NOISE REDUCTION BASED ON MEDIAN ε- FILTER

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Abstract: This paper describes a nonlinear filter, which can reduce the impulse noise with preserving the edge information labeled median ε-filter. ε-filter is a nonlinear filter, which can reduce the small amplitude noise with preserving the edge information. The algorithm is simple and it has many applications because it uses only switching and linear operations. Although it is difficult to reduce the impulse noise by using ε-filter due to its features, we can reduce the impulse noise effectively with preserving the edge information by combining the concept of median filter and ε-filter. Due to its simple design, the calculation cost is relatively small the same as ε-filter. To show the effectiveness of the proposed method, we also report the results of some comparative experiments concerning the filter characteristics.

1 INTRODUCTION

Noise reduction has an important role in image processing. Although there are many kinds of noise to be reduced, impulse noise reduction is one of the important topics because it appears in many practical cases due to errors generated in noisy sensors or communication channels. Although there are many studies to reduce the impulse type noise (Pitas and Venetsanopoulos, 1990), median filter is an attractive filter to reduce the impulse noise (Ko and Lee, 1991; Brownrigg, 1984; Lin and Willson, 1988). It is simple and effectively reduces the impulse noise. However, as median filter smooths not only the pixels which include impulse noise but also the undistorted pixels, it damages the good pixels.

Some researches aim to avoid the damages to the image by combining the impulse detector and median filter (Abreu et al., 1996; Wang and Zhang, 1999; Sun and Neuvo, 1994; Wang and Zhang, 1998; Muneyasu et al., 2000; Yamashita et al., 2006). Although these approaches generally work well, the estimation error degrades the filter performance. As these filters also employ the learning algorithms or detecting process, it requires much calculation cost compared to the simple median filter. They also need the noise free image for learning.

To solve the problem, we look to a nonlinear filter labeled ε-filter (Harashima et al., 1982; Arakawa et al., 2002; Arakawa and Okada, 2005). ε-filter is a nonlinear filter, which can reduce the small amplitude noise with preserving the edge the same as the bilateral filter (Tomasi and Manduchi, 1998). The algorithm is simple and the calculation cost is small compared to bilateral filter because it requires only switching and linear operation. Although it is difficult to reduce the impulse noise by using ε-filter due to its design, the concept of ε-filter can be expanded to design the improved median filter. Based on the aspects, we propose a nonlinear filter combining the concepts of ε-filter and median filter labeled *median ε-filter*. The algorithm is simple and it can effectively reduce the impulse noise. As it does not employ the impulse detector, it does not require the noise free image or learning process. In the next section, we first describe the algorithm of the conventional ε-filter to clarify its feature. In Sec.3, we describe the algorithm of the median ε-filter. In Sec.4, we show our experimental results of evaluating the filter characteristics of median ε-filter. We also compare the filter performance with some other filters. Conclusions follow in Sec.5.

2 ε-FILTER

We firstly explain the algorithm of ε-filter. To clarify the feature of ε-filter, we first describe the one dimensional case. Let us define $x(k)$ as the input signal (For instance, the signal including speech signal with

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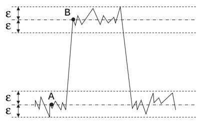
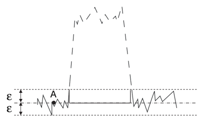
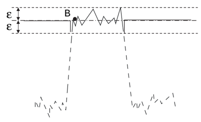


(a) Input signal



(b) In the case of applying ε-filter to the point A



(c) In the case of applying ε-filter to the point B

Figure 1: Basic concept of ε-filter.

noise) at time k. Let us also define $y(k)$ as output signal of ε-filter at time k as follows:

$$y(k) = x(k) + \sum_{i=-M}^{M} a(i)F(x(k+i) - x(k)), \quad (1)$$

where $a(i)$ represents the filter coefficient. $a(i)$ is usually constrained as follows:

$$\sum_{i=-M}^{M} a(i) = 1. \quad (2)$$

The window size of ε-filter is $2M+1$. $F(x)$ is the nonlinear function described as follows:

$$|F(x)| \leq \varepsilon : -\infty \leq x \leq \infty, \quad (3)$$

where ε is the constant number. This method can reduce small amplitude noise while preserving the signal. For example, we can set the nonlinear function $F(x)$ as follows:

$$F(x) = \begin{cases} x & (-\varepsilon \leq x \leq \varepsilon) \\ 0 & (else) \end{cases}. \quad (4)$$

Figure 1 shows the basic concept of ε-filter in case that we utilize Eq.4 as $F(x)$. Fig.1(a) shows the waveform of the input signal. Executing ε-filter at the point A in Fig.1(a), we replace all the points whose difference from A is larger than ε by the value of the point A. We then summate the signals in the same window. Fig.1(b) shows the basic concept of this procedure. In Fig.1(b), the dotted line represents the points whose difference from A is larger than ε. In Fig.1(b), the solid line represents the values replaced through this procedure. As a result, if the difference between the point A and the other point is large, the point is ignored. On the other hand, if the difference between the point A and the other point is small, the point is smoothed. Because of this procedure, ε-filter reduces the noise with preserving the precipitous attack and decay of the speech signal, In the same way, executing ε-filter at the point B in Fig.1(a), we replace all the points whose difference from B is larger than ε by the value of the point B. The point is ignored if the difference from the point B is large, while the point is smoothed if the difference from the point B is small. Consequently, we can reduce the small amplitude noise near by the processed point while preserving the speech signal.

ε-filter can easily be improved not only for one dimension but also for two dimension. Let us define $x(k,l)$ as the two dimensional input signal at (k,l). When we apply ε-filter to two dimensional data such as image, ε-filter is designed as follows:

$$\begin{aligned} y(k,l) &= x(k,l) \\ &+ \sum_{i=-M}^{M} \sum_{j=-M}^{M} a(i,j)F(x(k+i,l+j) - x(k,l)), \end{aligned} \quad (5)$$

where $a(i,j)$ represents the filter coefficient. $a(i,j)$ is usually constrained as follows:

$$\sum_{i=-M}^{M} \sum_{j=-M}^{M} a(i,j) = 1. \quad (6)$$

The feature of two dimensional ε-filter is similar to that of one dimensional ε-filter. We can smooth the small amplitude noise near by the processed point while preserving the edge. It requires less calculation when it is compared to bilateral filter because it requires only switching and linear operation. However, it is difficult to reduce the impulse noise because of its feature.

3 MEDIAN ε-FILTER

To reduce the impulse noise with preserving the edge information, we combine the concept of median filter and ε-filter. In median filter, we set the adequate window and replace the filtered point to the median value in the window. The median filter can reduce the impulse noise. However, the edge information is sometimes damaged. To solve the problems, median ε-filter is designed as follows:

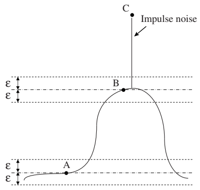
$$y(k) = med_{i=-M}^{M}[v(k+i)], \quad (7)$$

where $med_{i=-M}^{M}[v(k+i)]$ represents the median function of $v(k)$ from $v(k-M)$ to $v(k+M)$. $v(k+i)$ is described as follows:

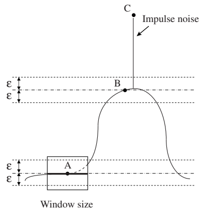
$$v(k+i) = x(k+i) + F(x(k) - x(k+i)), \quad (8)$$

where $F(x)$ is the same nonlinear function as Eq.(3) constrained as follows:

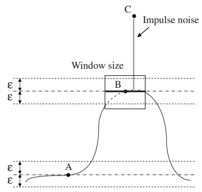
$$|F(x)| \leq \varepsilon : -\infty \leq x \leq \infty, \quad (9)$$



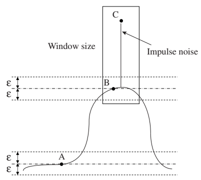
(a) Input signal



(b) In the case of applying median ε-filter to the point A



(c) In the case of applying median ε-filter to the point B



(d) In the case of applying median ε-filter to the point C

Figure 2: Basic concept of median ε-filter.

where ε is the constant number. For example, we can set the nonlinear function $F(x)$ similar to ε-filter as follows:

$$F(x) = \begin{cases} x & (-\varepsilon \le x \le \varepsilon) \\ 0 & (else) \end{cases} . \quad (10)$$

In this case, the output of median ε-filter can be described as:

$$y(k) = med_{i=-M}^{M}[v(k+i)], \quad (11)$$

where

$$v(k+i) = \begin{cases} x(k) & (|x(k+i)-x(k)| \le \varepsilon) \\ x(k+i) & (|x(k+i)-x(k)| > \varepsilon) \end{cases} . \quad (12)$$

Figure 2 shows the basic concept of median ε-filter in case that we utilize Eq.(10) as $F(x)$. Fig.2(a) shows the waveform of the input signal. Executing ε-filter at the point A in Fig.2(a), we replace all the points where the difference from A is smaller than ε by the value of the point A. We then apply the median filter to the signals in the same window. Fig.2(b) shows the basic concept of this procedure. In Fig.2(b), the dotted line represents the points where the difference from A is smaller than ε. In Fig.2(b), the solid line represents the values replaced through this procedure. As the median filter outputs the median value in the window, the input signal is basically intact when there are many signals in the window within ε as shown in Fig.2(b). In a similar fashion, executing ε-filter at the point B in Fig.2(a), we replace all the points where the difference from B is smaller than ε by the value of the point B. In this case, although the point C is included in the window, as there are also many signals in the window within ε, the input signal is also intact. Executing ε-filter at the point C (Impulse noise point) in Fig.2(a), as the differences between the point C and the other points are larger than ε in the window, the points within the same window are not replaced. Hence, at the point C, median ε-filter works as a simple median filter and reduces this impulse noise. Due to the above features, the median ε-filter can reduce the impulse noise without damaging the good pixels. To improve median ε-filter for two dimensional data, we only have to design it as follows:

(a) Original image of Lena.

(b) Original image with noise.

Figure 3: Original image (Lena) and input image with impulse noise.

$$y(k,l) = med_{i=-M,j=-M}^{M,M}[v(k+i,l+j)], \quad (13)$$

where

$$v(k+i,l+j) = x(k+i,l+j) + F(x(k,l) - x(k+i,l+j)). \quad (14)$$

The feature of two dimensional median ε-filter is similar to that of one dimensional median ε-filter. The calculation cost of median ε-filter is small the same as ε-filter because it requires only the switching and replacing operation. We can also reduce both impulse noise and small amplitude noise with preserving edge information by combining the conventional ε-filter and median ε-filter.

4 EXPERIMENTS

To show the effectiveness of the proposed method, we first show the output images after the filtering. As examples, we show the results using Lena and Boat listed in standard image database (SIDBA). To compare the effectiveness of the proposed methods to the other methods, we also show the filter outputs of the simple median filter and progressive switching median filter (PSMF) (Wang and Zhang, 1999). In the experiments, we set the window size to 5 pixels × 5 pixels. Figure 3 and Figure 5 show the examples of Lena and Boat, respectively. They show the original images and the input images with impulse noise


(a) Output image of median filter.


(b) Output image of progressive switching median filter.


(c) Output image of median ε-filter.

Figure 4: Filter outputs using Lena.


(a) Original image of Boat.


(b) Original image with noise.

Figure 5: Original image (Boat) and input image with impulse noise.

whose noise level is 10%, i.e. 10% pixels of the image are added the impulse noise. Figure 4 and Figure 6 show the filter outputs of the simple median filter, PSMF and the median ε-filter concerning Lena and Boat, respectively. As shown in Figs.4(a) and 6(a), when we employed the simple median filter, as all the pixels were filtered, the edge information was also damaged. When we employed the PSMF, although the edge was preserved compared to the simple median filter, some impulse noises remained due to the estimation error as shown in Figs.4(b) and 6(b). On the other hand, the median ε-filter could reduce the impulse noise without damaging the original image information.

To clarify the effectiveness of the proposed method, we also calculated the mean absolute error (MAE). In the experiments, we used ten images listed in SIDBA


(a) Output image of median filter.


(b) Output image of progressive switching median filter.


(c) Output image of median ε-filter.

Figure 6: Filter outputs using Boat.

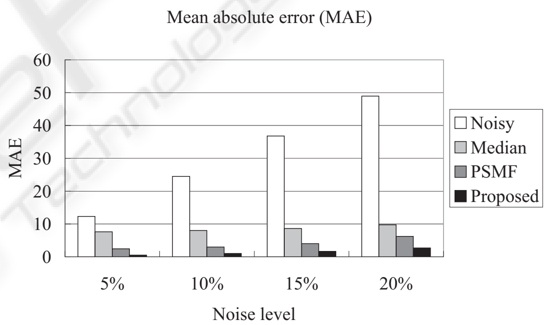


Figure 7: Mean absolute error of input signal, median filter and median ε-filter with changing the noise level from 5% to 20%.

and changed the noise level from 5% to 20% with 5% interval. Tables 1, 2, 3, and 4 show the MAE results when we used the input image with the impulse noise whose noise level was 5%, 10%, 15% and 20%, respectively. To clarify the effectiveness of the proposed method compared to the other methods, we also show the average MAE. Figure 7 shows the average MAEs of input signal, median filter, PSMF and median ε-filter. As shown in Fig.7, the MAE values of the proposed filter are much smaller than those of median filter in all the cases in spite of its simple design.

Table 1: MAE results when we added impulse noise whose noise level is 5%.

	Noisy	Median	PSMF	Proposed
Lena	12.1282	3.9409	0.7034	0.2803
Mandrill	12.2644	12.1395	4.7043	0.6881
Airplane	12.4084	5.9639	1.8183	0.4003
Barbara	12.3033	12.4223	4.8263	0.7331
Boat	11.922	4.7659	1.0482	0.2938
Bridge	12.3889	13.1145	4.7397	0.749
Building	12.0309	7.5519	1.4239	0.453
Girl	12.3734	4.0123	0.4649	0.2812
Lax	12.5095	12.1752	5.5711	0.8165
Woman	12.3111	5.2059	1.1797	0.364

Table 2: MAE results when we added impulse noise whose noise level is 10%.

	Noisy	Median	PSMF	Proposed
Lena	24.6922	4.3668	1.1233	0.5985
Mandrill	24.0891	12.474	5.1548	1.388
Airplane	24.4821	6.444	2.4129	0.8411
Barbara	24.3965	12.7899	5.4746	1.4835
Boat	24.2603	5.1177	1.5635	0.6371
Bridge	24.7934	13.5914	5.6281	1.6209
Building	24.6961	8.0907	1.9957	1.0329
Girl	24.8829	4.4298	0.8757	0.5783
Lax	24.4549	12.4003	5.9064	1.4839
Woman	24.0269	5.5859	1.7034	0.7206

Table 3: MAE results when we added impulse noise whose noise level is 15%.

	Noisy	Median	PSMF	Proposed
Lena	36.7582	4.8799	2.1492	1.006
Mandrill	36.3613	13.052	6.1686	2.2151
Airplane	37.1239	7.1861	3.3719	1.4578
Barbara	37.3263	13.3853	6.7379	2.4936
Boat	36.2212	5.7219	2.5915	1.0588
Bridge	37.1201	14.4864	6.6049	2.666
Building	36.6181	8.849	3.2259	1.6762
Girl	36.6804	4.9845	1.7034	1.0209
Lax	37.0422	13.0186	7.0353	2.3751
Woman	36.5364	6.1773	2.6824	1.1469

Table 4: MAE results when we added impulse noise whose noise level is 20%.

	Noisy	Median	PSMF	Proposed
Lena	49.0226	5.947	4.0882	1.8503
Mandrill	49.2094	14.178	8.5962	3.4303
Airplane	49.2094	8.2621	5.7692	2.3816
Barbara	49.0226	14.3624	8.8842	3.7084
Boat	49.0109	6.637	4.6289	1.8491
Bridge	48.3533	15.9184	9.1091	4.1476
Building	48.4701	9.9992	5.6002	2.7381
Girl	49.0148	5.9428	3.5904	1.7574
Lax	49.0421	14.428	9.3909	3.957
Woman	48.2444	7.2482	4.8524	1.997

5 CONCLUSIONS

In this paper, we proposed a nonlinear filter which can reduce the impulse noise with preserving the image information labeled median ε-filter. The proposed filter is simple and can reduce the noise effectively compared to the simple median filter or more complicated median filters. It does not require the noise free image or learning process. For future works, we would like to employ the median ε-filter for musical noise reduction by applying it to the acoustical signal in time-frequency domain.

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