AN EXPERIMENTAL COMPARISON OF NONHOLONOMIC CONTROL METHODS

Kang-Zhi Liu
Department of Electrical and Electronics Engineering, Chiba University, Chiba, Japan

Keywords: Parking benchmark, Khennouf-Wit method, Astolfi's method, Sordalen-Egeland method, Ikeda-Nam-Mita method, Jiang's method, Liu-Sampei method.

Abstract: Although numerous nonholonomic control methods have been proposed, few is known about the advantages and disadvantages of each method. So in this paper an automatic parking system is used as a benchmark to test several typical nonholonomic control approaches experimentally. The emphasis is put on the applicability and control performance.

1 INTRODUCTION

Since the last decade nonholonomic control has been studied extensively and numerous methods have been proposed. However, no experimental comparison was reported up to date to the knowledge of the author. So in this paper the automatic parking problem will be used as a benchmark to test several typical nonholonomic control approaches experimentally.

The methods to be tested are (1) Khennouf-Wit method (H.Khennouf and Wit, 1996), (2) Astolfi's method (Astolfi, 2000), (3) Sordalen-Egeland method (Sordalen and Egeland, 1995), (4) Ikeda-Nam-Mita method (Ikeda et al., 2000), (5) Jiang's Method (Jiang, 2000) and (6) Liu-Sampei method (Sampei and et al., 1995; Liu et al., 2006).

The following specifications are used for comparison: (1) applicability to automatic parking subject to steering angle and parking space constraints, (2) safety, (3) convergence performance of each variable, (4) oscillatory behaviour during the parking control process.

2 MODEL AND EXPERIMENT SET-UP

The plant is a rear-drive 4-wheeled car illustrated in Fig. 1. Subject to the assumption that no side slip occurs, the kinematic model is described by

$$\dot{x} = u_1 \cos\theta,\ \dot{y} = u_1 \sin\theta,\ \dot{\theta} = u_1 \frac{1}{L} \tan\phi \qquad (1)$$

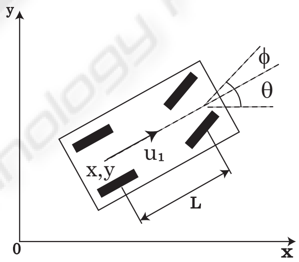


Figure 1: Model of 4-wheeled car.

in which L denotes the wheel base, (x,y) is the position of the center of rear wheel, θ is the orientation angle with respect to x axis. Further, ϕ and u_1 denote the steering angle and driving velocity respectively. Here

$$\eta = \tan\phi$$

and the driving velocity u_1 are regarded as the control input.

As real cars are subject to limitation of steering angle, the steering angle will saturate when the designed steering angle surpasses this limitation. That is, when the limit of steering angle is given by

$$\phi \in [-\phi_{max}, \phi_{max}],\ \phi_{max} > 0 \qquad (2)$$

the real input η becomes

$$\eta = \begin{cases} \eta^*, & |\phi| \le \phi_{max} \\ sgn(\eta^*)|\eta_{max}|, & |\phi| > \phi_{max} \end{cases} \qquad (3)$$

where $\eta^* = \tan\phi^*$ denotes the designed input and $\eta_{max} = \tan\phi_{max}$.

Liu K. (2009).
AN EXPERIMENTAL COMPARISON OF NONHOLONOMIC CONTROL METHODS.
In *Proceedings of the 6th International Conference on Informatics in Control, Automation and Robotics - Robotics and Automation*, pages 208-213
DOI: 10.5220/0002205802080213

Applying the following variable transformation

$$z_0 = x,\ z_1 = y,\ z_2 = \tan\theta \tag{4}$$

as well as input transformation

$$\begin{cases} v_0 = u_1 \cos\theta \\ v_1 = \frac{1}{L}\eta^*(1+\tan^2\theta)u_1 \quad (\eta^* = \tan\phi) \end{cases} \tag{5}$$

to the system (1) leads to a 3rd order chained system:

$$\dot{z}_0 = v_0,\ \dot{z}_1 = z_2 v_0,\ \dot{z}_2 = v_1. \tag{6}$$

Most of the 6 methods are built with respect to this chained system.

The prototype motor car used in experiment is shown in Fig.2, in which the garage and road are indicated by the white lines.

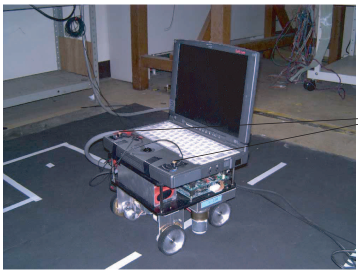


Figure 2: Experiment system.

Numerous parking experiment have been conducted and two sets of them will be shown (Table 1).

Table 1: Initial values.

	$x(0)$	$y(0)$	$\theta(0)$
Experiment 1	37[cm]	20[cm]	85[deg]
Experiment 2	41[cm]	16[cm]	33[deg]

In experiments, the designed driving velocity and steering angle are applied to their closed loop systems as reference input. Also in all figures of responses the solid, dotted lines show the measured data and the computed reference, respectively.

3 KHENNOUF-WIT METHOD

The input is given by

$$v = \begin{bmatrix} v_0 \\ v_1 \end{bmatrix} = 2f\frac{S(z)}{W(z)}\begin{bmatrix} -z_2 \\ z_0 \end{bmatrix} - k\begin{bmatrix} z_0 \\ z_2 \end{bmatrix} \tag{7}$$

in which $S(z)$ and $W(z)$ are

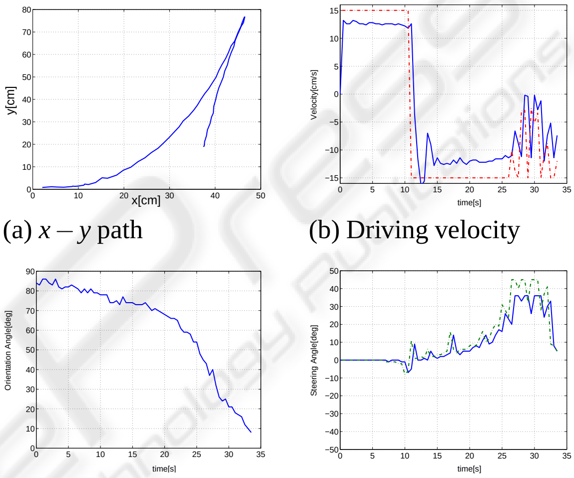
$$S(z) = z_1(t) - \frac{1}{2}z_0(t)z_2(t),\ W(z) = z_0^2 + z_2^2. \tag{8}$$

The closed loop system satisfies

$$W = W(z(0))\exp(-2kt),\ S = S(z(0))\exp(-ft) \tag{9}$$

when this input is applied to system (6). Therefore, W, S, z_0, z_1, z_2 converge to zeros.

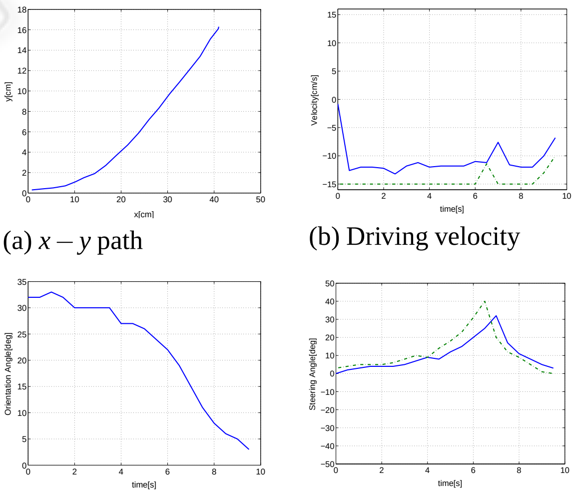
The results are illustrated in Fig. 3, 4. This method is good at controlling x, y, θ to zeros. However, in experiment 1 the car moved away from the garage to the position $(46, 77)$ before backing into the garage. This causes a safety problem. The phenomenon happens because the driving velocity is determined automatically and it can not be predicted where the car will make a turn. In this sense this method cannot be applied to parking control when the initial orientation angle is large.



(a) $x-y$ path (b) Driving velocity

(c) Orientation angle (d) Steering angle

Figure 3: Khennouf-Wit method: Experiment 1.



(a) $x-y$ path (b) Driving velocity

(c) Orientation angle (d) Steering angle

Figure 4: Khennouf-Wit method: Experiment 2.

4 ASTOLFI'S METHOD

This approach is proposed by Astolfi(Astolfi, 2000). First, the next coordinate transformation (σ-process)

$$y_1 = z_0,\ y_2 = z_2,\ y_3 = z_1/z_0 \tag{10}$$

is introduced to make the chained system (6) discontinuous. The transformed system is

$$\dot{y_1} = v_0,\ \dot{y_2} = v_1,\ \dot{y_3} = \frac{y_2 - y_3}{y_1} v_0. \tag{11}$$

When

$$v_0 = -ky_1,\ \ k > 0 \tag{12}$$

is applied, y_1 is stabilized and

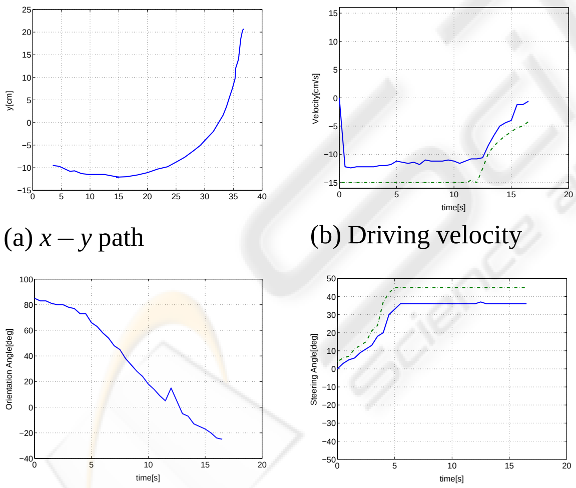
$$\begin{bmatrix} \dot{y_2} \\ \dot{y_3} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -k & k \end{bmatrix} \begin{bmatrix} y_2 \\ y_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_1 \tag{13}$$

is controllable. Hence, a linear feedback

$$v_1 = -f_2 y_2 + f_3 y_3, \quad f_2 > k,\ f_3 > f_2 \tag{14}$$

can stabilize $y_2,\ y_3$. As a result, the original states z_1, z_2, z_3 are also stabilized.

The responses are shown in Figs. 5 and 6. As is seen from the measured data, the (x,y) path is pretty smooth, but the steering angle does not converge to zero. Further, the y coordinate moves to the opposite side when the initial orientation angle θ is over 80[deg] (Fig. 5), which may cause a safety problem.



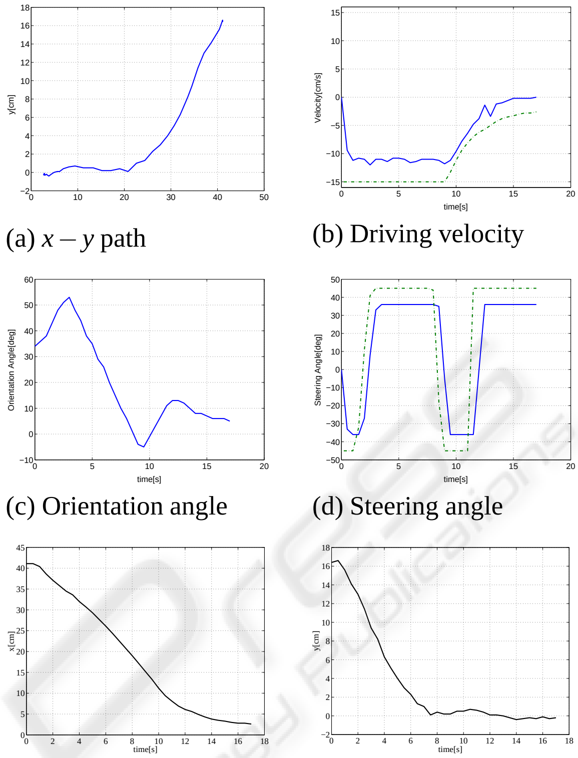
(a) $x-y$ path (b) Driving velocity

(c) Orientation angle (d) Steering angle

Figure 5: Astolfi's method: Experiment 1.

5 SORDALEN-EGELAND METHOD

Sordalen-Egeland method uses a periodic v_0 to drive the car and during this motion a time-varying $v_1(t)$ is applied to attenuate z_1, z_2 exponentially.



(a) $x-y$ path (b) Driving velocity

(c) Orientation angle (d) Steering angle

(c) x coordinate (d) y coordinate

Figure 6: Astolfi's method: Experiment 2.

The control law is as follows: let the period be T and $Z_2 = (z_1, z_2)^T$, set $v_0(t)$ in $iT \le t < (i+1)T$ as

$$v_0(t) = k(z(iT))f(t) \tag{15}$$

in which $f(t) = \frac{1}{2}(1 - \cos 2\pi t/T),\ G(Z_2) = c\|Z_2\|_2^{1/2}$,

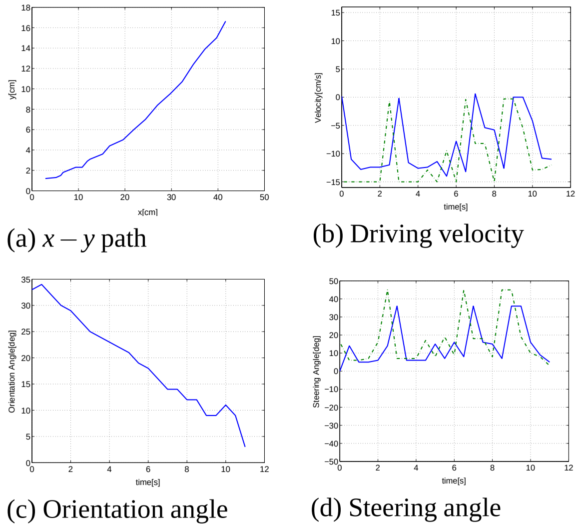
$k(z(iT)) = \mathrm{sat}(-[z_0(iT) + G(Z_2)\mathrm{sgn}(z_0(iT))]\beta,\ K)$.

Here $\beta > 0$,$K > 0$,$c > 0$ are design parameters and $\mathrm{sat}(x, K)$ is a saturation function of x with peak value K. On the other hand, in $iT \le t < (i+1)T$ $v_1(t)$ is determined as 0 when $z_0(iT) = 0$, and

$$v_1(t) = [\gamma_2, \gamma_3]Z_2,\ (z_0(iT) \neq 0) \tag{16}$$

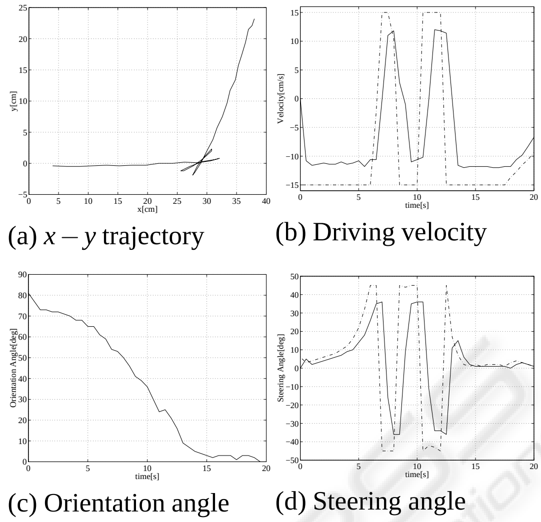
where $\gamma_2 = -\lambda - f^3(t)\lambda$, $\gamma_3 = [-\lambda^2 f(t) - 2\lambda\dot{f}(t)]f(t)/k(z(iT))$ and $\lambda > 0$ is a parameter.

The results are shown in Fig. 7. The experiment failed when the initial orientation angle θ is around 80[deg] because the car moves out of the sensing range of PSD camera with an approximately 0[deg] steering angle. Moreover, the tuning of parameter is rather difficult since there are many parameters in the control law. The oscillation in response in intrinsic to this method. So this method is not suitable for parking control.



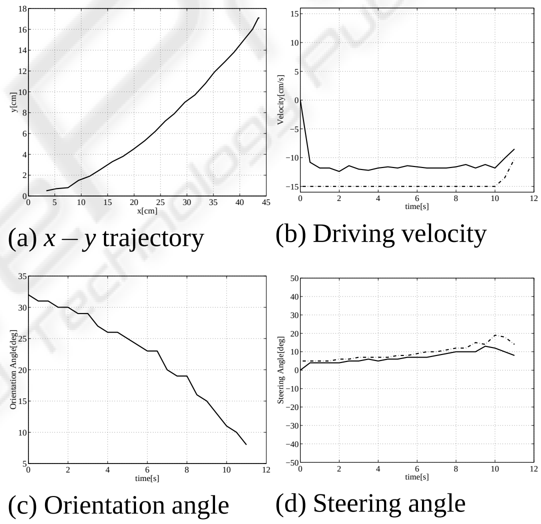
(a) $x-y$ path (b) Driving velocity (c) Orientation angle (d) Steering angle

Figure 7: Sordalen-Egeland method: Experiment 2.



(a) $x-y$ trajectory (b) Driving velocity (c) Orientation angle (d) Steering angle

Figure 8: Ikeda-Nam-Mita method: Experiment 1.



(a) $x-y$ trajectory (b) Driving velocity (c) Orientation angle (d) Steering angle

Figure 9: Ikeda-Nam-Mita method: Experiment 2.

6 IKEDA-NAM-MITA METHOD

The control procedure of this method is divided into two steps. In step 1, the input is determined as

$$v_0 = -\lambda_2 \frac{z_1}{z_2},\ v_1 = -\lambda_1 z_2,\ \lambda_2 > \lambda_1. \tag{17}$$

Then the closed loop system becomes

$$\dot{z}_0 = -\lambda_2 \frac{z_1}{z_2},\ \dot{z}_1 = -\lambda_2 z_1,\ \dot{z}_2 = -\lambda_1 z_2 \tag{18}$$

and $(z_2, z_1) \to 0$. Note that z_1/z_2 is bounded if $\lambda_2 > \lambda_1$. In step 2, the input is switched to

$$v_0 = -\lambda_3 z_0,\ v_1 = -\lambda_1 z_2 \tag{19}$$

once z_2 is sufficiently close to zero. The corresponding closed loop system changes to

$$\dot{z}_0 = -\lambda_3 z_0,\ \dot{z}_1 = -\lambda_3 z_0 z_2,\ \dot{z}_2 = -\lambda_1 z_2 \tag{20}$$

and $(z_0, z_2) \to 0$. In this process z_1 will not deviate far away from 0 because the initial values of z_1, z_2 are sufficient small due to the control in step 1. In the experiment, the input is switched back to step 1 if z_2 deviates far away from zero due to disturbance.

In the experiments, the input of step 1 is used if $|\theta| \leq 0.1$[rad]. Otherwise the input of step 2 is used.

As can be seen from Fig.8 and 9, this method is able to control x, y, θ to zeros pretty good. But in the first experiment, the car moves back and forth four times around the x axis. However, compared with Khennouf-Wit Method the change of the direction of the driving velocity occurs only in positions that are far away from the origin, it is not so severe a drawback.

7 JIANG'S METHOD

In (Jiang, 2000) Jiang Proposed a robust exponential regulation method for a class of nonholonomic systems with uncertainty.

First, a rotation of (x, y) coordinate is introduced to avoid singularity in the transformation to canonical form

$$x_0 = \theta,\ x_1 = x\sin\theta - y\cos\theta$$
$$x_2 = x\cos\theta + y\sin\theta,\ u_0 = u_1 \frac{1}{L}\tan\phi. \tag{21}$$

This transformation brings (1) into

$$\dot{x}_0 = u_0,\ \dot{x}_1 = x_2 u_0,\ \dot{x}_2 = u - x_1 u_0. \tag{22}$$

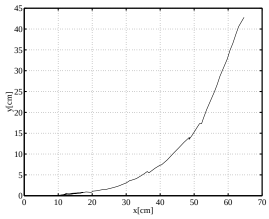
Further, a state scaling

$$z_1 = \frac{x_1}{x_0}, \quad z_2 = x_2 \tag{23}$$

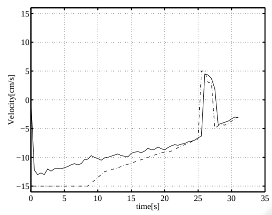
is introduced. Then based on backstepping, the following control input are obtained:

$$\begin{aligned} u_0 &= -\lambda_0 x_0 \quad (24) \\ u_1 &= -[\lambda_2 + \lambda_0(\lambda_1+1) + \frac{\lambda_0}{4}(x_0^2 - 1 + \lambda_1 + \lambda_1^2)] \\ &\quad \times (z_2 - (\lambda_1+1)z_1), \quad \lambda_0,\ \lambda_2 > 0,\ \lambda_1 > 1. \end{aligned}$$

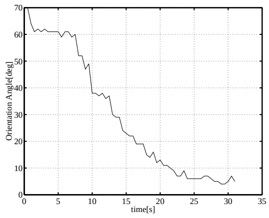
It is clear from the Figs. *Please place* `\label` ***after*** `\caption` and *Please place* `\label` ***after*** `\caption` that the car moves back and forth near the garage which may cause safety problem. Also, the steering angle is quite oscillatory and so is the orientation angle as its consequence.



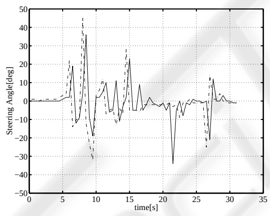
(a) $x-y$ trajectory



(b) Driving velocity



(c) Orientation angle



(d) Steering angle

Figure 10: Experiment 1(x(0)=64.7[cm], y(0)=42.8[cm], θ(0)=70[deg]).

8 LIU-SAMPEI METHOD

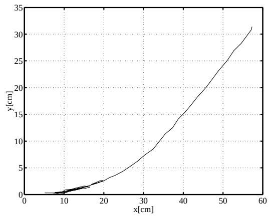
This method(Liu et al., 2006) evolved from Sampei's method(Sampei and et al., 1995). Its essence is to attenuate the orientation angle θ and y coordinate while drive the car back and forth on the allowed road, then finally park the car into the garage. A distinguishing feature of this method is that the driving velocity can be determined freely.

Let z_2^* be

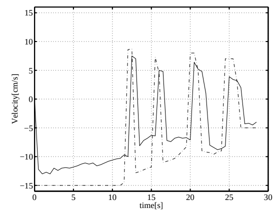
$$z_2^* = -C_1 sgn(v_0) z_1,\ C_1 > 0 \tag{25}$$

and determine the control input v_1 as

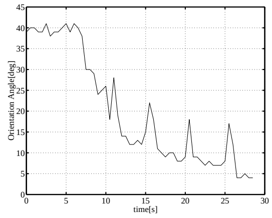
$$v_1 = -C_1 z_2 |v_0| - z_1 v_0 - C_2(z_2 - z_2^*)|v_0|. \tag{26}$$



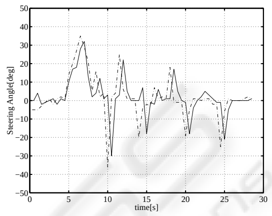
(a) $x-y$ trajectory



(b) Driving velocity



(c) Orientation angle



(d) Steering angle

Figure 11: Experiment 2(x(0)=57.3[cm], y(0)=31.4[cm], θ(0)=39[deg]).

Then the derivative of Lyapunov function $V = \frac{1}{2}z_1^2 + \frac{1}{2}(z_2 - z_2^*)^2$ satisfies $\dot{V} = -C_1 z_1^2 |v_0| - C_2(z_2 - z_2^*)^2 |v_0|$ which is negative semidefinite. Hence, the convergence of (z_1, z_2) is guaranteed.

The input v_0 is selected as follows: (1) Take v_0 arbitrarily if $z_1^2 + (z_2 - z_2^*)^2 > \gamma$. (2) $v_0 = -sgn(z_0)|U|$ when $z_1^2 + (z_2 - z_2^*)^2 < \gamma$ so as to stabilize z_0. Here, U is given by

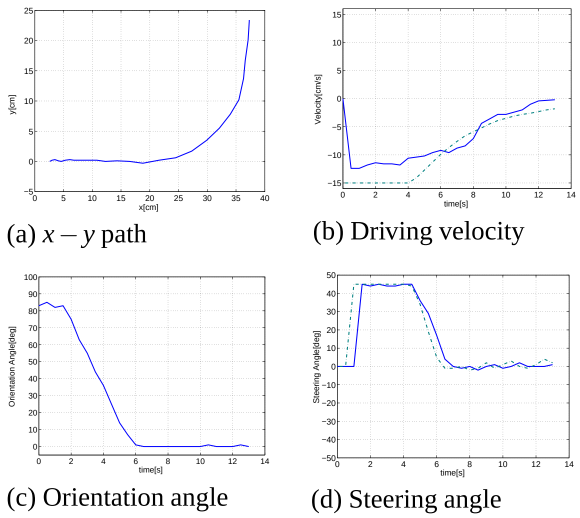
$$U = \begin{cases} u_{\max}\beta\cos(\theta) & if\ \sqrt{x^2+y^2} \geq \beta u_{\max} \\ \sqrt{x^2+y^2}\beta\cos(\theta) & if\ \sqrt{x^2+y^2} < \beta u_{\max} \end{cases} \tag{27}$$

$u_{\max}$ is the maximum of driving velocity and β is a deceleration factor.

The experiment data are illustrated in Fig. 12, 13. This method can stabilize x, y, θ from any initial state and provides the best performance for parking control.

9 CONCLUDING REMARKS

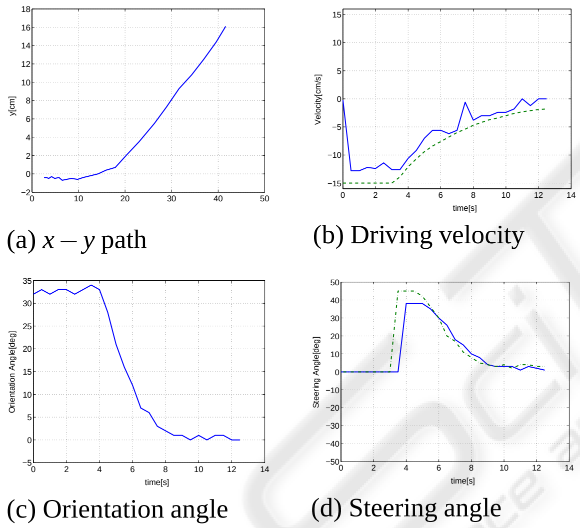
The applicability of 6 typical control methods for chained system has been tested experimentally by using an automatic parking benchmark. The results indicate that Astolfi's method(Astolfi, 2000) and Ikeda-Nam-Mita method(Ikeda et al., 2000) may be applied to parking control when the initial orientation angle is not too big. It is noted that in Astolfi's method the steering angle does not converge to zero. Liu(Liu et al., 2006)-Sampei(Sampei and et al., 1995) method is applicable to any situations. Meanwhile, Khennouf-Wit method and Jiang's method should be



(a) $x-y$ path (b) Driving velocity

(c) Orientation angle (d) Steering angle

Figure 12: Liu-Sampei method: Experiment 1.



(a) $x-y$ path (b) Driving velocity

(c) Orientation angle (d) Steering angle

Figure 13: Liu-Sampei method: Experiment 2.

used with care due to safety concern. Sordalen-Egeland method is not suitable for parking control under steering angle and parking space limitation.

It is also worth noting that all methods, except Liu-Sampei and Ikeda-Nam-Mita methods, guarantees asymptotic stability, but their performances are not as good as those of Liu-Sampei and Ikeda-Nam-Mita methods. The author feels that the degrading of performance is caused by killing the freedom of control (the driving velocity) in order to prove the asymptotic stability. In contrast, both Liu-Sampei method and Ikeda-Nam-Mita method use switching of control input which provides the control flexibility and leads to better performance, although it is very difficult to show their asymptotic stability. The author strongly believe that control design based on asymptotic/exponential stability point of view is not suitable for this class of control problems, the emphasis should be put on improving the performance instead.

REFERENCES

Astolfi, A. (2000). Discountinuous control of nonholonomic systems. *Systems and Control Letters*, 27(1):37–45.

H.Khennouf and Wit, C. (1996). Quasi-continuous exponential stabilizers for nonholonomic system. *Proc. of 13th IFAC World Congress*, 2b(174):49–54.

Ikeda, T., Nam, T.-K., and Mita, T. (2000). Nonholonomic variable constraint control of free flying robotics and its convergence (in japanese). *Journ. of Robotics Society of Japan*, 18.

Jiang, Z.-P. (2000). Robust exponential regulation of nonholonomic systems with uncertainties. *Automatica*, pages 189–209.

Liu, K.-Z., Dao, M.-Q., and Inoue, T. (2006). An exponential ε-convergent control algorithm for chained systems and its application to automatic parking. *IEEE Trans. on Control Systems Technology*, 14(6):1113–1126.

Sampei, M. and et al. (1995). Arbitrary path tracking control of articulated vehicles using nonlinear control theory. *IEEE Trans. on Control Systems Technology*, 3(1):125–131.

Sordalen, O. and Egeland, O. (1995). Exponential stabilization of nonholonomic chained systems. *IEEE Trans. Automat. Control*, 40:35–49.