FIXED POINT SVD COMPUTATION ERROR CHARACTERIZATION AND PERFORMANCE LOSSSES IN MIMO SYSTEMS

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Abstract: This paper is devoted to analyze the error met when computing the singular value decomposition (SVD) of the gain channel matrix in a MIMO communication system and using fixed-point arithmetic for the calculations. The study is focused in the case of the SVD implementation for modulation-mode and power assignment in non-frequency selective MIMO, and M-ary Quadrature Amplitude Modulation (QAM). It is demonstrated that not necessarily all MIMO layers must be activated. The combination of the CORDIC algorithm and look-up tables seems to be a good solution for this task since it can efficiently compute the singular value decomposition. The paper highlights the characterization of computation errors and shows the performance losses produced by the use of approximations in the eigenvalues and eigenvectors calculation, including the effects produced in the power assignment process.

1 INTRODUCTION

MIMO technology has attracted a lot of attention in wireless systems, since it offers significant increases in data transmission rate and link range without the need of providing larger bandwidths or transmitted power, and a significant BER improvement. The main goal of MIMO techniques is achieving higher spectral efficiency and increased reliability. The application of such techniques involves the appropriate data processing to obtain their expected advantages and can be considered as an essential part of increasing both the achievable capacity and integrity of future generations of wireless communication systems (Kühn, 2006), (Zheng and Tse, 2003).

The SVD performs an estimation of the eigenvalues and eigenvectors of the gains channel matrix. It allows transforming a MIMO channel into multiple single input single output (SISO) channels having unequal gains.

In order to avoid any signalling overhead, fixed transmission modes are investigated in (Ahrens and Lange, 2008) regardless of the channel quality. The study's results have shown that not all MIMO layers have to be activated in order to achieve the best bit error rate.

Assuming perfect channel state information (PCSI), the channel capacity can only be achieved by using water-pouring procedures. However, in practical application only finite and discrete transmission rates are possible. Therefore, in this contribution the efficiency of fixed transmission modes is studied regardless of the channel quality.

Furthermore, this paper focuses on the analysis of the error met when computing the SVD of the gain channel matrix in a MIMO system and using finite word length and fixed point arithmetic, specifically for modulation-mode and power assignment, using computationally efficient algorithms.

The paper remarks the feasibility of using appropriate approximations to implement the SVD

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providing low computational load operations with small performance degradation, focusing on MIMO channels using M-ary QAM.

2 SYTEM MODEL DESCRIPTION

We consider a single MIMO system with PCSI at both the transmission and reception sides. We model the channel as a flat independent and identically distributed (i.i.d.) Rayleigh channel. Given a MIMO system with n_T aerials at the transmitter and n_R at the receiver, the received signal can be expressed as

$$\mathbf{y}=\mathbf{H}\bullet\mathbf{s}+\mathbf{n}, \tag{1}$$

where **y** is the $n_R \times 1$ sized received signal vector, **s** is the $n_T \times 1$ transmitted signal vector containing the complex input symbols and **n** is the $n_R \times 1$ vector of the additive white Gaussian noise (AWGN) with variance var(Re[n_R]) = var(Im[n_R]) = $\sigma^2/2$.

H is the $n_R \times n_T$ complex channel gain matrix with entries also having unit magnitude variance. We assume that n_T equals n_R. In order to convert the MIMO system into several SISO channels, data vector **s** is multiplied by the matrix **V** before transmission. The received signal is preprocessed by multipling it by the matrix $\mathbf{U}^{\mathbf{H}}$ to obtain the received data vector $\hat{\boldsymbol{y}}$.

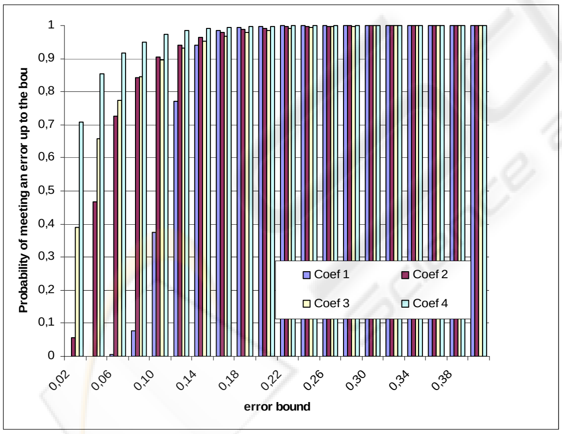

Figure 1: Probability of getting eigenvalues a magnitude error up to defined values, for n_T=n_R=4 MIMO case.

3 THE SVD PRINCIPLES

The SVD of a real $n_R \times n_T$ matrix **H** means factorizing into the product of three matrices,

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{H}, \tag{2}$$

where **U** and **V** are orthogonal $n_R \times n_T$ unitary matrices and $\mathbf{\Sigma}$ = diag($\sigma_1,\sigma_2,...,\sigma_n$) is an $n_R \times n_T$ diagonal matrix containing the singular values of **H**, where n equals $min(n_R,n_T)$. The columns of **U** and **V** are called respectively the left and right singular vectors and σ_i the ith singular value of **H**. Within this work, we assume that n_T equals n_R, i.e., **H** is assumed to be a non-singular square matrix.

4 FUNDAMENTALS OF THE CORDIC ALGORITHM

The CORDIC (COordinate Rotation Digital Computer) algorithm consists of an iterative algorithm that allows computing relatively complex functions by using simple operations, just addition and shift operations.

In the most general form, a CORDIC algorithm iteration used to perform a rotation is described by:

$$\begin{aligned} &z(k+1)=z(k)-\sigma \cdot atan(2^{-k}),\\ &x(k+1)=x(k)-y(k)\cdot \sigma\cdot(2^{-k}),\\ &y(k+1)=y(k)+x(k)\cdot\sigma\cdot(2^{-k}), \end{aligned} \tag{3}$$

where k ranges from 0 to N, being N the number of iterations, and σ takes the value +1 if $z(k)$ is equal or greater than 0 and –1 otherwise, and the samples x and y correspond to the coordinates. The target angle equals $z(0)$. Phase micro-rotations are determined by z that is forced to become zero in the iterative process. The application of the CORDIC algorithm using (3) has the immediate advantage that it does not require product units. Some efficient strategy must be used to compute the *atan* function.

Authors present more details on the implementation of the singular value decomposition using finite word length and fixed-point arithmetic in (Benavente-Peces et al., 2008).

5 ERROR DESCRIPTION AND ANALYSIS

This section analyses the error computing eigenvalus eith fixed point arithmetic to predict system performance and degradation. As an example we have considered a 4x4 (four transmitting and four receiving antennas) MIMO system and the channel is modeled as a flat independent and identically distributed (i.i.d.) Rayleigh one.

Figure 1 shows the computation of the probability of getting the eigenvalues an error up to a predefined bound. Eigenvalues are arranged from largest to smallest as Coef 1 to Coef 4. If we take into acount that eigenvectors are arranged in decreasing order, it is noticable that lower values are affected by larger errors.

The eigenvalues were computed using the Jacobi method and the CORDIC algorithm was applied with 16-bits. The main conclusion is that power allocation will be affected by the erros in the eigenvalues computation and degrading the system performance.

The Jacobi iterations search for largest off-diagonal element in **H** and eliminates it by rotations, finding one eigenvalue. So, the largest eigenvalue is obtained at first, being affected by smaller errors. Fortunately, the largest eigenvalue is the most influencing in the transmission, producing a lower effect on the performance degradation.

Figures 2 to 5 show the error probability distribution met for each eigenvalue (Coef 1 to Coef 4) in the interval we obtain the more significant values.

The distribution of errors for “Coef 1” and “Coef 3” seem to follow a gaussain distribution with the mean given by the mean value of the computation error.

On the other hand, the errors for “Coef 2” and “Coef 4” are far a way from a gaussian distribution. In the case of “Coef 2”, it could be considered as a gaussian distribution in a narrow interval due to the assimetry. Errors in “Coef 4” eigenvalue looks an exponential distribution function.

This behaviour suggests the possibility of modelling the computation of the eigenvalues as the given value disturbed by noise that could be approximated by a gaussian distribution. It allows the prediction of the system performance.

6 QUALITY CRITERIA

There are two possibilities, one based on the measurement of the channel capacity and compare the results to the traditional SISO (single input single output) system, and other based on the BER measurement obtained for various SNR and compare the gain of the MIMO system using full precision in SVD computation to the SISO case.

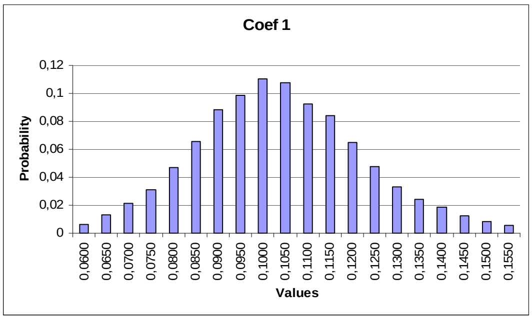

Figure 2: Probability error distribution for Coef 1.

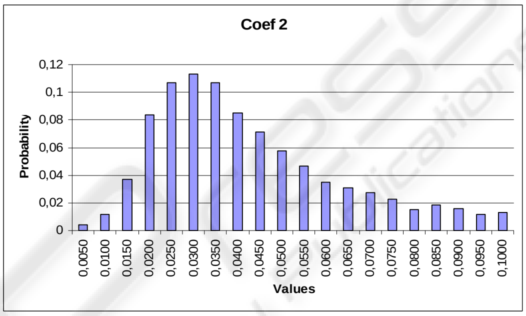

Figure 3: Probability error distribution for Coef 2.

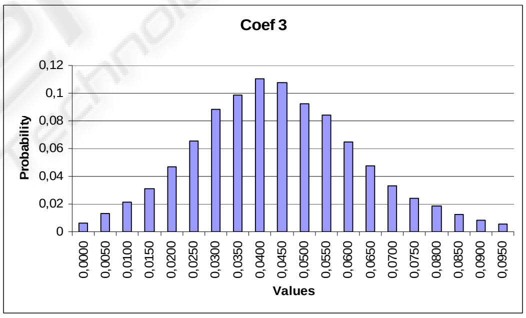

Figure 4: Probability error distribution for Coef 3.

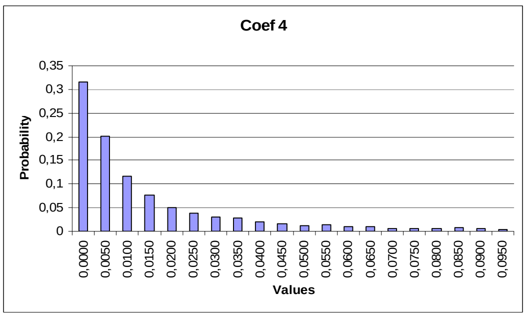

Figure 5: Probability error distribution for Coef 4.

6.1 Channel capacity

Figure 6 shows the channel capacity of the MIMO channel for various n_T and n_R combinations. The unmarked line corresponds to SVD full precision computation. Marked lines correspond to the MIMO channel capacity computed using the CODIC with fixed point arithmetic. As consequence of the errors, the power is not allocated appropriately provocating capacity losses. These effects are more remarkable for larger signal to noise ratios where computation noise is more noticiable. Besides, those errors are large for larger number of antennas.

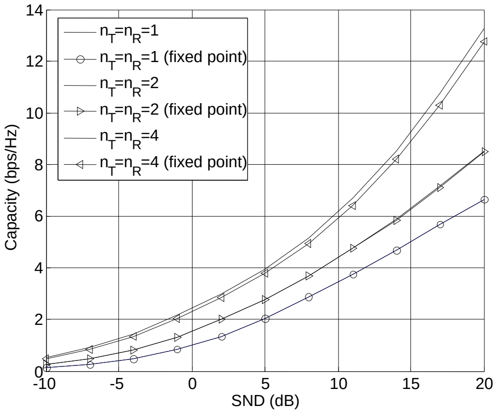

Figure 6: Channel capacity.

6.2 Bit Error Rate

In general, the quality can be informally assessed by using the (SNR) at the detector's (Ahrens and Lange, 2007).

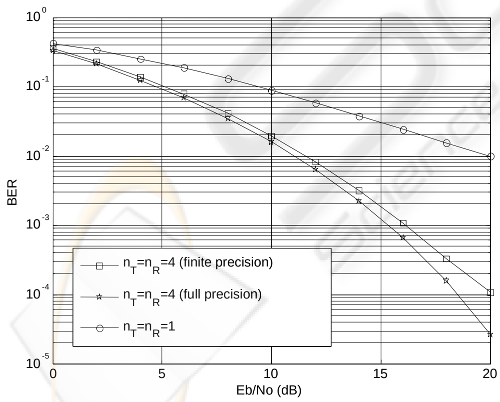

Figure 7: BER comparison for the case n_T=n_R=4.

Figure 7 shows the BER for various signal to noise ratios. QAM is used on all activated MIMO layers. In the figure we compare the results obtained using full precision with those obtained when using the CORDIC with finite word length and fixed point arithmetic to compute the SVD and these values are used to analyse the MIMO system performance.

7 CONCLUSIONS

The work reveals that larger errors are found for the smaller eigenvalues. This is due to the Jacobi algorithm application to perform the SVD decomposition using the CORDIC iterations. We conclude that the largest singular values are quite robust against computation errors and vice verse. Finite word length and fixed point arithmetic decreases system capacity. The loss results very little when using 16 bits for SVD computation and data processing. From the point of view of the BER, the use of finite word length and fixed point arithmetic increases the BER in relation to the full precision implementation. For 16-bits arithmetic the losses are moderate. The combined use of the CORDIC algorithm and look-up tables for computing the SVD of **H** in a non-frequency selective MIMO system applying the Jacobi algorithm and using fixed point arithmetic provides a very efficient tool with very low computational load and complexity with acceptable losses in the system performance.

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