# DESIGN OF A FULLY AUTOMATED ROBOTIC SPOT-WELDING LINE 

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Abstract: The mixed model assembly line design problem includes allocating operations to the stations in the robotic cell and satisfying the demand and cycle time within a desired interval for each model to be produced. We also ensure that assignability, precedence and tool life constraints are met. Each pair of spot welding tools can process a certain number of welds and must be replaced at the end of tool life. Tool replacement decisions not only affect the tooling cost, but also the production rate. Therefore, we determine the number of stations and allocate the operations into the stations in such a way that tool change periods coincide with the unavailability periods to eliminate tool change related line stoppages in a mixed model fully automated robotic assembly line. We provide a mathematical formulation of the problem, and propose a heuristic algorithm.

## 1 INTRODUCTION

This study focuses on a robotic cell mixed-model assembly line design problem in which automotive body components are produced. The problem is to determine the number of stations to be established, to allocate the welding operations to these stations with a constraint on the cycle time and, different from the literature, to determine the tool change periods in order to maximize the total profit. Each station consists of a single robot equipped with spot welding guns. Each gun has spot welding tools used to perform welding operations. These tools have limited life time which is represented by the total number of spot welds that they can process.

Numerous studies have been conducted by researchers, in which different aspects of assembly line design problems are considered such as the layout of the line, equipment selection and line balancing. The interested reader can see the recent surveys by Becker and Scholl (2006) and Boysen et al. (2007). In this study, different from the existing ones, we address a mixed-model assembly line problem with a profit maximizing objective. We aim to determine the number of stations to install, which effects the station installation costs and the throughput rate; the assignme-
nt of operations to the stations, which effects throughput rate and tool usage (as a consequence the tooling cost); and tool change periods, which effect cycle time and tooling costs. Therefore, the relevant terms for the total profit function are the individual profits from each manufactured component (revenue minus all the costs except the tooling and station installation costs), tooling cost, and station installation cost. As a consequence, we define the total profit function as the difference between the sum of individual profits gained by manufacturing components of the final products and the sum of investment costs for stations and tooling. Station investment costs include robot, fixture and space costs. Tooling costs are incurred over time as the tools are replaced with new ones. Prior studies do not consider the unavailability periods of assembly lines and assume that assembly lines work 24 hours a day continuously. In this study, we take such breaks into account to reflect a more realistic production environment.

In Gultekin et al. (2006), we dealt with the robotic cell scheduling problem with two identical CNC machines and a single material handling robot. We already showed that the previous theoretical results on the same problem were no longer valid when we added the tool life related constraints to the problem
setting since the cycle times depend on the allocation of the operations. Tool life has a number of implications in the current problem setting as well: Continuing to use a tool even after its life is over results in quality problems and, thus, the tools must be replaced before their lives end. On the other hand, replacing a tool before its life is over increases the tooling cost. Additionally, the life of a tool may end at a time instance when the assembly line is supposed to be operating and therefore may result in line stoppages. In order to overcome this problem, we aim to design the line so that the tools are changed in periods of unavailability in which the line is already not operating. At each break, if there exist welding tools that have a remaining number of spot welds less than the total number of spot welds that they have to perform until the next break, they must be replaced with new ones. With such an operating rule and by assigning the operations to the stations considering not only the cycle times but also the tool usage rates at each station, we prove that the total profit can be increased.

The design problem that we study in this paper is originated from a project that we conducted for one of the leading automotive manufacturing companies in Turkey. One of the important aspects of any design problem is the operational problems that can be faced during the actual implementation of the design. We first investigated the existing automotive body component robotic spot welding assembly line, where the line is fully automated to produce two models in any order. In that project, our aim was to develop a tool change decision policy for the assembly line where there was no systematic approach for scheduling the tool change periods. All of the tools were being replaced just at the end of their lives. As a consequence, the line was stopped regularly for tool changes. The company has allocated almost $10 \%$ of their available capacity to the line stoppages due to tool changes. This is the main reason why we study line design and operation allocation problems together with the scheduling of tool change periods in this study.

## 2 PROBLEM DEFINITION

We consider an assembly line that contains at most $m$ stations: $S_{j}, j \in M=\{1,2, \ldots, m\}$. Space restrictions in the production area and a limited budget for investment costs are among the reasons of such a restriction on the number of stations. We use parameter $V_{j}$ to denote the cost of setting up $S_{j}$. There are $g$ different models to be produced in this robotic assembly line. $O_{h i}$ represents operation $i$ of a part of model $h$; $i \in N_{h}=\left\{1,2, \ldots, n_{h}\right\}, h \in G=\{1,2, \ldots, g\}$. A num-
ber of spot welds are grouped together to form an operation. The selection is based on the closeness of the welds to each other and to satisfy the proper geometry of the assembled components. Let $W_{h i}$ be the number of spot welds required to perform operation $i$ on model $h$. The total number of spot welds that the welding tool in $S_{j}$ can process before its life is over is denoted by $B_{j}$.

Factors that affect the number of stations to be installed are the target cycle time, the station investment cost, and the tooling cost. Let us represent the yearly expected demand for the parts to be produced as $\theta_{h}$. We use the parameter $\gamma_{h}$ to denote the target cycle time for model $h$ to meet the yearly expected demand $\theta_{h}$. In general, let $T_{h}$ be the total time allocated for production of model $h ; f_{h}$ the actual cycle time of model $h$; and $\widehat{\theta}_{h}$ the corresponding production quantity. Then, we have the following: $\widehat{\theta}_{h}=\frac{T_{h}}{f_{h}} \quad \forall h$.

Producing model $h$ less than a specified quantity is not acceptable. Let this specified quantity be denoted by $\theta_{h}^{U}$ and the corresponding cycle time be denoted by $\gamma_{h}^{U}$, which specifies an upper bound on the cycle time. Additionally, if production exceeds a certain quantity for each model, it is not possible to make any additional profit. Let this production level be denoted by $\theta_{h}^{L}$ and the corresponding cycle time lower bound be denoted by $\gamma_{h}^{L}$. Clearly, we have $\theta_{h}^{U} \leq \theta_{h} \leq \theta_{h}^{L}$. Additionally, the individual profit gained from model $h$ is assumed to be a piecewise linear function. It is denoted by $P R_{h}$ for $\widehat{\theta}_{h} \leq \theta_{h}$. On the other hand, if the actual production amount of model $h$ exceeds the yearly demand of $\theta_{h}$ so that $\theta_{h}<\widehat{\theta}_{h} \leq \theta_{h}^{L}$, then the profit for each model $h$ produced as surplus is $P R_{h}^{\varepsilon}$, where $P R_{h}^{\varepsilon}=P R_{h}-\Delta_{h}, \quad 0 \leq \Delta_{h} \leq P R_{h}$, where $\Delta_{h}$ denotes the reduction in the individual profit for the excess parts when the production exceeds the expected demand. As a consequence, the individual profits from each model $h$ is a two-piece linear function where the slope of the second piece is less than that of first piece.

In this study, we consider the tool change periods as a decision variable. If the tools are only changed at scheduled breaks as we propose, the line will never stop for a tool change, which increases the throughput rate, but on the other hand, some tools will be changed before their lifetime ends. Therefore, the tooling cost will be increased. As a result, one aim of the current study is to determine the best solution that balances the increased tooling cost with the increased throughput rate. Let $C_{j}$ be the cost of the tools in $S_{j}$. We assume there are $q=1,2, \ldots, U$, total number of breaks in the planning horizon. We also need the following decision variable:
$\begin{aligned} z_{j q}= & \text { Binary variable indicating whether tools in } S_{j} \\ & \text { are changed in tool change period } q \text { or not. }\end{aligned}$

A huge number of spot welds is required to complete the production of a part. Depending on the location of these welds and the geometry of the parts, it may not be possible for a particular type of welding gun to reach every location on a part and the special characteristics of the weld such as the diameter and the thickness may require a different welding tool. Hence, several types of guns and tools can be loaded on the robots and hence we need the following parameter indicating the assignability of operations.
$a_{h i j}=1$, if operation $i$ of model $h$ is assignable to $S_{j}$;
0 , otherwise.
Additionally, we have a precedence relation among the operations which is indicated with the following parameter:
$p_{h i k}=1$, if operation $i$ precedes operation $k$ in model $h$;

## 0 , otherwise.

Let $t_{h i}$ be the time required to perform operation $i$ of model $h$. We calculate $t_{h i}$ as a function of the number of spot welds required to perform operation $i$ of model $h$ as follows: $t_{h i}=\alpha+\beta \cdot W_{h i} \quad \forall h, i$, where $\alpha$ is a constant that denotes the time required for the robot to reach to the location to perform operation $i$ of model $h$ and $\beta$ is another constant that corresponds to the time required to process a single spot weld.

We define the parameter $\psi_{h}$ as the ratio of the total yearly required time to process all demanded parts of model $h$ to total yearly required time to process all demanded parts of all models. This ratio is the time allocated for production of parts of model $h$ and can be calculated as follows:

$$
\psi_{h}=\frac{\theta_{h} \cdot \gamma_{h}}{\sum_{l=1}^{g} \theta_{l} \cdot \gamma_{l}} \quad \forall h .
$$

In most automotive companies (including the one with which we have been collaborating), the production time between two breaks is equal and lasts 6300 seconds within a shift. In order to calculate the remaining tool life at each break we need to calculate the time allocated for the production of parts of all models between two breaks as $6300 \cdot \psi_{h}$ seconds.

In addition to $z_{j q}$, the other decision variables that we will use to formulate our model are defined below:
$R_{j q}=$ remaining number of spot welds that the tool in $S_{j}$ can process after tool change period $q$.
$f_{h}=$ actual cycle time for model $h$.
$\sigma_{j}=1$, if $S_{j}$ is used in the assembly line; and 0 , o.w.
$x_{h i j}=1$, if operation $i$ of model $h$ is assigned to station $j$; and 0 , o.w.

The nonlinear mixed-integer mathematical model can be formulated as follows:

Model 1 (NLMIP):

$$
\begin{align*}
& \text { Max } \sum_{h=1}^{g} P R_{h} \cdot \min \left\{\frac{\theta_{h} \cdot \gamma_{h}}{f_{h}}, \theta_{h}\right\} \\
& +\sum_{h=1}^{g} P R_{h}^{\varepsilon} \cdot \min \left\{\max \left\{\frac{\theta_{h} \cdot \gamma_{h}}{f_{h}}-\theta_{h}, 0\right\}, \frac{\theta_{h} \cdot \gamma_{h}}{\gamma_{h}^{L}}-\theta_{h}\right\} \\
& -\rho \cdot \sum_{j=1}^{m} \sum_{q=1}^{U} C_{j} \cdot z_{j q}-\eta \cdot \sum_{j=1}^{m} V_{j} \cdot \sigma_{j} \tag{1}
\end{align*}
$$

Subject to

$$
\begin{equation*}
f_{h} \geq \tau_{j}+\sum_{i=1}^{n_{h}} t_{h i} \cdot x_{h i j} \quad \forall h, j \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
f_{h} \leq \gamma_{h}^{U} \quad \forall h \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
x_{h i j} \leq a_{h i j} \cdot \sigma_{j} \quad \forall h, i, j \tag{4}
\end{equation*}
$$

$$
\begin{equation*}
\sum_{j=1}^{m} x_{h i j}=1 \quad \forall h, i \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
p_{h i k} \cdot \sum_{l=1}^{j} x_{h i l}+\left(1-p_{h i k}\right) \geq x_{h k j} \quad \forall h, i, j, k \tag{6}
\end{equation*}
$$

$$
R_{j q}=B_{j} \cdot z_{j q}+\left[R_{j(q-1)}\right.
$$

$$
\begin{equation*}
\left.-\sum_{h=1}^{g} \frac{6300 \cdot \psi_{h}}{f_{h}} \cdot \sum_{i=1}^{n_{h}} W_{h i} \cdot x_{h i j}\right]\left(1-z_{j q}\right) \forall j, q \tag{7}
\end{equation*}
$$

$$
\begin{align*}
& R_{j 0}=B_{j} \forall j=1 \quad  \tag{8}\\
& R_{j q} \geq \sum_{h=1}^{g} \frac{6300 \cdot \psi_{h}}{f_{h}} \cdot\left(\sum_{i=1}^{n_{h}} W_{h i} \cdot x_{h i j}\right) \quad \forall j, q  \tag{9}\\
& f_{h} \geq 0, R_{j q} \geq 0, \quad x_{h i j}, \sigma_{j}, z_{j q} \in\{0,1\} \quad \forall h, i, j, q \tag{10}
\end{align*}
$$

The objective function is a piecewise linear function that includes the individual profits that can be earned by producing up to $\theta_{h}$ parts of model $h$ and the reduced profits for the situation of producing more than $\theta_{h}$ as well as the tooling and station investment costs. Tooling and station costs are converted to annual costs by the constants $\rho$ and $\eta$, which depend on the selection of the total number of tool change periods in the planning horizon, $U$, and the number of months that the production of the selected models will continue, respectively. Constraint (2) ensures that the cycle time is the maximum station time, which is the sum of the operation times allocated to that station plus a constant $\tau_{j}$ required for the robot in station $j$ to begin and finalize processing the allocated operations. (3) prevents the actual cycle time for model $h$ from exceeding $\gamma_{h}^{U}$. Constraint (4) restricts operations to be assigned only to the stations where they can be performed and to stations which are used in the assembly line. By (5), we ensure that an operation is assigned to exactly one station and none of the operations remain unassigned. Constraint (6) allows an operation to be assigned to a station if all its predecessor operations are assigned to the same or to a preceding station. Constraint (7) is a coupling constraint that connects different product models to compete for a limited tool life at each station. It also handles updates for the remaining number of spot welds
for welding tools. If the tool is replaced at that tool change period, the life time of the tool is reset. Otherwise, the remaining tool life is calculated by subtracting the number of spot welds performed during the last working period from the tool life at the beginning of this working period. The lifetimes of the tools are initialized in Constraint (8). By constraint (9), we guarantee that none of the welding tools finishes its lifetime between two breaks.

This is a nonlinear mixed integer programming formulation. We could use DICOPT, a nonlinear solver included in GAMS software, to solve this formulation. Initial tests proved that this nonlinear model is not efficient in solving the problem.

## 3 LINEARIZATION OF NLMIP

In this section we present the methodology for linearization of Model 1. We will linearize the objective function (e.g., Equation (1)), Constraints (2), (3), (7), and (9) of NLMIP with some well-known techniques. First, we replace $\frac{1}{f_{h}}$, which frequently occurs in Model 1 with a new variable $\omega_{h}$. As a result, the piecewise linear profit part of the objective function becomes

$$
\begin{gathered}
\sum_{h=1}^{g} P R_{h} \cdot \min \left\{\theta_{h} \cdot \gamma_{h} \cdot \omega_{h}, \theta_{h}\right\} \\
+\sum_{h=1}^{g} P R_{h}^{\varepsilon} \cdot \min \left\{\max \left\{\theta_{h} \cdot \gamma_{h} \cdot \omega_{h}-\theta_{h}, 0\right\}, \frac{\theta_{h} \cdot \gamma_{h}}{\gamma_{h}^{L}}-\theta_{h}\right\}
\end{gathered}
$$

In order to linearize this we introduce new positive variables $A O_{h}, \lambda_{h}^{1}, \lambda_{h}^{2}, B O_{h}$ and a binary variable $y_{h}$. The new objective function can be written as follows:

$$
\begin{aligned}
& \sum_{h=1}^{g} P R_{h} \cdot A O_{h}+\sum_{h=1}^{g} P R_{h}^{\varepsilon} \cdot B O_{h}-\rho \cdot \sum_{j=1}^{m} \sum_{q=1}^{U} C_{j} \cdot z_{j q} \\
& -\eta \cdot \sum_{j=1}^{m} V_{j} \cdot \sigma_{j}
\end{aligned}
$$

We also require the following constraints:

$$
\begin{array}{ll}
A O_{h} \leq \theta_{h} \cdot \gamma_{h} \cdot \omega_{h} & \forall h \\
A O_{h} \leq \theta_{h} & \forall h \\
\theta_{h} \cdot \gamma_{h} \cdot \omega_{h}-\theta_{h}=\lambda_{h}^{1}-\lambda_{h}^{2} & \forall h \\
\lambda_{h}^{1} \leq F \cdot y_{h} & \forall h \\
\lambda_{h}^{2} \leq F \cdot\left(1-y_{h}\right) & \forall h \\
B O_{h} \leq \lambda_{h}^{1} & \forall h \tag{1.6}
\end{array}
$$

$$
\begin{equation*}
B O_{h} \leq \frac{\theta_{h} \cdot \gamma_{h}}{\gamma_{h}^{L}}-\theta_{h} \quad \forall h \tag{1.7}
\end{equation*}
$$

Proposition 1. Constraints (1.1)-(1.7) correctly linearize the objective function of Model 1 given in Equation (1).

Replacing $\omega_{h}=\frac{1}{f_{h}}, e_{h i j}=x_{h i j} \cdot \omega_{h}$, and using a very large number $F$, Constraint (2) can be replaced by the following linear constraints:

$$
\begin{array}{ll}
e_{h i j} \leq \omega_{h} & \forall h, i, j \\
e_{h i j} \leq F \cdot x_{h i j} & \forall h, i, j \\
e_{h i j} \geq \omega_{h}-F \cdot\left(1-x_{h i j}\right) & \forall h, i, j \\
e_{h i j} \geq 0 & \forall h, i, j \\
1 \geq \tau_{j} \cdot \omega_{h}+\sum_{i=1}^{n_{h}} t_{h i} \cdot e_{h i j} & \forall h, j \tag{2.5}
\end{array}
$$

Proposition 2. Constraints (2.1)-(2.5) correctly linearize Constraint (2).

Since $\omega_{h}$ is strictly positive, Constraint (3) can be rewritten as follows:

$$
\begin{equation*}
\gamma_{h}^{U} \cdot \omega_{h} \geq 1 \quad \forall h \tag{3.1}
\end{equation*}
$$

Whereas Constraint (7) can be replaced with the following linear constraints:

$$
\begin{array}{ll}
O_{j q} \leq R_{j(q-1)} & \forall j, q \\
O_{j q} \leq F \cdot z_{j q} & \forall j, q \\
O_{j q} \geq R_{j(q-1)}-F \cdot\left(1-z_{j q}\right) & \forall j, q \\
O_{j q} \geq 0 & \forall j, q \\
\pi_{h i j q} \leq e_{h i j} & \forall i, j, q \\
\pi_{h i j q} \leq F \cdot z_{j q} & \forall i, j, q \\
\pi_{h i j q} \geq e_{h i j}-F \cdot\left(1-z_{j q}\right) & \forall i, j, q \\
\pi_{h i j q} \geq 0 & \forall i, j, q \\
R_{j q}=B_{j} \cdot z_{j q}+R_{j(q-1)}-O_{j q} & \\
-\sum_{h=1}^{g} 6300 \cdot \psi_{h} \cdot\left(\sum_{i=1}^{n_{h}} W_{h i} \cdot e_{h i j}\right) & \\
+\sum_{h=1}^{g} 6300 \cdot \psi_{h} \cdot\left(\sum_{i=1}^{n_{h}} W_{h i} \cdot \pi_{h i j q}\right) & \forall j, q
\end{array}
$$

Proposition 3. Constraints (7.1)-(7.9) correctly linearize Constraint (7). Replacing $e_{h i j}=x_{h i j} \cdot \omega_{h}$ as in Constraint (7) also linearizes Constraint (9):

$$
\begin{equation*}
R_{j q} \geq \sum_{h=1}^{g} 6300 \cdot \psi_{h} \cdot\left(\sum_{i=1}^{n_{h}} W_{h i} \cdot e_{h i j}\right) \quad \forall j, q \tag{9.1}
\end{equation*}
$$

Having linearized the objective function and the necessary constraints, we finally have the following mixed integer model(MIP):

$$
\begin{array}{ll}
\operatorname{Max} & \sum_{h=1}^{g} P R_{h} \cdot A O_{h}+\sum_{h=1}^{g} P R_{h}^{\varepsilon} \cdot B O_{h} \\
& -\rho \cdot \sum_{j=1}^{m} \sum_{q=1}^{U} C_{j} \cdot z_{j q}-\eta \cdot \sum_{j=1}^{m} V_{j} \cdot \sigma_{j}
\end{array}
$$

$$
\begin{array}{ll}
\text { S.t. } & \text { Constraints (1.1)-(1.7) } \\
& \text { Constraints (2.1)-(2.5) } \\
& \text { Constraint (3.1) } \\
& \text { Constraints (4)-(6) } \\
\text { Constraints (7.1)-(7.9) } \\
\text { Constraint (8) } \\
\text { Constraint (9.1) } \\
& \text { Constraint (10) } \\
& \omega_{h}, A O_{h}, B O_{h}, \lambda_{h}^{1}, \lambda_{h}^{2}, R_{j q} \geq 0, y_{h} \in\{0,1\} \forall h, i, j, q
\end{array}
$$

This MIP formulation can be solved using any commercial LP solver. However, for large scale problems, in order to get solutions in reasonable times we also developed a two-stage heuristic algorithm. The first stage of this algorithm finds a set of feasible solutions for a given $\gamma_{h}^{U}$ value and the second stage is an improvement algorithm to obtain stronger results from the given initial feasible solutions.

Example. An automotive company is planning to set up a robotic cell assembly line to produce body components for two different models of cars. Next year, the company plans to sell 150000 cars of the first model and 75000 cars of the second model. To achieve this production amount, the company sets a target cycle time of 76 seconds for both models. In order to meet the orders received so far, a cycle time of at most 80 seconds should be satisfied for both models. Market research shows that the company can not sell cars more than the amount that can be produced when the cycle time is 60 seconds for both models. Cost analysis shows that up to the production amount of 150000 for the first model and 75000 for the second model, each body component produced will contribute a profit of $\$ 10$. In case of producing more than the expected demand, each excess body component produced will contribute an expected profit of $\$ 7$ for both models. In the production area, there is available space for at most ten stations. The cost of setting up one station in the assembly line is $\$ 192500$. The welding tools have a tool life of 3200 spot welds and

each welding tool costs $\$ 10$. Both models require 15 spot welding operations and the number of spot welds required to perform these operations are $10,8,8,7$, $10,9,8,7,10,9,7,6,13,12$ and 10 , respectively. The time required for a robot in a station to reach to the position to perform an operation, the time required to perform a single spot weld and the time required for a robot to begin and finalize processing the allocated operations are calculated to be 1,2 and 3 seconds, respectively. Figure 1 depicts the components of the objective function one at a time as well as the total profit with respect to the cycle time. As seen in this figure, revenue, which is the sum of the profits that each product contributes, strictly increases as the cycle times decrease, until to $\gamma_{h}^{L}$ values. Station cost is a nondecreasing function as the cycle times decrease because lower cycle times require larger number of stations and it exhibits a stepping structure due to the breakpoints that correspond to particular cycle times.

## 4 COMPUTATIONAL STUDY

In this section, we will evaluate the performances of the proposed heuristic algorithm and the mathematical formulations (CPLEX version 10.1 is used as the MIP solver). There are two important factors that affect the size of our problem and optimal allocations in the solutions. The first one is the number of different models that will be produced in the robotic cell assembly line, denoted as $g$. As this parameter increases, our problem becomes harder to be solved by the CPLEX. The second one is the ratio of the profit contribution of the models to the station investment cost, denoted as $\frac{P R_{h}}{V_{j}}$. This ratio is very important to evaluate the tradeoff between the revenue and the number of stations that will be used in the assembly line. We set two levels as low and high for both of
these factors. For the number of models $g=2$ is used for the low and $g=4$ is used for the high levels. The cost of setting up one station is a constant and selected as $V_{j}=180000$ for $j=1,2, \ldots, m$. When the ratio $\frac{P R_{h}}{V_{j}}$ is at its low level, $P R_{h}$ is a uniform random number from the interval [5,7]. When this ratio is at its high level, this interval is changed to [9,11]. We take 5 replications for each combination resulting in 20 different randomly generated runs.

The value of the parameter $n_{h}$, the number of spot welding operations required for model $h$, is a uniform random number from the interval $[8,15]$. The value of the parameter $W_{h i}$, the number of spot welds required to perform operation $i$ of model $h$, is also a uniform random number from the interval $[6,15]$. When the number of models is at the low level, we use $\theta_{1}=150000$ and $\theta_{2}=75000$ and at the high level, we use $\theta_{1}=100000, \theta_{2}=50000, \theta_{3}=50000$ and $\theta_{4}=25000$ for the values of expected demands. We set the value of the expected profit for each excess production of model $h$ as $P R_{h}^{\varepsilon}=\left\lceil P R_{h} \cdot 0.7\right\rceil$. An operation can be allocated to any station. The precedence relationship matrix is not included due to space limitations, but can be obtained from the authors.

We now continue with some numerical results of our computational study. First, we solve each run with the first stage of our algorithm which finds a set of feasible solutions. Then we use these solutions as initial feasible solutions for both the MIP model and the second stage of our heuristic which is the improvement step. The reason that we insert initial solutions to CPLEX is to improve the quality of the solutions. Table 1 summarizes the results. Note that, since the CPLEX runs are terminated after a time limit of 2 hours, the proposed algorithm provided better results than the CPLEX for all runs.

## 5 CONCLUSIONS

In this study, we considered a mixed-model assembly line design problem with the objective of maximizing the total profit. We considered unavailability periods and finite tool lives. We first formulated the problem as an NLMIP model and then provided the linearized MIP version of it. We also developed a heuristic algorithm that obtains a set of feasible solutions and improved these solutions by incorporating a surrogate problem. The results of the computational study indicate that this heuristic is very efficient in terms of CPU time and the quality of the solutions found in comparison to CPLEX. Our study is the first one to consider unavailability periods and tool changes in the assembly lines, and to eliminate the tool change

Table 1: Total Profit Values and CPU Times.

|  | Algorithm <br> Run \# |  |  | Algorithm | CPU Time |
| :---: | :---: | :---: | :---: | :---: | :---: | MIP | Best Possible |
| :---: |
| (for MIP) | | MIP |
| :---: |
| CPU Time |

related line stoppages. For future research, flexibility can be inserted to the scope of our study by considering controllable processing times instead of using deterministic processing times for the assembly line operations as discussed in Gultekin et al. (2008).

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