

# INDUCING COOPERATION IN FUZZY CLASSIFICATION RULES USING ITERATIVE RULE LEARNING AND RULE-WEIGHTING

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Abstract: Fuzzy Rule-Based Classification Systems (FRBCSs) focus on generating a compact rule-base from numerical input data for classification purposes. Iterative Rule Learning (IRL) has been proposed to reduce the search space for learning a rule-set for a specific classification problem. In this approach, a rule-set is constructed by searching for an appropriate fuzzy rule and adding it to the rule-set in each iteration. A major element of this approach is the requirement of an evaluation metric to find the best rule in each iteration. The difficulty in choosing the best rule is that the evaluation metric should be able to measure the degree of cooperation of the candidate rule with the rules found so far. This poses a major difficulty when dealing with fuzzy rules; because unlike crisp rules, each pattern is compatible with a fuzzy rule only to a certain degree. In this paper, the cooperation degree of a candidate rule is divided into the following two components: I)- The cooperation degree of the rule with other rules of the same class, II)- The cooperation degree of the rule with rules of the other classes. An IRL scheme to generate fuzzy classification rules is proposed that induces cooperation among the rules of the same class. Cooperation between the rules of different classes is handled using our proposed rule-weighting mechanism. Through a set of experiments on some benchmark data sets from UCI-ML repository, the effectiveness of the proposed scheme is shown.

## 1 INTRODUCTION

The main application area of fuzzy rule-based systems has been control problems (Sugeno, 1985). Fuzzy rule-based systems for control problems can be viewed as approximators of nonlinear mappings from non-fuzzy input vectors to non-fuzzy output values. Recently, fuzzy rule-based systems have often been applied to classification problems where non-fuzzy input vectors are to be assigned to one of a given set of classes. Many approaches have been proposed for generating and learning fuzzy if-then rules from numerical data for classification problems. For instance, FRBCSs are created by simple heuristic procedures (Ishibuchi et al., 1992), (Abe, 1995), neuro-fuzzy techniques (Nauck and R. Kruse, 1997), clustering methods (Abe and Thawonmas, 1997), genetic algorithms (Ishibuchi et

al., 2005), etc.

Pattern classification has been the main issue in machine learning. Classification is to acquire knowledge from a set of training patterns and use this knowledge to predict the class of a new pattern. FRBCSs use fuzzy rules as a mean to perform classification tasks. A rule is an if-then relation from the  $n$ -dimensional pattern space to the set of classes. In a single winner rule approach (Ishibuchi and Nakashima, 2001), to classify an unknown pattern, one rule is selected and used to classify the pattern. In this paper, a single winner rule approach is used which will be discussed later. In the broadest sense, any method that incorporates information from training samples in the design of a classifier employs learning. Therefore, designing classifiers involves some type of learning to learn or estimate unknown parameters using a set of labeled patterns.

In this paper, an IRL approach for fuzzy rule selection is presented in which the degree of cooperation of each candidate rule with other rules of the same class is estimated. In this approach, the final rule-set for classification is constructed by searching for an appropriate fuzzy rule and adding it to the rule-set in each step. Then, a simple rule-weighting mechanism is proposed to reach some degrees of cooperation/competition among the rules of different classes. Four UCI ML driven data sets are then used to evaluate the proposed fuzzy classification method.

## 2 FUZZY CLASSIFICATION RULES

In the design of fuzzy rule-based systems, we face two conflicting objectives: error minimization and interpretability maximization. Error minimization has been used in many applications of fuzzy rule-based systems in the literature while the interpretability was not usually taken into account in those applications. Recently, the tradeoff between these two objectives has been discussed in some studies. When fuzzy rule-based systems are used for two-dimensional problems, fuzzy rules can be represented in a tabular form (Ishibuchi and Yamamoto, 2004). Figure 1 shows an example of a fuzzy rule table for a two-dimensional pattern classification problem. In this figure, we have the following four fuzzy rules:

- If  $X_1$  is *small* and  $X_2$  is *small* then Class 1,
- If  $X_1$  is *small* and  $X_2$  is *large* then Class 2,
- If  $X_1$  is *large* and  $X_2$  is *small* then Class 3,
- If  $X_1$  is *large* and  $X_2$  is *large* then Class 4,

where *small* and *large* are linguistic values defined by triangular membership functions.

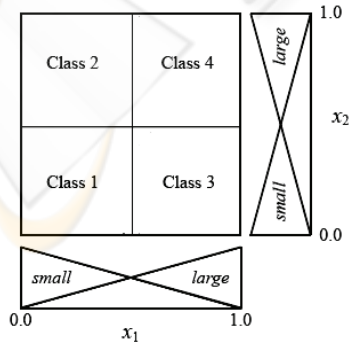


Figure 1: Four fuzzy rules in the 2-dimensional pattern space  $[0,1] \times [0,1]$ .

As shown in Figure 1, fuzzy rules for 2-dimensional problems can be written in a human understandable manner using the tabular form representation. When fuzzy rule-based systems are applied to high-dimensional problems, their interpretability is significantly degraded due to the two difficulties: the increase in the number of fuzzy rules and the increase in the number of antecedent conditions of each fuzzy rule.

Assume that we have  $m$  labeled patterns  $X_p=(x_{p1}, \dots, x_{pn})$ ,  $p=1,2,\dots,m$  from  $M$  classes in an  $n$ -dimensional continuous pattern space is given. For classification problems with  $n$  number of attributes, as in (Ishibuchi and Yamamoto, 2004), we use fuzzy rules of the following form:

$$\text{Rule } R_i : \text{If } x_1 \text{ is } A_{i1} \text{ and } \dots \text{ and } x_n \text{ is } A_{in} \quad (1)$$

then class  $C_i$  with  $CF_i$

where  $R_i$  is the  $i$ -th rule,  $X=(x_1, \dots, x_n)$  is an  $n$ -dimensional pattern vector,  $A_{ij}$  is an antecedent fuzzy set (i.e., linguistic value such as *small* or *large* in Figure 1),  $C_i$  is the class label of  $R_i$ , and  $CF_i$  is the weight of  $R_i$ . It should be noted that the consequent part of our fuzzy rule for classification problems is totally different from standard fuzzy rules for function approximation problems. The consequent of our fuzzy rule is a non-fuzzy class label and the rule weight  $CF_i$  is a real number in the unit interval  $[0, 1]$ . The rule weight is used as the strength of each fuzzy rule when a new pattern is classified by a fuzzy rule-based classification system (see (Ishibuchi and Nakashima, 2001) for details).

The compatibility grade of a training pattern  $X_p$  with the antecedent part  $A_i=(A_{i1}, \dots, A_{in})$  of fuzzy rule  $R_i$  is calculated using product operator as,

$$\mu_{A_i}(X_p) = \mu_{A_{i1}}(X_{p1}) \times \dots \times \mu_{A_{in}}(X_{pn}) \quad (2)$$

where  $\mu_{A_{ij}}(\cdot)$  is the membership function of the antecedent fuzzy set  $A_{ij}$ .

## 3 CANDIDATE RULE GENERATION

In our approach, fuzzy if-then rules are generated from numerical data. Then, the generated rules are used as candidate rules from which a small number of fuzzy if-then rules are selected in an iterative manner. The domain interval of each attribute  $x_i$  is discretized into  $K_i$  fuzzy sets. Figure 2 shows some examples of fuzzy discretization.

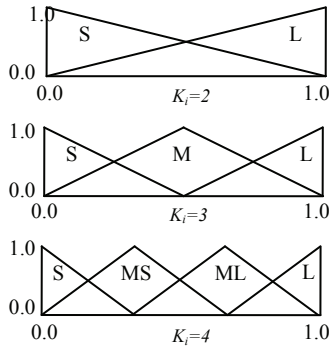


Figure 2: Some typical examples of fuzzy partitions of the domain interval  $[0, 1]$ .

The meaning of each label is as follows:

S: small, MS: medium small, M: medium, ML: medium large, and L: large. The superscript of each label denotes the granularity of the corresponding fuzzy partition.

Each antecedent fuzzy set in a fuzzy rule can be one of  $K_i$  fuzzy sets or “don't care”. Therefore the total number of possible antecedent combinations is  $(K_i+1) \times \dots \times (K_n+1)$ .

To determine the consequent part of a rule, we use a concept in data mining called confidence degree. The confidence of a fuzzy association rule is defined as (Ishibuchi and Yamamoto, 2004):

$$c(A_i \Rightarrow \text{class } h) = \frac{\sum_{x_p \in \text{class } h} \mu_{A_i}(X_p)}{\sum_{p=1}^m \mu_{A_i}(X_p)} \quad (3)$$

The consequent class  $C_i$  of the fuzzy rule  $R_i$  is specified by identifying the class with the maximum confidence. If the maximum confidence of a rule is zero or the difference between the first and second maximum confidences is zero, the rule is not generated.

To avoid coping with a large number of candidate rules in the rule selection procedure, some prescreening criterion is needed. Several criteria is used in the previous works (Gonzalez and Perez, 1999). In this paper we use the following criterion:

$$\text{value}(A_i \Rightarrow \text{class } h) = \sum_{x_p \in \text{class } h} \mu_{A_i}(X_p) - \sum_{x_p \in \text{class } h} \mu_{A_i}(X_p) \quad (4)$$

## 4 RULE SELECTION

After generating the candidate rules, a set of rules must be selected to construct the rule-base of the classifier. The rules are selected in an iterative

manner. The generated fuzzy if-then rules are divided into  $M$  groups according to their consequent classes. Fuzzy if-then rules in each group are sorted in descending order of the evaluation criterion (4).

In the first step of the rule selection the best rule of each class is added to the rule-base. To build a rule-base with  $N$  rules ( $N \geq M$ ), the remaining  $N-M$  rules are selected one by one. A major element of this approach is the need of an evaluation metric to find the best rule in each iteration.

The difficulty in choosing the best rule is that the evaluation metric should be able to measure the degree of cooperation of the candidate rule with the rules found so far. This is a major difficulty when dealing with fuzzy rules, due to the fact that each pattern is compatible with a fuzzy rule to a certain degree.

For the rules found so far, a measure called “fuzzy accuracy measure” of the rule-base is defined as:

$$F_{\text{rule-base}} = \sum_{x_p \in \text{class } h} \max_{R_i \in \text{rule-base}} (\mu_{R_i}(x_p)) - \sum_{x_p \in \text{class } h} \max_{R_i \in \text{rule-base}} (\mu_{R_i}(x_p)) \quad (5)$$

The aim of this measure is to calculate the overall effectiveness of the rules of the same class that are found so far. To add the rule  $R_w$  from the set of candidate rules to the rule-base, the rule that improves  $F_{\text{rule-base}}$  the most is chosen:

$$R_w = \arg \max_{R_i \in \text{Candidate\_Rules}} \left\langle F_{\text{rule-base} \cup \{R_i\}} - F_{\text{rule-base}} \right\rangle \quad (6)$$

The process of rule selection is continued iteratively as long as there are further improvements in  $F_{\text{rule-base}}$ . The proposed scheme both induces cooperation among the rules of the same class and avoids including redundant rules in the final rule-base which results in having a compact rule-base.

## 5 INDUCTING COOPERATION WITH RULE WEIGHTING

The first component of the cooperation of the newly added rule is its degree of cooperation with the rules of the same class. This component is considered in the rule selection phase. The second component of the cooperation is the degree of cooperation between rules of different classes. This component is also called *competition*. Competition among the rules of different classes is handled by assigning a weight to each different rule.

In (Nauck & Kruse, 1998), the effect of rule weights in fuzzy rule-based systems for function

approximation problems is discussed. They also showed how the modification of the membership functions of antecedent or consequent fuzzy sets can be equivalently replaced by the learning of rule weights. Several heuristic criteria for rule-weighting have been introduced in earlier works done by Ishibuchi et al (Ishibuchi and Yamamoto, 2004) which are briefed here:

$$CF_1 = c(A_i \Rightarrow C_i) \quad (7)$$

$$CF_2 = c(A_i \Rightarrow C_i) - \frac{1}{M} \sum_{\substack{t=1 \\ t \neq C_i}}^M c(A_i \Rightarrow C_t) \quad (8)$$

$$CF_3 = c(A_i \Rightarrow C_i) - \max\{c(A_i \Rightarrow \text{Class } t | t=1, 2, \dots, M; t \neq C_i)\} \quad (9)$$

$$CF_4 = c(A_i \Rightarrow C_i) - \frac{\sum_{x_p \in \text{Class } C_i} \mu_{R_i}(x_p)}{\sum_{x_p} \mu_{R_i}(x_p)} \quad (10)$$

where  $c(A_i \Rightarrow C_i)$  is the confidence of a fuzzy rule  $R_i$ , and  $\mu_{R_i}(X_p)$  is the compatibility grade of a training pattern  $X_p$  with the antecedent part of fuzzy rule  $R_i$ .

In the following, a simple rule-weighting criterion is presented. In our suggested method, it is tried to reach some degrees of cooperation /competition among the rules of different classes. To calculate the weight of the fuzzy rule  $R_i$ , first a value is calculated named as *contrast* for each training data point  $X_p$ :

$$\text{Contrast}_{R_i}(X_p) = \frac{\max_{\substack{j=1, \dots, N \\ \text{label}(R_j) \neq C_i}} (\mu_{R_j}(X_p))}{\mu_{R_i}(X_p) + \max_{\substack{j=1, \dots, N \\ \text{label}(R_j) \neq C_i}} (\mu_{R_j}(X_p))}, \quad (11)$$

where  $R_i$  is the rule that is being weighted. If a data point is covered by the rules of other classes, the *contrast* value of this data point, with respect to the rule in hand, is close to one; otherwise it is closer to zero.

Data points are sorted in ascending order of their *contrast* values. The next step is to find a threshold of the *contrast* values,  $\omega$ , that best separates the data points of the same class from the data points of other classes. In this way, each data point  $X_p$  for which  $\text{Contrast}_{R_i}(X_p) < \omega$  is assumed to be of the same class as  $R_i$ . The threshold is then altered from the list

*contrast* value to the greatest and accuracy of the classifier with respect to the current threshold is measured. The weight of rule  $R_i$  is obtained from the the value of the best threshold (i.e. leading to the highest accuracy) normalized in the range of [0, 1] as follows,

$$CF_i = \frac{\omega}{1 + \omega} \quad (12)$$

## 6 EXPERIMENTAL RESULTS

In our experiments, we used four data sets in Table 1 available from the UCI ML repository (Merz and Murphy, 1996).

Table 1: Statistics of the data sets used in our experiments.

Data set	# of attributes	# of patterns	# of Classes
Pima	8	768	2
Wine	13	178	3
Cancer Wis.	9	699	2
Glass	9	214	6

All attribute values of the four data sets were normalized into real numbers in the unit interval [0, 1] before extracting fuzzy rules. Since we did not know an appropriate fuzzy partition for each attribute of each test problem, we simultaneously used three different fuzzy partitions in Figure 2. One of the 9 triangular fuzzy sets was used as an antecedent fuzzy set. To generate simple fuzzy rules (i.e., short fuzzy rules with a small number of antecedent conditions), we also used “*don’t care*” as an antecedent fuzzy set. The membership function of “*don’t care*” is defined as  $\mu_{\text{“don’t care”}}(X) = 1$ . The total number of combinations of antecedent fuzzy sets is  $10^n$  for an  $n$ -dimensional problem.

In our computational experiments we only examined fuzzy rules with three or less antecedent conditions (i.e., with  $n-3$  or more “*don’t care*” conditions). The restriction on the number of antecedent conditions is to generated interpretable fuzzy rules as well as for decreasing the CPU time.

In Tables 2-5, the results of the fuzzy classification system using the proposed fuzzy rule selection method with different rule-weighting methods are shown on the data sets of Table 1. All the reported results are the average of ten trials of ten-fold cross validation. The first column of each Table is the number of rules used to classify the data points in the selected data set. The other five columns represent the classification accuracy of the

four mentioned weighting methods proposed in (Ishibuchi and Yamamoto, 2004) compared to our proposed method. As it can be seen in the results, the proposed method led to the best results among the rule-weighting methods. In each row of the Table 2-5, the method which had the best result is bolded.

Table 2: Test data classification rates of Glass dataset.

# of rules	No Weight	CF1	CF2	CF3	CF4	Our Method
6	49.61	48.95	49.42	48.71	54.15	<b>56.99</b>
12	55.72	58.03	58.80	60.56	60.37	<b>63.89</b>
18	57.81	59.43	58.37	59.41	63.08	<b>66.29</b>
24	61.25	60.85	63.18	60.47	62.38	<b>67.18</b>
30	61.35	63.05	61.47	61.33	63.78	<b>67.47</b>
36	62.08	62.37	63.68	62.13	65.51	<b>68.11</b>
42	61.21	60.28	61.75	63.53	64.18	<b>68.29</b>
45	62.98	61.63	63.14	64.22	65.01	<b>68.62</b>

Table 3: Test data classification rates of Wine dataset.

# of rules	No Weight	CF1	CF2	CF3	CF4	Our Method
3	84.90	87.27	87.82	86.99	85.97	<b>85.54</b>
6	91.55	92.53	93.31	91.69	91.85	<b>93.14</b>
9	93.14	91.89	92.28	92.86	94.14	<b>91.97</b>
12	92.88	94.81	94.96	93.77	93.38	<b>92.11</b>
15	93.94	93.16	94.84	94.69	93.44	<b>95.51</b>
18	94.57	93.86	93.78	93.60	92.73	<b>95.48</b>
51	95.33	95.00	94.56	94.64	93.34	<b>95.60</b>
56	95.18	94.37	94.66	94.53	94.42	<b>95.64</b>

Table 4: data classification rates of Cancer dataset.

# of rules	No Weight	CF1	CF2	CF3	CF4	Our Method
2	81.84	83.29	80.81	81.06	<b>83.16</b>	83.13
3	91.79	91.25	91.65	<b>92.67</b>	<b>92.04</b>	91.16
4	89.61	91.41	92.34	<b>92.44</b>	92.36	91.61
5	92.87	91.34	90.35	90.57	<b>93.08</b>	92.20
6	93.16	<b>93.66</b>	93.32	92.55	92.59	90.81
9	90.44	94.55	91.98	91.00	91.14	<b>94.82</b>
12	92.66	92.87	90.70	91.63	92.60	<b>94.91</b>
17	93.49	91.66	92.34	92.25	91.73	<b>95.44</b>

Table 5: Test data classification rates of Pima dataset.

# of rules	No Weight	CF1	CF2	CF3	CF4	Our Method
2	68.53	69.80	69.34	68.10	69.20	<b>68.85</b>
5	69.1	71.66	68.64	70.22	68.22	<b>73.64</b>
7	68.15	70.28	71.05	69.20	70.34	<b>76.03</b>
10	70.52	69.59	68.47	70.52	70.38	<b>74.23</b>
18	71.79	70.08	70.59	70.36	70.49	<b>74.92</b>
27	73.11	70.1	70.73	70.46	70.99	<b>75.24</b>
37	71.40	70.53	72.32	71.67	70.39	<b>75.78</b>
50	70.97	72.56	71.32	71.86	71.47	<b>76.22</b>

Although the classification accuracy has always been the main concern in classification problems, interpretability also have to be considered. There are two factors that heavily affect the interpretability of a rule-based system: number of the generated rules and number of antecedent conditions of each generated rule. As shown, our proposed method is highly interpretable in terms of both number the generated fuzzy classification rules and their number of antecedent conditions.

In Table 6, we compared our results to the results obtained by another successful rule-based method as benchmark results called C4.5 reported by (Elomaa and Rousu, 1999). As shown in Table 6, except in one case, the proposed classifier in this paper shows higher classification rates.

Table 6: Accuracy of the proposed classifier compared to C4.5. The best result in each row is highlighted by boldface.

Data set	The proposed classifier (%)	C4.5 classifier	
		Worst (%)	Best (%)
Pima	<b>76.2</b>	72.8	75.0
Cancer	<b>95.4</b>	94.0	94.9
Wine	<b>95.6</b>	92.2	94.4
Glass	68.6	68.8	<b>72.7</b>

## 7 CONCLUSIONS

In this paper, the cooperation degree of the fuzzy classification rules was divided into the two components: I)- The cooperation degree of the rules with other rules of the same class, II)- The cooperation degree of the rules with rules of the other classes. We proposed an IRL method for fuzzy rule selection. Using the proposed criterion, it was possible to estimate the degree of cooperation of a candidate rule with other rules of the same class in

the final rule-base. Furthermore, a simple rule-weighting mechanism was proposed to reach some degrees of cooperation/competition among the rules of different classes. The experimental results on real problems like speech data classification showed the effectiveness of the proposed method to generate fuzzy classification rules with high degrees of cooperation among them.

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