NORMALIZATION PROCEDURES ON MULTICRITERIA DECISION MAKING
An Example on Environmental Problems

Ethel Mokotoff, Estefanía García and Joaquín Pérez
Departamento de Fundamentos de Economía e Hª Eª, Universidad de Alcalá, Alcalá de Henares, Spain

Keywords: Strategic Decision Support System, Multicriterio Decision Making, Normalization Procedures, Sustainable Development.

Abstract: In Multicriteria Decision Making, a normalization procedure is required to conduct the aggregation process. Even though this methodology is widely applied in strategic decision support systems, scarce published papers detail this specific question. In this paper, we analyze the results of the influence of normalization procedures in the weight sum aggregation in Multicriteria Decision problems devoted to sustainable development.

1 INTRODUCTION

Multicriteria Decision Making (MDM) methodology has reached a high level of maturity and its applications pervade nowadays to almost every field of human activity. In fact, there is a growing demand for systems specifically designed to support such a kind of analysis, even by casual users who do not have a deep understanding of its theoretical foundations.

This situation is especially true concerning the so called Discrete Multicriteria Decision Making problem, *i.e.* the branch of MDM devoted to problems where there are a finite, and usually small, number of alternatives competing for one to be finally selected or which have to be ranked. Problems of this kind are everywhere: selection of research projects, biddings to a public contest, candidates for a job, locations of a new facility, investments, etc. In the last few years, the increasing concern on environmental problems has created a need of including these issues and developing more effective decision tools.

Whatever the problem in question is, the criteria used to evaluate alternatives usually respond to different issues. In particular, dealing with sustainable development problems, the selected criteria respond to questions not just economic but also social and ecological. Thus, original data are not measurable by the same units for the whole set of criteria, and a normalization procedure (that converts all the criteria values into non-dimensional, *i.e.* comparable quantities) is required to make possible the aggregation procedure. MDM exercises use sometimes a particular normalization procedure regardless the influence of this procedure on the results. Therefore, the objective of this article is to point out the fact that prior normalization of data is not neutral, and, more important, the final ranking of alternatives may well depend on the normalization procedure used.

We have conducted an experiment comparing the results obtained by varying the normalization procedure in a real environmental application (Pasanen, et al., 2005, and Hiltunen, et al., 2009). We have employed the weighted sum method included in the SMC package (Barba-Romero and Mokotoff, 1998).

This paper is organized as follows. Section 2 briefly introduces the theoretical foundations of MDM and the normalization procedures. Section 3 presents the example and the experiment that serves to thoroughly illustrate the influence of the normalization procedure. Section 4 provides some concluding remarks.

Mokotoff E., García-Vázquez E. and Pérez Navarro J. (2010).
NORMALIZATION PROCEDURES ON MULTICRITERIA DECISION MAKING - An Example on Environmental Problems.
In *Proceedings of the 12th International Conference on Enterprise Information Systems - Artificial Intelligence and Decision Support Systems*, pages 206-211
DOI: 10.5220/0002896102060211

2 MDM: THEORETICAL BASIS

2.1 MDM Matrix

In any multicriterion analysis, the first step is the information gathering phase that will apply to the whole of the problem at hand, involving a survey of the criteria and possible alternatives. Then, the design phase consists of constructing the choice sets, *i.e.* the *alternatives*, a finite and discrete set, in this case.

Let us suppose there are m alternatives, constituting the *choice set* $A = \{A_1, A_2, \ldots, A_m\}$ and n *criteria,* $C_1, C_2, \ldots, C_j, \ldots, C_n$. Thus, an mxn matrix of evaluations, $[a_{ij}]$, characterizes a MCDM instance. Each line of the matrix expresses the performance of the alternative A_i according to the n criteria, while each column, C_j, expresses the evaluations of all the alternatives according to the criterion C_j.

Leaving aside the problem of how to construct the criteria proper, it is not an easy matter to evaluate each alternative A_i relative to a given criterion C_j, to obtain a coefficient a_{ij}. In this paper, we assume that these evaluations are known with certainty.

Each of the referred criteria is originally measured in its inherent unit (even we can have not only numerical attributes). Thus, the MDM matrix may presents evaluations of different nature. These evaluations must be aggregated, taking into account the preferences of the decision maker, to achieve a global evaluation value for each alternative, on which the overall ranking is based.

In our study, we consider the most widely known aggregation method, *i.e.* the *weighted (linear) sum,* also known as *simple additive weight*, whose main advantage is that it is both, intuitive and simple to apply. Although the method is very simple, we must nevertheless carefully specify the starting data and the transformation it undergo.

Weighted sum is a compensatory method. Compensatory aggregation methods require the different criteria evaluations and weights to be settled down in compatible scale. This means that a normalization procedure has to be executed to transform figures of the matrix on a comparable scale.

We suppose that the evaluations, a_{ij}, result from the nature of the attribute that criterion C_j measures on a numerical scale. We also assume that the decision maker's preferences can be stated by means of positive weights, w_j, which should be associated to each criterion, C_j.

For weights there is no problem because the normalization is achieved making their sum equal to 1, dividing each weight, w_j, by $\sum_j w_j$.

With respect to the evaluations, two classical normalization procedures are considered for comparison in the present study: *proportionality preservation* and *natural thresholds*, which are briefly described below.

For a given criterion C_j, a normalization procedure transforms the evaluations of the m alternatives, $(a_{1j}, a_{2j}, \ldots, a_{mj})$, into a new vector, $(v_{1j}, v_{2j}, \ldots, v_{mj})$, where v_{ij} is the normalization of a_{ij}.

Without loss of generality, we assume that all criteria are going to be maximized and that values of a_{ij} are strictly positive (since, in the example of this paper indeed it is).

2.2 MDM Matrix Normalization Procedures

2.2.1 Proportionality Preservation

This procedure transforms the evaluation vector, $(a_{1j}, a_{2j}, \ldots, a_{mj})$, of each criterion, C_j, into a normalized one by making

$$v_{ij} = \frac{a_{ij}}{\max_i a_{ij}}, \quad \forall i \tag{1}$$

Therefore, for each criterion, C_j, the normalized value of the best alternative is 1, and all the rest are percentages of the maximum value, resulting in the interval $0<v_{ij} \leq 1$, (we assume $a_{ij}>0$).

It is the most widely used normalization procedure. The main advantage of the method is that the original proportion existing between the evaluations of every pair of alternatives is preserved after normalization, *i.e.* $a_{ij} / a_{i'j}$ is equal to $v_{ij}/v_{i'j}$. This is a very desirable property in many circumstances, but it is not trivial to obtain, especially when dealing with minimizing criteria. Even it may be impossible to apply when there are evaluations with a zero value or with different signs, because proportionality is then not defined. The drawback of the result vector is that the evaluations obtained by this procedure are not forced to cover the complete interval [0, 1].

2.2.2 Natural Thresholds

This procedure transforms the evaluation vector, $(a_{1j}, a_{2j}, \ldots, a_{mj})$, of each criterion, C_j, into a normalized one by making

$$v_{ij} = \frac{a_{ij} - \min_i a_{ij}}{\max_i a_{ij} - \min_i a_{ij}}, \quad \forall i \qquad (2)$$

Therefore, for each criterion, C_j, the normalized value of the best alternative is 1, while the normalized value of the worst alternative is 0. The rest ones take values $0 \le v_{ij} \le 1$, (if a_{ij} should take the same value $\forall i$, then v_{ij}=1, $\forall i$).

The main advantage of the method is that it ensures that the evaluations cover the entire range [0, 1], through a simple linear interpolation between the extreme points. If the criterion is to minimize, the transformation is inverse, in an obvious way. This procedure respects cardinality but it does not preserve proportionality.

2.3 Aggregation

Once coefficients and weights have been normalized, for each alternative, A_i, the global evaluation is computed as follows

$$GE(A_i) = \sum_{j=1}^{n} w_j v_{ij} \qquad (3)$$

Alternatives are then ranked in descending order of their global evaluation values. In case of ties, the *rank average* of Kendall is applied.

3 ILLUSTRATIVE EXAMPLE OF THE INFLUENCE OF THE NORMALIZATION PROCEDURES

3.1 Model

To illustrate the normalization problem we have chosen the example presented by Pasanen et al. (2005), named MESTA, to provide support to landowners in the forest planning process. The model considers four alternative forest plans for 10 years:

- A_1: *Status Quo*
- A_2: *Cuttings*
- A_3: *Recreation*
- A_4: *Nature Protection*

The forest owners have different objectives as regards forest utilisation. Their individual owner goals will be not only including economic, but ecological, and social aims too. The model proposes the following five criteria to evaluate the alternative plans:

- C_1: *Old forest area* (%): Percentage of land preserved to old-growth forest conservation. It captures biodiversity values including endangered species protection.
- C_2: *Cutting removal* (1000 m^3): Timber extraction.
- C_3: *Scenery forests* (ha): Land area preserved to landscape and recreation activities.
- C_4: *Job opportunities* (men/years): Local employment in any of the alternative plans.
- C_5: *Turnover* (mill €): Monetary return from the various activities in the area.

Clearly, C_2 and C_5 are economical criteria. C_1 is a pure ecological criterion. C_4 is a social criterion, and social sustainability aspects can also be found in the recreation and landscape criterion, C_3.

Table 1 presents the decision matrix with the corresponding evaluations that establishes the correspondence between alternative plans and criteria. (The matrix data has been extracted from www.metla.fi/hanke/3292/metsauunnittelu/). It is easy to realize, from the figures on this matrix, that starting from the *status quo* alternative, A_1, the different values in corresponding evaluations are based on the objectives each plans pursued. Thus, A_2 can be named as an economic option, A_3 as a social one, and A_4 as a conservative or ecological plan.

Table 1: MESTA Decision Matrix.

ALT/CRIT	C_1	C_2	C_3	C_4	C_5
A_1	27.2	749	138504	350	33
A_2	26	984	122213	440	42
A_3	29.3	535	138504	269	25
A_4	32	156	138110	124	10

3.2 Experiment

For a better understanding, we have organized the criteria into three different groups: Environmental Conservation, Economic and Social criteria. This allows us to make a balanced allocation of weights among these three "super-criteria", assigning 0.33 to each of them. Within each group, weights had been distributed the way we subjectively consider most appropriate. (We decided not to include the sensitivity analysis of weights in this paper, by limited extension thereof). These model data are then completely determined (see Table 2). Obviously, these evaluations must be converted to comparable units in order to get the final aggregation result.

At this point, we have employed the SMC because it offers the possibility to choose the normalization procedure and the aggregation method. We have chosen proportionality preservation and natural threshold procedures because of their automatism, without necessity to set up any parameters. As aggregation method, we have chosen the weighted sum method because it probably is the best to clearly show the essence of evaluation in MDM.

3.2.1 Proportionality Preservation

This normalization procedure converts evaluations into numbers in the (0, 1] interval. Table 3 shows the normalized evaluations for the model we present. When preserving proportionality, transformed figures maintain the original dispersion. The best alternative always presents 1, while 0 does not appears (unless an original evaluation is null).

Figure 1 shows results after computing global evaluations by the weighted sum method. The final order of the alternatives turned out to be, $A_2 > A_1 > A_3 > A_4$.

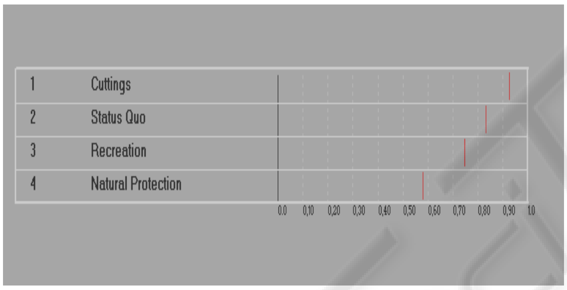

Figure 1: Ranking and Global Evaluations computed by Proportionality Preservation.

3.2.2 Natural Thresholds

Table 4 shows the normalized evaluations using Natural Threshold. We can observe that, regardless of the dispersion of the original figures, each criterion numbers are distributed along the closed interval [0, 1]. There are 0 and 1 evaluation values, corresponding to the worst and best alternatives, for all criteria.

Aggregated evaluations give the results showed in Figure 2. Alternatives are now ordered as $A_1 > A_2 > A_3 > A_4$, though there is no ties, we can realize that differences between A_2 and A_3 are negligible, even A_1 is quite close by A_2 and A_3, however, A_4 is notably the most underprivileged.

In the example the social and ecological alternatives, A_3 and A_4, respectively, present relatively good evaluations with respect to C_1 and C_3. When

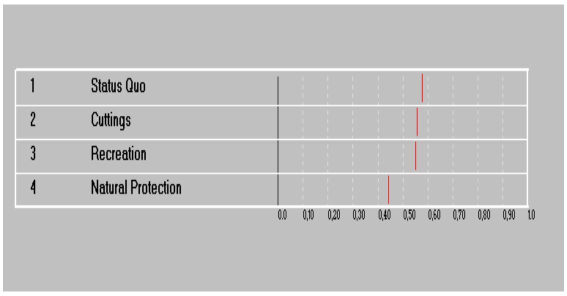

Figure 2: Ranking and Global Evaluations computed by Natural Thresholds.

proportionality is preserved for all criteria, the social and ecological alternatives will never be well ranked because, keeping proportionality on values from criteria C_1 and C_3, where data are sparsely dispersed, makes negligible the differences between the values of these attributes.

After proving the great influence of the normalization procedure on results (global evaluations and ranking of the alternatives), we have essayed two other possible normalization schemes, which emerged from the analysis of each criterion and the corresponding figures.

3.2.3 Normalization Procedures According to Each Criterion

In this model, an alternative is a possible plan to be carried out by the owner of a small portion of land. Although each alternative plan that one individual owner can implement will directly generate a certain amount of cuttings, income and job opportunities, the incidence of her own decision on the global environment can be quite small. That is why, Nature Protection plan, A_4, is not significantly differentiated from the rest (not even Cuttings option, A_2) when considering Environment Protection criteria. Something similar occurs with the Recreation option, A_3, and the Scenery Forests criterion.

To prevent loss of discrimination between different plans in Old Forest Area and Scenery Forests, we have applied the Natural Threshold procedure, while the other three criteria have been normalized by the Proportionality Preservation procedure.

Results for this model are shown in Table 5 and Figure 3. Alternatives are now ordered as $A_3 > A_1 > A_4 > A_2$, We can realize that global evaluations are still less disperse. Even with this normalization, A_2 is notably the most underprivileged. A_3 and A_4 are now ranked in better position than before.

Table 2: Model with original evaluation and weights.

Weights	0,33	0,33	0,06	0,27	0,33	0,22	0,11	0,33
ALT/CRIT	C_1	Environment	C_2	C_3	Economics	C_4	C_5	Social
A_1	27,2	-	749	33	-	350	138504	-
A_2	26,0	-	984	42	-	440	122213	-
A_3	29,3	-	535	25	-	269	138504	-
A_4	32,0	-	156	10	-	124	138110	-

Table 3: Model with evaluations normalized by Proportionality Preservation.

Weights	0,33	0,33	0,06	0,27	0,33	0,22	0,11	0,33
ALT/CRIT	C_1	Environment	C_2	C_3	Economics	C_4	C_5	Social
A_1	0,850	-	0,761	0,786	-	0,796	1,000	-
A_2	0,813	-	1,000	1,000	-	1,000	0,882	-
A_3	0,916	-	0,544	0,595	-	0,611	1,000	-
A_4	1,000	-	0,159	0,238	-	0,282	0,997	-

Table 4: Model with evaluations normalized by Natural Thresholds.

Weights	0,33	0,33	0,06	0,27	0,33	0,22	0,11	0,33
ALT/CRIT	C1	Environment	C_2	C_3	Economics	C_4	C_5	Social
A_1	0,200	-	0,716	0,719	-	0,715	1,000	-
A_2	0,000	-	1,000	1,000	-	1,000	0,000	-
A_3	0,550	-	0,458	0,469	-	0,459	1,000	-
A_4	1,000	-	0,000	0,000	-	0,000	0,977	-

Table 5: Model with C_1 and C_3 normalized by Natural Thresholds, and C_2, C_4 and C_5 by Proportionality Preservation.

Weights	0,33	0,33	0,06	0,27	0,33	0,22	0,11	0,33
ALT/CRIT	C1	Environment	C_2	C_3	Economics	C_4	C_5	Social
A1	0,200	-	0,761	0,786	-	0,796	1,000	-
A2	0,000	-	1,000	1,000	-	1,000	0,000	-
A3	0,550	-	0,544	0,595	-	0,611	1,000	-
A4	1,000	-	0,159	0,238	-	0,282	0,976	-

Table 6: Model with evaluations normalized by Satiation Thresholds.

Weights	0,33	0,33	0,06	0,27	0,33	0,22	0,11	0,33
ALT/CRIT	C1	Environment	C_2	C_3	Economics	C_4	C_5	Social
A_1	0,272	-	0,749	0,330	-	0,350	0,770	-
A_2	0,260	-	0,984	0,420	-	0,440	0,444	-
A_3	0,293	-	0,535	0,250	-	0,269	0,770	-
A_4	0,320	-	0,156	0,100	-	0,124	0,762	-

3.2.4 Satiation Thresholds

In this procedure, thresholds are not automatically determined, but rather we have to set them up. Indeed, it has been originally developed to avoid the "irrelevant alternatives dependence" effect (Barba-Romero and Mokotoff, 1998). Thresholds can be fixed independently of the evaluations values. In this case, we have essayed settling down a wide range. This way, there are neither 0 nor 1 evaluation values, as we can see in Table 6. Results show (Figure 4) the same ranking as when proportionality preservation is applied.

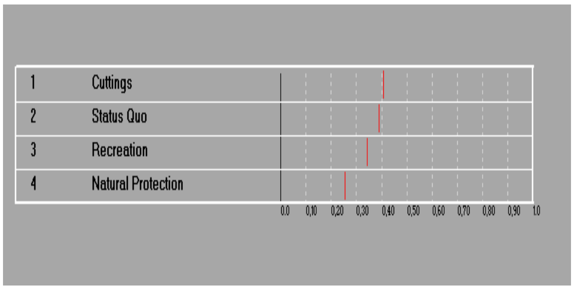

Figure 4: Ranking and Global Evaluations computed by Satiation Thresholds.

4 CONCLUSIONS

Regarding normalization procedures, there is no doubt about the relevance of preserving the original proportion existing between the evaluations of every pair of alternatives. However, we have observed that, when the evaluation values for a criterion are not widely dispersed, maintaining proportionality (by applying proportionality preservation as normalization procedure) implies that the normalized evaluation vectors remain with the same dispersion and, therefore, it does not help to differentiate alternatives. When evaluation values of different alternatives are very close together, it is possible to gain dispersion, applying natural thresholds normalization. In this way, the normalization procedure helps to distinguish alternatives with apparently similar attribute values.

We can conclude that the decision maker may miss the importance of choice of a criterion under certain normalization procedures. It is unrealistic to hope for general normalization procedures that performs equally well for different type of criterion. The analysis of the results obtained by this experiment give support to the hypothesis which states that the normalization procedure should be specially chosen in accordance with every criterion in a MCDM model.

Concerning to the preferences, we can claim that, to request the decision maker to express the criterion weights, disregarding the normalized evaluation values, makes the decision process not valid.

ACKNOWLEDGEMENTS

This research was supported by the Research Projects ECO2008-05895-C02-02, Ministerio de Ciencia e Innovación, Spain.

REFERENCES

Barba-Romero, S., and Mokotoff, E. (1998). A System to Support Discrete MultiCriteria Evaluations and Decisions: the SMC Package. In: *2nd IMACS International Multiconference. Computational Engineering in Systems Applications,* Nabeul-Hammamet, Túnez.

Hiltunen, V., Kurttila, M., Leskinen, P., Pasanen, K., and Pykäläinenet, J. (2009). Mesta: An internet-based decision-support application for participatory strategic-level natural resources planning. *Forest Policy and Economics* 11, 1–9.

Pasanen, K., Kurttila, M., Pykäläinen, J., Kangas, J., and Leskinen, P. (2005). Mesta: nonindustrial private forest owners' decision-support environment for the evaluation of alternative forest plans over the Internet. *International Journal of Information Technology & Decision Making*, 4 (4), 601–620.