SKELETON REPRESENTATION BASED ON COMPOUND BEZIER CURVES

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Abstract: A new method to describe the skeleton of a polygonal figure is presented. The skeleton is represented as a planar graph, whose edges are linear and quadratic Bezier curves. The description of a radial function in Bezier splines form is given. An algorithm to calculate control polygons of Bezier curves is proposed. Also, we introduce a new representation of skeleton as a straight planar control graph of a compound Bezier curve. We show that such skeleton representation allows simple visualization and easy-to-use skeleton processing techniques for image processing.

1 INTRODUCTION

A closed domain on Euclidean plane R^2 such that its boundary consists of one or more simple nonintersecting polygons is called *a polygonal figure*. The set of polygonal figure points that have two or more closest boundary points of figure is called *the skeleton* or *medial axis*. Polygonal figures and their skeletons are widely used in image shape analysis and recognition (Pfaltz, Rosenfeld, 1967).

To construct the skeleton of a polygonal figure the concept of a Voronoi diagram of line segments is commonly used (Drysdale, Lee, 1978, Kirkpatrick, 1979). The polygonal figure boundary is a union of linear segments and vertices, which are considered as the Voronoi sites. The Voronoi diagram of these sites is generated and the skeleton is extracted as a subset of the diagram. The skeleton of a polygonal figure with n sides can be obtained from the Voronoi diagram taking $O(n)$ time. By-turn, there are known effective $O(n \log n)$ algorithms to construct the Voronoi diagram for the general set of linear segments (Fortune, 1987, Yap, 1987) as well as for the sides of a simple polygon (Lee, 1982) or multiply-connected polygonal figures (Mestetskiy, Semenov, 2008).

Geometric construction of a polygonal figure skeleton is simple enough: it is a planar graph with straight-line and parabolic edges (figure 1).

However, such analytical description of skeletons presents some difficulties. Presence of parabolic edges gives rise to certain problems in constructing, storing, processing, and utilizing skeletons in image analysis. The general form for a parabola is described by an implicit equation. This is not handy for calculation of parabolas intersections, for drawing and analysis.

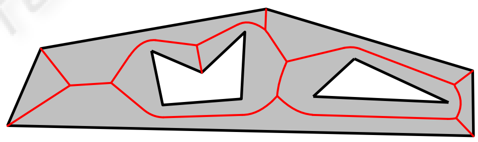

Figure 1: A polygonal figure and its skeleton.

This shortcoming generates the tendency to handle skeletons having no parabolic edges. This idea is implemented in the concept of *straight skeleton* (Aichholzer, Aurenhammer, 1996). But the straight skeleton suffers from certain shortcomings, videlicet: complexity of mathematical definition, low algorithmic efficiency, regularization complexity if noise effects are available.

In this paper, we propose a different method of describing a skeleton in the form of a planar graph with straight edges. To construct such a graph, computing parabolic edges is not necessary either at the step of the Voronoi diagram computing, or at the steps of skeleton storing, drawing and processing, respectively. This can be achieved as follows.

1. The skeleton of a polygonal figure is the union of a set of the first and second order elementary

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Bezier curves. This union we call *the compound Bezier curve*.

2. A compound Bezier curve is defined by its control graph, which is obtained from the control polygons of elementary Bezier curves. Every control graph has linear edges.

Thus, to describe the skeleton, a straight-line control graph is needed (figure 2).

The set of control graph vertices consists of two subsets. The first subset is formed by vertices of polygonal figure skeleton. And the second one consists of the certain control points called *handles* of Bezier curves.

2 STRUCTURE OF THE SKELETON

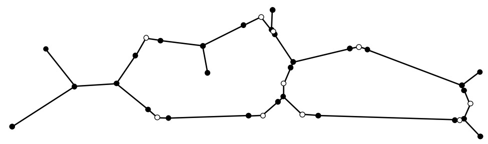

Figure 2: The control graph of the skeleton from figure 1. Terminal vertices are black and handle vertices are white.

Assume that M is a polygonal figure on R^2 with the Euclidean distance $d(p,q)$, $p,q \in R^2$. The boundary of the figure ∂M consists of several simple polygons.

An *empty disk* of the figure M of radius $r \geq 0$ centered at a point p is the closed point set

$$K_r(p) = \{q : q \in R^2, d(p,q) \leq r\}$$

such that $K_r(p) \subset M$.

A *maximal empty disk* (or, *inscribed disk*) $K_r^{\max}(p)$ of the figure M is the empty disk that is not contained in any other empty disks.

The skeleton S of the figure M is the set of all centers of maximal empty disks of the figure

$$S = \{p : p \in M, K_r^{\max}(p) \neq \varnothing\}.$$

This definition of the skeleton is more accurate as comared to the one given in the introduction since terminal vertices of skeletal graph are determined. According to this definition all convex vertices of the figure are terminal vertices of the skeleton. A non-degenerate maximal empty disk touches the figure boundary at least at two points. Every point of the figure can be considered as a degenerate disk of zero radius. These disks are empty ones because they do not contain internal points and therefore, the boundary points of the figure. Degenerate disks centered at convex vertices of figure are maximal empty disks because they are not contained in other empty disks. Consequently, convex vertices of polygonal figure are part of the skeleton.

A *radial function* is determined at every point of skeleton. Radial function is equal to a radius of the inscribed disk centered at this point. The radial function assigns “the width” of figure relative to the points of the skeleton.

Let S be the skeleton of the polygonal figure M. The total number of points in the set S is infinite, but it occurs that all these points are located at the finite set of the straight-line and quadratic parabolic segments. Let $s \in S$ be a point of a skeleton and $g_1, g_2 \in \partial M$ be the two closest boundary points of $s \in S$. The points g_1 and g_2 may have different positions on the figure boundary. We shall name the boundary point by a *corner point* if it is the vertex of polygonal figure, and *simple point* otherwise. Three cases of g_1 and g_2 type combinations are possible: g_1 and g_2 make a pair of corner points, a pair of simple points or a corner and a simple point.

If both g_1, g_2 are corner points then the point $s \in S$ lies on the medial perpendicular of the straight line segment $[g_1, g_2]$ (figure 3a).

If both points g_1, g_2 are simple and lie on different sides of the figure then s is equidistant from these sides. Then the point s lies on the bisector of the angle, formed by these sides (figure 3b). If these sides are parallel then s lies on the straight line equidistant from these sides (figure 3c).

But if one of the points (for example, g_1) is corner and the other (g_2) is simple then s is equidistant from g_1 and from the side of polygon, which contains g_2. In this case s lies on the parabola having a focus g_1. And the directrix of parabola is the side of polygon such that g_2 lies on this side (figure 3d).

Thus, we distinguish three types of lines. The first line (straight line) is defined by the pair “vertex-vertex”, the second one (bisector) is defined by the pair “side-side” and the third one (parabola) is defined by the pair “vertex-side”. Every point of the skeleton lies on one of these lines.

Let us use the following terminology. Vertices and sides of polygonal figure are called *sites*. The maximal connected subset of the skeleton equidistant from the pair of sites is called *middle axis* or, *bisector*. There are *vv-bisectors*, *ss-bisectors* and *vs-bisectors* for the pairs of sites "vertex-vertex", "side-side" and "vertex-side", respectively.

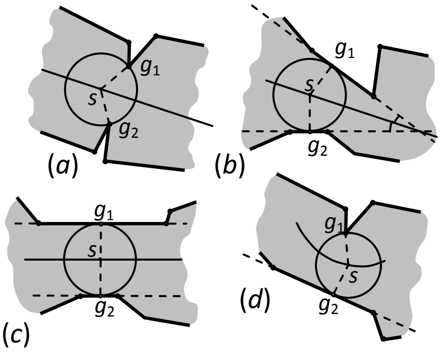

Figure 3: Bisector types of polygonal figure.

3 SKELETON VERTICES

We aim to propose a method describing the skeletal graph such that calculation of the equations of parabolic bisectors is not needed.

The skeleton vertices are equidistant to three or more sites. To find these vertices tangent circles can be constructed for the triplets of sites. Calculation of such circles involves a number of geometric tasks (figure 4) related to the following combinations:

1) three vertex-sites (figure 4*a*);
2) two vertex-sited and one segment-site (figure 4 *b,c*);
3) two segment-sites and one vertex-site (figure 4 *d,e*);
4) three segment-sites (figure 4 *f*).

The second and third combinations involve two cases depending on whether the vertex-sites match the terminal points of segments.

Assume that the tangent circle exists and the sequence of tangent points is defined. Then the tangent circle is unique. To compute the center t of the circle tangent three sites s_1, s_2, s_3, the following system of equation is to be solved:

$$\begin{cases} d^2(t,s_1) = d^2(t,s_2) \\ d^2(t,s_1) = d^2(t,s_3) \end{cases}$$

In the cases in figures 4*a,c,e,f* both equations are linear. But in the cases in figures 4*b,d* one equation is linear, and the other is quadratic. After expressing the Y-coordinate of the point t through the X-coordinate in the linear equation it become possible to reduce the second equation to the usual quadratic equation, which is easily solved.

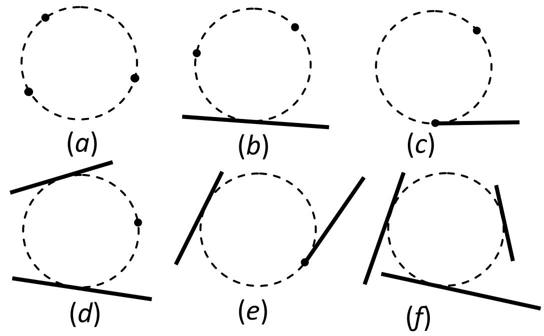

Figure 4: Tangent circles for the triplets of sites.

The obtained solution has to satisfy two auxiliary conditions, which are easily checked. The first condition requires the projections of t onto the segment-sites to lie on these segments themselves. The second condition requires the tangent circle to lie inside the figure. This means the center of tangent circle is required to lie to the left of the segment-site.

4 SKELETON EDGES AS BEZIER CURVE

Explicit description of the parametric curve $V(t) = (x(t), y(t)),\quad t \in [0,1]$ provides handy tools to deal with parabolic edges of skeleton. $V(t)$ determines the skeleton edge with the vertices $V(0)$ and $V(1)$.

The main idea of our solution is that every parabolic edge of the skeleton can be represented by a quadratic Bezier curve

$$V(t) = V_0 B_0^2(t) + V_1 B_1^2(t) + V_2 B_2^2(t), \quad t \in [0,1],$$

where $B_0^2(t) = (1-t)^2$, $B_1^2(t) = 2t(1-t)$, $B_2^2(t) = t^2$ are *Bernstein polynomials*. This curve is determined by its *control triangle* $\{V_0, V_1, V_2\}$. The points V_0 and V_2 are called the *terminal points*, and the point V_1 is *handle point* of the Bezier curve. V_0 and V_2 are vertices of skeleton, bat V_1 is not a skeleton vertex.

Such a way of edge description is compact and easy-to-use since the only point together with two terminal ones defines every edge. Also skeleton drawing and handling becomes very simple since various effective algorithms to handle Bezier curves are known.

Generalized description is based on representation of linear edges of the skeleton in the form of first order Bezier curves

$V(t) = V_0 B_0^1(t) + V_1 B_1^1(t)$, $t \in [0,1]$.

Here points V_0, V_1 denote terminal points of bisector. $B_0^1(t) = 1 - t$ and $B_1^1(t) = V_1 \cdot t$ are Bernstein polynomials.

A parabolic skeleton edge is a *vs*-bisector. Let A and B be a pair of sites that assign this bisector. Moreover, let A be a vertex and B be a side of polygonal figure connecting vertices B_1 and B_2. We shall denote the side B itself as well as a line containing it by B_1B_2. Without loss of generality, let us assume the polygonal figure be left to the side B_1B_2. Let us drop the perpendicular from the point A to the straight line B_1B_2 calling the intersection D. Denote the middle of AD by O. The sites A and B designate a rectangular Cartesian coordinate system originated at O and having DA as its Y-axis and a line parallel to B_1B_2 as its X-axis (figure 5).

Given the sites A and B, the bisector of A and B consists of the centers of the circles touched both B_1B_2 and A. Let V_0 and V_2 be the terminal points of this bisector. And let C_0 and C_2 be the projections of V_0 and V_2 onto the straight line B_1B_2 (figure 5).

Let us examine a point $V = (x, y)$ on the bisector and its orthogonal projection U on the straight line B_1B_2. The point A coordinate pair is $(0, p)$. Since $AV^2 = UV^2$ we obtain $x^2 + (y - p)^2 = (y + p)^2$. Then the bisector parabolic equation is $y = \frac{1}{4p}x^2$.

Given the points $V_0 = (x_0, y_0)$ and $V_2 = (x_2, y_2)$, consider two lines tangent parabola at V_0 and V_2.

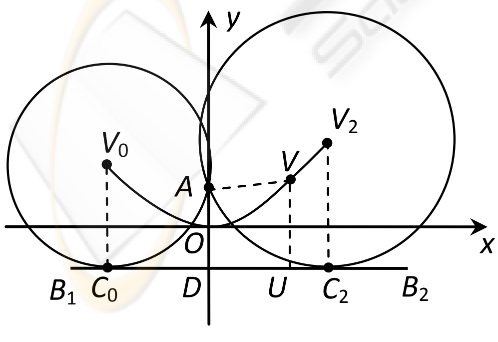

Figure 5: Parabolic curve for *vs*-bisector.

As is known, the equation of a tangent line for a curve $F(x, y) = 0$ at point $(\hat{x}, \hat{y})$ is

$$F'_x(\hat{x}, \hat{y}) \cdot (x - \hat{x}) + F'_y(\hat{x}, \hat{y}) \cdot (y - \hat{y}) = 0 .$$

In our case, we have $F(x, y) = x^2 - 4py$. Then the equations for tangent lines at the points V_0 and V_2 on the curve are the following:

$$2x_0 \cdot (x - x_0) - 4p \cdot (y - y_0) = 0 \tag{1}$$

$$2x_2 \cdot (x - x_2) - 4p \cdot (y - y_2) = 0 \tag{2}$$

Since

$$y_0 = \frac{1}{4p}x_0^2 \ , \ y_2 = \frac{1}{4p}x_2^2 \tag{3}$$

the solution of the system (1)-(2) is

$$x_1 = \frac{1}{2}(x_0 + x_2) , \tag{4}$$

$$y_1 = \frac{1}{4p}x_0 x_2 \tag{5}$$

Thus, we have obtained the point of tangent lines intersection $V_1 = (x_1, y_1)$.

Permutation of Bernstein polynomials to the quadratic Bezier curve equation gives the parametric equations for Bezier curve $V(t)$:

$$x(t) = (x_0 - 2x_1 + x_2)t^2 - 2(x_0 - x_1)t + x_0 \tag{6}$$

$$y(t) = (y_0 - 2y_1 + y_2)t^2 - 2(y_0 - y_1)t + y_0 \tag{7}$$

$t \in [0,1]$.

Permutation of (4) to (6) presents

$$x(t) = x_0 + (x_2 - x_0) \cdot t \tag{8}$$

And permutation of (3) and (5) to (7) presents

$$\begin{aligned} y(t) &= \frac{1}{4p}\left[(x_0^2 - 2x_0x_2 + x_2^2)t^2 - 2(x_0^2 - x_0x_2)t + x_0^2\right] = \\ &= \frac{1}{4p}\left[(x_0 - x_2)^2 \cdot t^2 - 2x_0(x_0 - x_2)t + x_0^2\right] = \\ &= \frac{1}{4p}\left[(x_0 - x_2)t - x_0\right]^2 \end{aligned} \tag{9}$$

From (8) and (9) we have $y(t) = \frac{1}{4p}[x(t)]^2$, that is the equation of the parabola of *vs*-bisector.

Thus, we have a parabolic bisector described as a quadratic Bezier curve. This curve is assigned by a control triangle $\{V_0, V_1, V_2\}$. Two vertices V_0, V_2 are terminal points of the bisector, and V_1 is the point of intersection of tangents lines.

Consequently, in order to obtain bisector as the Bezier curve it is necessary to calculate tangent lines at the terminal points of bisector and to find their intersection. Let us consider the solution of this problem.

5 CONTROL TRIANGLE OF SKELETON EDGE

Let $A = (0, p)$ be the focus of a parabola and $y = -p$ be the directrix of the parabola. Assume that the point $V = (\hat{x}, \hat{y})$ lies on the parabola and $C = (\hat{x}, -p)$ is the projection of V onto the directrix (figure 6). Let us show that a tangent line to a parabola at the point $V = (\hat{x}, \hat{y})$ is orthogonal to the vector $\overrightarrow{AC}$.

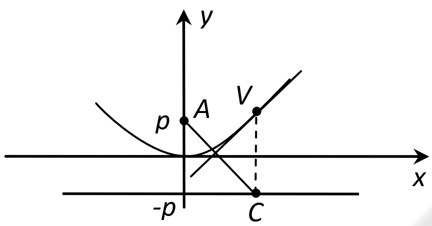

Figure 6: Orthogonality of tangent and direction from the focus into the point of projection.

The equation of the tangent line to the parabola $x^2 - 4py = 0$ at the point $V = (\hat{x}, \hat{y})$ is

$$2\hat{x} \cdot (x - \hat{x}) - 4p \cdot (y - \hat{y}) = 0 \tag{10}$$

We have that the vector $(2\hat{x}, -4p)$ is a normal vector of the tangent line and is collinear to the vector $\overrightarrow{AC}$.

This property makes it possible to find tangent lines at the terminal points V_0 and V_2 of a skeleton parabolic edge. This requires the projections C_0 and C_2 of V_0 and V_2, respectively, onto the straight line B_1B_2 to be calculated first. Then the vectors $\overrightarrow{AC_0}$ and $\overrightarrow{AC_2}$ are to be calculated. These vectors are orthogonal to the corresponding tangent lines.

The source data to identify tangent lines to the bisector at its terminal vertices is the following.

Given the pair of the sites A, B and two terminal points $V_0 = (x_0, y_0)$, $V_2 = (x_2, y_2)$ of bisector, let us find the handle vertex V_1 of the control triangle $\{V_0, V_1, V_2\}$. Without loss of generality, assume that the site A is a vertex, the site $B = [B_1, B_2]$ a side of the polygonal figure and the polygonal figure lies to the left of B.

Let us introduce the following notation. Let $\overrightarrow{PQ}$ denote the vector with an initial point P and a terminal point Q. By $[\overrightarrow{P_1Q_1} \times \overrightarrow{P_2Q_2}]$ denote the cross product, by $\left(\overrightarrow{P_1Q_1}, \overrightarrow{P_2Q_2}\right)$ denote the scalar product, by $V + \overrightarrow{PQ}$ denote a shift of point V by vector $\overrightarrow{PQ}$, by $\left|\overrightarrow{PQ}\right|$ denote length of the vector.

The algorithm to solve the problem is following:

Algorithm steps

1. To find the parameter p of the parabola:

$$p = \frac{[\overrightarrow{B_1B_2} \times \overrightarrow{B_1A}]}{2 \cdot \left|\overrightarrow{B_1B_2}\right|}.$$

2. To find points C_0, C_2 which are projections of V_0, V_2, respectively:

$$C_0 = B_1 + \overrightarrow{B_1B_2} \cdot \frac{\left(\overrightarrow{B_1B_2}, \overrightarrow{B_1V_0}\right)}{\left|\overrightarrow{B_1B_2}\right|},$$

$$C_2 = B_1 + \overrightarrow{B_1B_2} \cdot \frac{\left(\overrightarrow{B_1B_2}, \overrightarrow{B_1V_2}\right)}{\left|\overrightarrow{B_1B_2}\right|}.$$

3. To find vectors $\overrightarrow{AC_0}$ and $\overrightarrow{AC_2}$:

$$\overrightarrow{AC_0} = (C_0.x - A.x, C_0.y - A.y) = (a, b)$$

$$\overrightarrow{AC_2} = (C_2.x - A.x, C_2.y - A.y) = (c, d)$$

(a, b) and (c, d) are coordinate pairs of $\overrightarrow{AC_0}$ and $\overrightarrow{AC_2}$, respectively.

4. To solve the system of equations

$$\begin{cases} a \cdot (x - x_0) + b \cdot (y - y_0) = 0 \\ c \cdot (x - x_2) + d \cdot (y - y_2) = 0 \end{cases}$$

5. The solution of the system gives the coordinates of the handle point $V_1 = (x_1, y_1)$ of the control triangle.

6 SKELETAL GRAPH AS A COMPOUND BEZIER CURVE

We showed that each parabolic edge of the skeleton (*vs*-bisector) can be described by its quadratic Bezier curve. For generality we can consider linear edges (*vv*-bisectors and *ss*-bisectors) to be linear Bezier curves $V(t)=V_0B_0^1(t)+V_1B_1^1(t)$, $t\in[0,1]$. Here points V_0, V_1 denote terminal points of bisector. From $B_0^1(t)=1-t$ and $B_1^1(t)=V_1\cdot t$ we have $V(t)=V_0\cdot(1-t)+V_1\cdot t$.

Thus, the skeleton is a union of Bezier curves of first- and second-order. We call this union the "compound Bezier curve" analogously to the related font design concept, where compound curves describe the closed outlines of font symbols. In this paper, curves describe more complex structure that is a connected planar graph.

Planarity of the control graph of the compound Bezier curve is an important property of the control graph. This property can be proved as follows.

Let us examine the vertex-site A and the segment-site B connected with the parabolic edge.

If points V_0 and V_2 lie on the same side of the Y-axis, i.e., x_0 and x_2 are of the same sign, then from (5) it follows that $y_1\ge 0$ and the point V_1 lies above the segment B.

Assume that x_0 and x_2 have different signs (figure 7). Since the focus A of the parabola is the concave vertex of polygonal figure then the angle α, formed by incident sides of A, belongs to the interval $\pi<\alpha<2\pi$. Let us examine the angle $\angle V_0AV_2$ between vectors $\overrightarrow{AV_0}$ and $\overrightarrow{AV_2}$. It is obvious that

$$\angle V_0AV_2\le 2\pi-\left(\alpha+2\cdot\frac{\pi}{2}\right)=\pi-\alpha<\pi$$

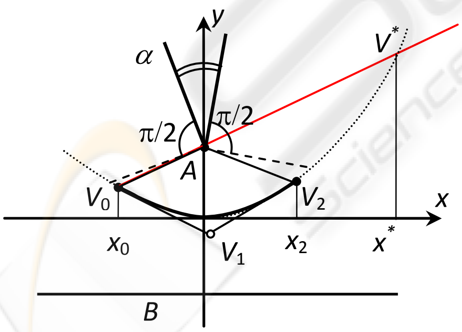

Figure 7: Planarity of the control graph.

Consequently, the point V_2 lies below the straight line AV_0 passing through the focus A, and point V_0. This straight line intersects parabola at the point V_0 and at the point V^* with the coordinates (x^*,y^*), moreover $x^*>x_2$. The equation of the straight line AV_0 is $y=p+a\cdot x$, where a is the angular coefficient. The points of intersection of this straight line with the parabola can be found from the equation $p+ax=\frac{1}{4p}x^2$.

This quadratic equation has two roots:

$$x_0=2p\cdot\left(a-\sqrt{a^2+1}\right),\ x^*=2p\cdot\left(a+\sqrt{a^2+1}\right).$$

Intersection point V_1 of the tangent lines has an ordinate y_1. From the equation (5) and the condition $x^*>x_2$ we obtain the following estimation $y_1=\frac{1}{4p}x_0x_2>\frac{1}{4p}x_0x^*=\frac{1}{4p}x_0x_2=$

$$=\frac{1}{4p}\left[2p\cdot\left(a-\sqrt{a^2+1}\right)\right]\cdot\left[2p\cdot\left(a+\sqrt{a^2+1}\right)\right]=$$

$$=\frac{1}{4p}4p^2\cdot\left[a^2-(a^2+1)\right]=-p.$$

We obtain $y_1>-p$ and the point V_1 lies above the segment B, too. Thus, we have that the control triangle of a parabolic edge does not intersect its own segment-site and lies inside the union of empty circles centered at the points of a parabolic segment. Consequently, the sides of a control triangle do not have intersections with the remaining edges of control graph. But this means that the control graph of skeleton is planar.

7 RADIAL FUNCTION OF SKELETON

To each point of a skeleton a radial function assigns a radius to an inscribed empty disk centered at this point. Let us examine representation of the radial function if the skeleton is represented by the compound Bezier curve.

Given the terminal points V_0 and V_1 of a linear *ss*-bisector together with r_0 and r_1, we can find the radius of the empty disk centered at any inner point of the edge $V_0\,V_1$ (r_0 and r_1 are radii of the disks centered at V_0 and V_1, respectively). The radius of empty disk centered at the point $V(t)=V_0\cdot(1-t)+V_1\cdot t$ is

$$r(t)=r_0\cdot(1-t)+r_1\cdot t \qquad (11)$$

Let us consider the *vs*-bisector case. In the local coordinate system (figure 7) we have simple relation between radii of disks and ordinates of the points of bisector $r(t) = y(t) + p$. From the property of Bernstein polynomials $B_0^2(t) + B_1^2(t) + B_2^2(t) = 1$ we obtain

$$r(t) = y_0 B_0^2(t) + y_1 B_1^2(t) + y_2 B_2^2(t) + p =$$
$$= (y_0 + p)\cdot B_0^2(t) + (y_1 + p)\cdot B_1^2(t) + (y_2 + p)\cdot B_2^2(t) =$$
$$= r_0 \cdot B_0^2(t) + (y_1 + p)\cdot B_1^2(t) + r_2 \cdot B_2^2(t).$$

Therefore

$$r(t) = r_0 B_0^2(t) + r_1 B_1^2(t) + r_2 B_2^2(t) \tag{12}$$

Let us consider the disc centered at the handle point V_1. For radius of this disk we have $r_1 = y_1 + p$. This disk is called a *handle disk*. As it follows from geometric analysis (figure 7), r_1 is the distance from the point V_1 to the line B_1B_2. We obtain: $r_1 = \dfrac{[\overrightarrow{B_1B_2} \times \overrightarrow{B_1V_1}]}{\left|\overrightarrow{B_1B_2}\right|}$.

Thus, the formulas (11) and (12) look like Bezier splines.

Now let us consider the *vv*-bisector. All empty disks centered at this bisector inner points touch the common vertex of polygonal figure. Therefore the radius of an empty disk centered at the point $V(t) = V_0 \cdot (1-t) + V_1 \cdot t$ is defined as distance from the point $V(t)$ to A.

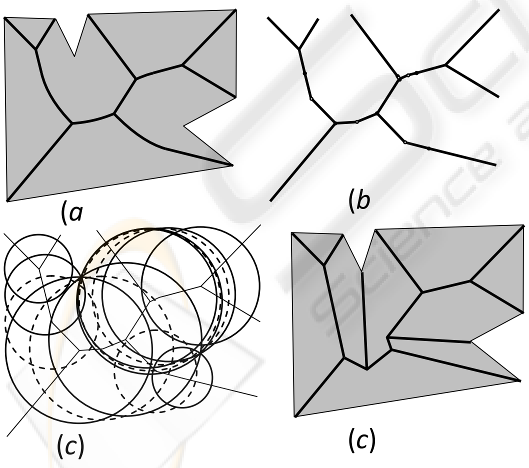

Figure 8: Polygonal figure, its skeleton, control graph, radial function, and straight skeleton.

We see that within the *vv*-bisector the raidus of an empty disk can not be presented in Bezier spline form. Thus, in order to compute the radial function for any point of *vv*-bisector, coordintaes of related concave vertices of polygonal figure should be stored in the skeleton data structure. At the same time, *vs*-bisector and *ss*-bisector require coordinates of centers of handle disks as well as radii of handle disks to be stored in the skeleton data structure.

The example in figure 8 shows the polygonal figure and its skeleton (*a*), control graph of the skeleton (*b*), and control disks of radial function (*c*). Figure (*d*) shows the stright skeleton (Aichholzer, Aurenhammer, 1996) for this polygon.

8 APPLICATIONS

Skeleton representation based on the compound Bezier curve is a handy tool for visualization, storage and image shape analysis in computer vision.

To visualize the skeleton it is enough to utilize standard graphic applications supporting drawing of straight-line segments and Bezier curves as well. Generally, graphic libraries are supplied with the tools to draw cubic Bezier curves. To exploit such programs in order to draw quadratic Bezier curves the known conversion of control polygons is to be carried out. A quadratic Bezier curve with the control triangle $\{V_0, V_1, V_2\}$ matches the cubic Bezier curve with the control quadrangle $\{W_0, W_1, W_2, W_3\}$ if and only if

$$W_0 = V_0,\ W_1 = V_0 \cdot \frac{1}{3} + V_1 \cdot \frac{2}{3},\ W_2 = V_1 \cdot \frac{2}{3} + V_2 \cdot \frac{1}{3},\ W_3 = V_2$$

Thus, obtaining the control polygon of the cubic Bezier curve matching the quadratic Bezier curve can be represented.

Example in the figure 9 presents an application of our method to a natural image (binary bitmap with the silhouette of Lomonosov Moscow university).

Bezier representation of the polygonal figure skeleton and family of its maximal empty disks provide us with the opportunity to modify shape of the figure. Modifying a figure shape based on adjusting the skeleton and its radial function can be used in computer graphics (Mestetskiy, 2000) and image recognition to measure similarity of flexible objects (Mestetskiy, 2007).

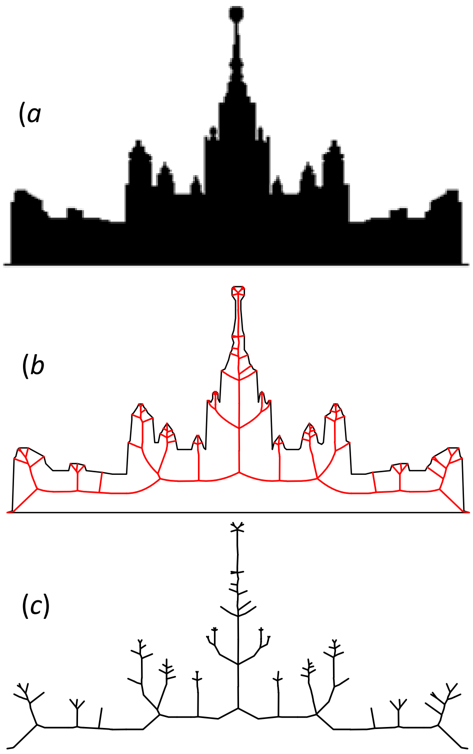

Figure 9: Skeleton representation of natural image: (*a*) binary bitmap, (*b*) polygonal figure approximation and the skeleton, (*c*) the control graph of the skeleton.

9 CONCLUSIONS

In this paper, we have presented a new approach to describe the skeleton of polygonal figure by stright line control graph of compound Bezier curves. One major advantage is the simplicity of this description. Another advantage is the independence from the algorithm of skeleton construction. The worst-case running time for skeletal graph transformation to compound Bezier curve is $O(n)$. Proposed form of skeleton presents the tool for storing skeletons in geographical databases and computer graphics systems. We are currently working on extending the above results to the segment Voronoi diagrams.

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