounts receivable / average sum of accounts receivable)

In the simulation, the number of companies was taken to be 260 (one thousandth of the actual number of businesses), and measurements were performed by repeating the transactions over two years. In real situations, not necessarily all the credit is replaced with electronic receivables, and not necessarily all the electronic receivables are subject to being transferred, so the simulation was performed by making a few changes to the ratio of transferable sums with regard to the credit sums of the electronic receivables belonging to a business. The simulation environment parameters were as follows: MPU: Xeon¹ 2.8 GHz, Memory: 3 GByte, Windows XP², JDK 1.6.0.01³, LpSolve 5.5.0.10 (Berkelaar et al., 2004). In the evaluation results shown in the next section, measurements were also performed by varying some conditions of the other parameters (variation in redemption periods of electronic receivables, variation in frequency of issue of electronic receivables, variation in monetary value of electronic receivables, number of companies simulated), but the effects of these changes were smaller than those of the parameters shown in Table 1 and thus these results are omitted.

¹Xeon is a registered trademark of Intel Corporation.

²Windows XP is a registered trademark of Microsoft Corporation.

³Java is a trademark of Sun Microsystems, Inc.

Table 1: Financial Statement of Average Japanese Company and Assumed Issue Condition of Electronic Receivables.

Total Sales(k¥)	20,483
Total Receivables(k¥)	82,887
Account Receivables(k¥)	66,287
Note Receivables(k¥)	16,600
Average Amount of Account Receivables	2,000
Average Amount of Note Receivables	4,000
Assumed Average Redemption Period of Electronic Receivables(Day)	59
Assumed Average Issue Cycle of Electronic Receivables(Times/Day)	0.102
Assumed Average Amount of Electronic Receivables(k¥)	2222

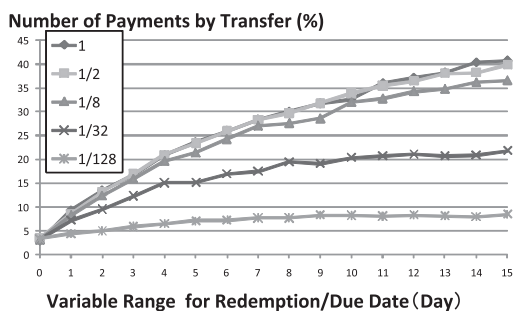


Figure 2: Ratio of Payments by Transfer.

4.2 Measurement Results

We will first use the due date modification function to investigate the extent to which payments are promoted by transfers. Next, by optimizing the combinations of payments and electronic receivables, we will verify the extent to which it is possible to achieve the participants' fluctuation targets. Finally, we will investigate the computational load required for optimization.

The graph in Fig. 2 shows the ratio of all payments in which payment was made by assignment of claim. The horizontal axis shows the number of days by which the payment due date and redemption due date can be varied in either direction. The multiple measurement results in this graph correspond to measurements made while varying the criteria regarding the extent to which the electronic receivables owned by a business are transferred on a monetary basis. This evaluation shows the results obtained when the matching of electronic receivables and payments is optimized.

In these results, the case where the modifiable date is 0 corresponds to the case where no changes can be made to the redemption due date of the electronic receivables. In this case, the ratio of the payment that

Ratio of Adjusted Asset to Target

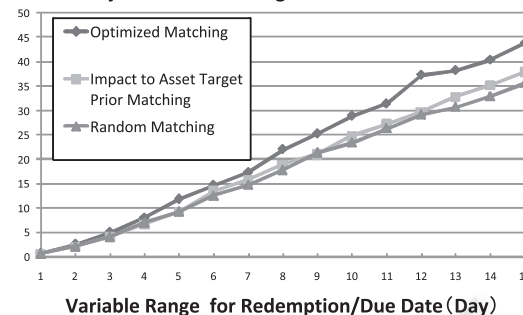


Figure 3: Ratio of Adjusted Asset to Target.

can be assigned in the transfer of electronic receivables is about 3.3% of the total. On the other hand, when the redemption due date and payment due date are set with the ability to be varied forwards or backwards by up to 3 days, the ratio of the payment in the transfer increases to 16.7% (about 5 times larger) when all the electronic receivables are transferred. Also, even when the ratio of the electronic receivables to be transferred is reduced to about 1/32 of the total electronic receivables, an increase of 12.3% (about 3.7 times) is seen. These results show that by providing a framework for making adjustments of a few days in the redemption due date or payment due date, each business can make a relatively large improvement to the ratio of payments made by transfers.

The optimization results are shown next. Figure 3 shows the extent to which it is possible to achieve the fluctuation targets of current assets set by the participants for each method of matching a number of electronic receivables and payments. The specific matching methods used to make these measurements were as follows:

- Randomly ordered Matching. Matching is performed in random order from among candidates consisting of electronic receivables and payments that are capable of being combined.
- Matching in Order of Effects on the Level of Achievement. Matching is performed in order of the magnitude of how the achievement of the fluctuation targets is affected by candidates consisting of electronic receivables and payments that are capable of being combined.
- Matching based on Optimal Pattern Searching. Matching is performed by searching for optimal patterns that are closer overall to the fluctuation targets.

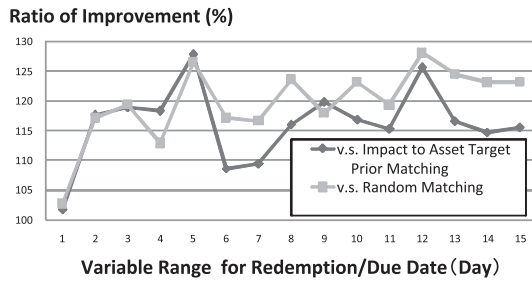


Figure 4: Effect of Optimization.

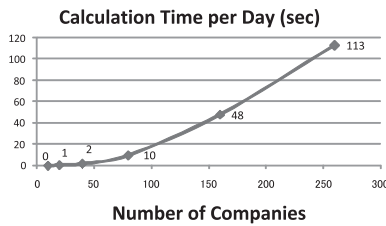


Figure 5: Calculation Time for Daily Optimization.

The horizontal axis in the graph of Fig. 3 shows the number of days by which the payment due date and redemption due date can be varied in either direction. The vertical axis shows the extent to which the fluctuation targets preset by each business could be achieved. With regard to these measurements, the fluctuation targets of the businesses were set randomly in the range of the sums of electronic receivables owned by these businesses. Figure 4 compares the improvement rate of optimization between random matching and matching based on the effects on the level of achievement. These results show that although there is some degree of variation when there are few modifiable days, the average ratio of improvement is approximately 18.1% for matching based on the effects on the level of achievement, and approximately 19.6% for random matching.

Next, the computation time needed to optimize the combinations of electronic receivables and payments is investigated using the graph shown in Fig. 5. In this graph, the number of businesses is shown on the horizontal axis and the time required for the optimization computations per day is shown on the vertical axis. In a simulation of 260 companies, the computation time required for optimization was 113 seconds per day on average. This remains future work to make this method in practice because this simulation done with 1/1000 of actual number of companies and the computation time grows exponentially according to increase the number of companies. We discuss this problem in section 5.

Finally, Table 2 shows the extent to which the

Table 2: Fragmented Number of the Electronic Receivables.

	Average	Standard Deviation	Max
Random Matching	2.09	1.05	10
Impact to Asset Target Prior Matching	1.38	0.57	7
Optimized Matching	2.06	1.93	9

electronic receivables are fragmented by the matching of due dates. The number of fragments is a value that shows on average how many electronic receivables the original electronic receivables are divided into at the time of redemption. The average number of fragments was 2.08 when matching was performed in random order, 1.38 when matching was performed in order of the effects on the effects on the level of target achievement, and 2.06 when matching was performed based on optimal pattern searching. The reason for the small number of fragments obtained when matching in order of the effects on the effects on the level of target achievement is thought to be because in this algorithm, matching is performed preferentially on groups of electronic receivables and payments involving larger sums and greater differences in due dates, and these larger sums suppress the fine fragmentation of electronic receivables.

5 CONCLUSIONS

In this study, we have proposed a scheme for optimizing the matching of redemption due dates and payment due dates for electronic receivables to promote the transfer of electronic receivables. Specifically, we have demonstrated an optimization scheme that makes every effort to make the fluctuations in the current assets of each business (caused by changes of due date) approach their fluctuation targets based on criteria specified by each business regarding the fluctuation targets of current assets, the range of alterations to due dates, and the rates of interest. We have also shown that this problem can be classified as a type of integer linear programming problem. By performing simulations based on the financial circumstances of average Japanese businesses, we have shown that this technique is capable of promoting transfers involving changes of due date and improving the degree to which fluctuation targets are achieved by optimization.

One issue for further study is the problem of electronic receivables being finely fragmented by the optimization process. In the method of this study, if the fluctuation targets of the participants' assets are im-

proved, then the electronic receivables can be arbitrarily fragmented and a number of payments will be assigned to a number of new due dates. This fragmentation of electronic receivables leads to increased administration costs, so to actually put this method into practice, it is important to investigate how to control the fragmentation of electronic receivables.

A second issue is that of partitioning the optimization regions. In this study, optimization was performed by using a single target function to represent the criteria of all the companies concerned, but this is inefficient with regard to increasing the scale of businesses to which the method is applied. In practice, there is considered to be some degree of locality in the transaction relationships between businesses, so if groups of businesses can be split into suitable ranges, then it should be possible to split the optimization problem into multiple sub-problems with fewer variables and constraints. In this way it should be possible to reduce the computational cost and speed up the computation time by employing parallel processing.

A third issue is that not necessarily all of the optimal matching results demonstrated by this algorithm are the best matching results from the viewpoint of each individual business. For actual operations, an important issue is therefore to somehow present alternative proposals in cases where a business rejects the matching results.

REFERENCES

- Aardal, K., Nemhauser, G., and Weismantel, R. (2005). *Optimization: Handbooks in Operations Research and Management Science*, volume 12. Elsevier.
- Berkelaar, M., Eikland, K., and Notebaert, P. (2004). Ip-Solve: Open Source (Mixed-Integer) Linear Programming System. GNU LGPL (Lesser General Public License).
- Fairchild, A. (2003). Possible Distintermediation: What Role for Banks in Electronic Invoicing(EIPP). In *16th Bled eCommerce Conference eTransformation*, pages 107–118. <http://domino.fov.unim-b.si/ECOMFrames.nsf/pages/bled2003>.
- FSA (2005). Summary of Discussion on Electronic Receivable Legislation from a Financial System Perspective. *Newsletter of Financial Services Agency, The Japanese Government*, pages 2–5. <http://www.fsa.go.jp/en/newsletter>.
- Ikeda, M. (2006). Electronic receivables - examination and essay for legislation - (in japanese). *Banking Law Journal, Kinzai Institute for Financial Affairs*, 1781:8–19.
- JSBRI (2007). *White Paper on Small and Medium Enterprises in Japan*. Japan Small Business Research Institute. http://www.chusho.meti.go.jp/sme_english/whitepaper/whitepaper.html.
- KFTC (2002). Check clearings. *KFTC 2002*, pages 18–19. <http://www.kftc.or.kr/english/statistics/KFTCAR.pdf>.
- Oogaki, H. (2006). Finance in secondary loan market and electronic receivables (in japanese). *Banking Law Journal, Kinzai Institute for Financial Affairs*, 1781:20–28.
- Schrijver, A. (1986). *Theory of Linear and Integer Programming*. Wiley-Interscience.
- Shimamura, A., Moritsu, T., and Someya, H. (2006). Delivery path length and holding tree minimization method of securities delivery among the registration agencies connected as non-tree (in japanese). *IEEJ Trans.*, 126-C(4):506–512.