Interactive Fuzzy Stochastic Multi-level 0-1 Programming through Probability Maximization

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Abstract: This paper considers multi-level 0-1 programming problems involving random variable coefficients both in objective functions and constraints. Following the probability maximization model together with the concept of chance constraints, the formulated stochastic multi-level 0-1 programming problems are transformed into deterministic ones. Taking into account vagueness of judgments of the decision makers, we present interactive fuzzy programming. In the proposed interactive method, after determining the fuzzy goals of the decision makers at all levels, a satisfactory solution is derived efficiently by updating satisfactory levels of the decision makers with considerations of overall satisfactory balance among all levels. An illustrative numerical example for a three-level 0-1 programming problem is provided to demonstrate the feasibility of the proposed method.

1 INTRODUCTION

The Stackelberg soluiton has been usually employed as a solution concept to multi-level programming problems (Sakawa and Nishizaki, 2009). To describe the concept of the Stackelberg solution, consider a two-level programming problem. There are two decision makers (DMs); each DM completely knows objective functions and constraints of the two DMs, and the DM at the upper level (leader) first make a decision and then the DM at the lower level (follower) specifies a decision so as to optimize an objective function with full knowledge of the decision of the leader. According to to the rule, the leader also make a decision so as to optimize the leader's objective function. Then a solution defined as the above-mentioned procedure is called the Stackelberg solution.

Lai (Lai, 1996) and Shih et al. (Shih et al., 1996) proposed solution concepts for two-level linear programming problems or multi-level ones such that decisions of DMs in all levels are sequential and all of the DMs essentially cooperate with each other. In their methods, the DMs identify membership functions of the fuzzy goals for their objective functions, and in particular, the DM at the upper level also specifies those of the fuzzy goals for the decision variables. The DM at the lower level solves a fuzzy programming problem with a constraint with respect to a satisfactory degree of the DM at the upper level. Unfortunately, there is a possibility that their method leads a final solution to an undesirable one because of inconsistency between the fuzzy goals of the objective function and those of the decision variables. In order to overcome the problem in their methods, by eliminating the fuzzy goals for the decision variables, Sakwa et al. have proposed interactive fuzzy programming for two-level or multi-level linear programming problems to obtain a satisfactory solution for DMs (Sakawa et al., 1998; Sakawa and Uemura, 2000).

In actual decision making situations, however, we must often make a decision on the basis of vague information or uncertain data. For such decision making problems involving uncertainty, there exist two typical approaches: probability theoretic approach and fuzzy-theoretic one. Stochastic programming, as an optimization method based on the probability theory, have been developing in various ways (Stancu-Minasian, 1990), including two stage problems considered by Dantzig (Dantzig, 1955) and chance constrained programming proposed by Charnes et al. (Charnes and Cooper, 1959).

Under these circumstances, in this paper, we present interactive fuzzy programming for multi-level 0-1 programming problems involving random variable coefficients both in objective functions and constraints. Using the concept of chance constraints, stochastic constraints are transformed into deterministic ones. Following the probability maximization model, the minimization of each stochastic objec-

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tive function is replaced with the maximization of the probability that each objective function is less than or equal to a certain value. Under some appropriate assumptions for distribution functions, the formulated stochastic multi-level 0-1 programming problems are transformed into deterministic ones. In our interactive method, after determining the fuzzy goals of the DM at all levels, a satisfactory solution is derived efficiently by updating the satisfactory degrees of the DMs at the upper level with considerations of overall satisfactory balance among all levels. An illustrative numerical example for a three-level 0-1 programming problem is provided to demonstrate the feasibility of the proposed method.

2 STOCHASTIC MULTI-LEVEL 0-1 PROGRAMMING PROBLEMS

In this paper, we consider stochastic multi-level 0-1 programming problems where each of the DMs at all levels takes overall satisfactory balance among all levels into consideration and tries to optimize each objective function. Such a stochastic multi-level 0-1 programming problem is formulated as

$$\left.\begin{array}{ll}\underset{\text{DM1 (Level 1)}}{\text{minimize}} & z_1(x)=c_{11}(\omega)x_1+\cdots+c_{1K}(\omega)x_K\\ \vdots & \qquad\vdots\\ \underset{\text{DM}K\text{ (Level }K)}{\text{minimize}} & z_K(x)=c_{K1}(\omega)x_1+\cdots+c_{KK}(\omega)x_K\\ \text{subject to} & A_1x_1+\cdots+A_Kx_K\le b(\omega)\\ & x_1\in\{0,1\}^{n_1},\ldots,x_K\in\{0,1\}^{n_K}\end{array}\right\} \quad (1)$$

where $x_l, l=1,\ldots,K$, is an n_l-dimensional 0-1 decision variable column vector; $\overline{c}_{lj}, l=1,\ldots,K, j=1,\ldots,K$, is an n_j-dimensional random variable row vectors. Here, we assume that $\overline{c}_{lj}$ is expressed as $\overline{c}_{lj}=c^1_{lj}+\overline{t}_l c^2_{lj}$ where $\overline{t}_l, l=1,2,\ldots,K$ are mutually independent random variables with mean M_l and their distribution functions $T_l(\cdot), l=1,2,\ldots,K$ are continuous and strictly increasing, and that $\overline{\alpha}_l, l=1,2,\ldots,K$ are random variables expressed as $\overline{\alpha}_l=c^1_l+\overline{t}_l\alpha^2_l$. This definition of random variables is one of the simplest randomization modeling of coefficients using dilation and translation of random variables, as discussed by Stancu-Minasian (Stancu-Minasian, 1984). In addition, $\overline{b}_i, i=1,2,\ldots,m$ are mutually independent random variables whose distribution functions are also assumed to be continuous and strictly increasing.

Since (1) contains random variable coefficients, solution methods for ordinary mathematical programming problems cannot be applied directly. Consequently, we first deal with the constraints in (1) as chance constraints (Charnes and Cooper, 1959) which mean that the constraints need to be satisfied with a certain probability (satisficing level) and over. Namely, replacing constraints in (1) by chance constraints with a satisficing level β, the problem can be transformed as:

$$\left.\begin{array}{ll}\underset{\text{DM1 (Level 1)}}{\text{minimize}} & c_{11}(\omega)x_1+\cdots+c_{1K}(\omega)x_K\\ \vdots & \qquad\vdots\\ \underset{\text{DM}K\text{ (Level }K)}{\text{minimize}} & c_{K1}(\omega)x_1+\cdots+c_{KK}(\omega)x_K\\ \text{subject to} & \Pr\{a_{i1}x_1+\cdots+a_{iK}x_K\le b_i(\omega)\}\\ & \qquad\ge\beta_i, i=1,\ldots,m\\ & x_1\in\{0,1\}^{n_1},\ldots,x_K\in\{0,1\}^{n_K}\end{array}\right\}. \quad (2)$$

The first constraint in (2) is rewritten as:

$$\begin{array}{ll} & \Pr\{a_{i1}x_1+\cdots+a_{iK}x_K\le b_i(\omega)\}\ge\beta_i\\ \Leftrightarrow & 1-\Pr\{a_{i1}x_1+\cdots+a_{iK}x_K\ge b_i(\omega)\}\ge\beta_i\\ \Leftrightarrow & 1-F_i(a_{i1}x_1+\cdots+a_{iK}x_K)\ge\beta_i\\ \Leftrightarrow & F_i(a_{i1}x_1+\cdots+a_{iK}x_K)\le 1-\beta_i\\ \Leftrightarrow & a_{i1}x_1+\cdots+a_{iK}x_K\le F_i^*(1-\beta_i)\end{array}$$

where F_i^* is a pseudo-inverse function of F_i.

When the DM wants to maximize the probability that the profit is greater than or equal to a certain permissible level, probability maximization model (Charnes and Cooper, 1959) is recommended. In this paper, assuming that the DM wants to maximize the probability that the profit is greater than or equal to a certain permissible level for safe management, we adopt the probability maximization model as a decision making model.

In the probability maximization model, the minimization of each of objective function $\overline{z_l}(x)$ in (2) is substituted with the maximization of the probability that $\overline{z_l}(x)$ is less than or equal to a certain permissible level h_l under the chance constraints. Through probability maximization, problem (2) can be rewritten as:

$$\left.\begin{array}{ll}\underset{\text{DM1iLevel 1j}}{\text{maximize}} & \Pr\{z_1(x_1,\ldots,x_K,\omega)\le h_1\}\\ \vdots & \qquad\vdots\\ \underset{\text{DM}K\text{iLevel }K\text{j}}{\text{maximize}} & \Pr\{z_K(x_1,\ldots,x_K,\omega)\le h_K\}\\ \text{subject to} & \Pr\{a_{i1}x_1+\cdots+a_{iK}x_K\le b_i(\omega)\}\\ & \qquad\ge\beta_i, i=1,\ldots,m\\ & x_1\in\{0,1\}^{n_1},\ldots,x_K\in\{0,1\}^{n_K}\end{array}\right\}. \quad (3)$$

Supposing that c for any feasible solution x to (3), from the assumption on the distribution function $T_l(\cdot)$ of each random variable $\overline{t_l}$, we can rewrite objective functions in (3) as follows.

$$
\begin{aligned}
&\Pr\{z_l(x_1,\ldots,x_K,\omega)\le h_l\}\\
&=\Pr\{c_{l1}(\omega)x_1+\cdots+c_{lK}(\omega)x_K\le h_l\}\\
&=\Pr\left\{\frac{c_{l1}(\omega)x_1+\cdots+c_{lK}(\omega)x_K-(\bar{c}_{l1}x_1+\cdots+\bar{c}_{lK}x_K)}{\sqrt{(x_1^T,\ldots,x_K^T)V_l(x_1^T,\ldots,x_K^T)^T}}\right.\\
&\qquad\left.\le\frac{h_l-(\bar{c}_{l1}x_1+\cdots+\bar{c}_{lK}x_K)}{\sqrt{(x_1^T,\ldots,x_K^T)V_l(x_1^T,\ldots,x_K^T)^T}}\right\}\\
&=\phi_l\left(\frac{h_l-(\bar{c}_{l1}x_1+\cdots+\bar{c}_{lK}x_K)}{\sqrt{(x_1^T,\ldots,x_K^T)V_l(x_1^T,\ldots,x_K^T)^T}}\right)
\end{aligned}
$$

Hence, (3) can be equivalently transformed into the following deterministic multi-level programming problem.

$$
\left.\begin{array}{l}
\underset{\text{DM1iLevel 1j}}{\text{maximize}}\\
\phi_1\left(\dfrac{h_1-(\bar{c}_{11}x_1+\cdots+\bar{c}_{1K}x_K)}{\sqrt{(x_1^T,\ldots,x_K^T)V_1(x_1^T,\ldots,x_K^T)^T}}\right)\\
\quad\vdots\qquad\qquad\qquad\vdots\\
\underset{\text{DM}K\text{iLevel }K\text{j}}{\text{maximize}}\\
\phi_K\left(\dfrac{h_K-(\bar{c}_{K1}x_1+\cdots+\bar{c}_{KK}x_K)}{\sqrt{(x_1^T,\ldots,x_K^T)V_K(x_1^T,\ldots,x_K^T)^T}}\right)\\
\text{subject to}\quad x\in X
\end{array}\right\}\quad(4)
$$

3 INTERACTIVE FUZZY PROGRAMMING

In the previous section, we have dealt with randomness involved in the objective functions and constraints in the original stochastic multi-level programming problem (1), and transformed into the deterministic multi-level programming problem (4) through the ideas of chance constraint and probability maximization model. In this section, we take account of fuzziness of human judgments by introducing fuzzy goals for objective function values obtained in the previous section.

To be more specific, in order to consider the imprecise nature of the DMs' judgments for the probabilities $p_l(x), l=1,2,\ldots,K$ in (4), it seems natural to assume that the DMs have fuzzy goals such as "$p_l(x)$ should be substantially greater than or equal to some specific value." Then, (4) can be rewritten as:

$$
\left.\begin{array}{ll}
\underset{\text{DM1iLevel 1j}}{\text{maximize}} & \mu_1(p_1(x_1,\ldots,x_K))\\
\quad\vdots & \qquad\vdots\\
\underset{\text{DM}K\text{iLevel }K\text{j}}{\text{maximize}} & \mu_K(p_K(x_1,\ldots,x_K))\\
\text{subject to} & A_1x_1+\cdots+A_Kx_K\le\hat{b}\\
 & x\in X
\end{array}\right\}\quad(5)
$$

where $\mu_l(\cdot)$ is a membership function to quantify a fuzzy goal for the l th objective function in (4) and it is assumed to be nondecreasing.

Although the membership function does not always need to be linear, for the sake of simplicity, we adopt a linear membership function. To be more specific, if the DM feels that $p_l(x)$ should be greater than or equal to at least $p_{l,0}$ and $p_l(x)\ge p_{l,1}(>p_{l,0})$ is satisfactory, the linear membership function $\mu_l(p_l(x))$ is defined as:

$$
\mu_l(p_l(x))=
\begin{cases}
0 & ,\quad \mu_l(p_l(x))<p_{l,0}\\
\dfrac{\mu_l(p_l(x))-p_{l,0}}{p_{l,1}-p_{l,0}} & ,\quad p_{l,0}\le\mu_l(p_l(x))\le p_{l,1}\\
1 & ,\quad \mu_l(p_l(x))>p_{l,1}
\end{cases}
$$

and it is depicted in Figure 1.

Now we are ready to propose interactive fuzzy programming for deriving a satisfactory solution by updating the satisfactory degree of the DM at the upper level with considerations of overall satisfactory balance among all the levels.

Interactive Fuzzy Programming

Step 1: Ask the decision maker at the upper level, DM1, to subjectively determine a satisficing level $\beta\in(0,1)$ for constraints. Go to Step 2.

Step 2: In order to determine permissible levels $h_l, l=1,2,\ldots,K$, the following problems are solved to find the minimum values $z^E_{l,\min}$ and $z^E_{l,M}$ of objective functions $z^E_l(x)$ under the chance constraints with satisficing levels $\beta_i, i=1,2,\ldots,m$.

$$
\left.\begin{array}{ll}
\text{minimize} & \bar{c}_{l1}x_1+\cdots+\bar{c}_{lK}x_K\\
\text{subject to} & x\in X
\end{array}\right\},\ l=1,\ldots,K\qquad(6)
$$

If the set of feasible solutions to these problems is empty, the satisficing levels $\beta_i, i=1,2,\ldots,m$ must be reassessed and return to step 1. Otherwise, let $z^E_{l,\min}$ be optimal objective function values to (6). Since (6) are 0-1 programming problems, they can be solved by tabu search based on strategic oscillation. Ask DM1 to determine permissible levels

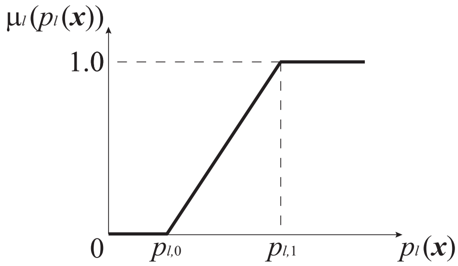

Figure 1: Linear membership function.

$h_l, l = 1, 2, \ldots, K$ for objective functions in consideration of $z^E_{l,\min}$ and $z^E_{l,M}$. Go to Step 3.

Step 3: Solve the following problems to find the maximum values $p_{l,\max}$ and $p_{l,M}$ of objective functions $p_l(x)$ under the chance constraints with satisficing levels $\beta_i, i = 1, 2, \ldots, m$.

$$\left.\begin{array}{l} \text{maximize} \\ \phi_l\left(\dfrac{h_l - (\bar{c}_{l1}x_1 + \cdots + \bar{c}_{lK}x_K)}{\sqrt{(x_1^T, \ldots, x_K^T)V_l(x_1^T, \ldots, x_K^T)^T}}\right) \\ \text{subject to} \quad x \in X \end{array}\right\} \quad (7)$$

Then, identify the linear membership function $\mu_l(z^P_l(x)), l = 1, 2, \ldots, K$ of the fuzzy goal for the corresponding objective function. Go to step 4.

Step 4: Solve the following corresponding maxmin problem.

$$\underset{x \in X}{\text{maximize}} \min_{l=1,\ldots,K} \left\{\mu_l(Z^P_l(x))\right\} \quad (8)$$

Go to step 5.

Step 5: Ask DM1 to subjectively set the minimal satisfactory level $\hat{\delta}_1$. Then, solve the following maxmin problem.

$$\left.\begin{array}{l} \underset{x \in X}{\text{maximize}} \min_{l=2,\ldots,K} \left\{\mu_l(Z^P_l(x))\right\} \\ \text{subject to } \mu_1(p_1(x)) \geq \hat{\delta}_1 \end{array}\right\} \quad (9)$$

Set $\lambda := 2C\lambda' := 1$. Go to step 6.

Step 6: Ask DMλ to set the membership function $\mu_{\Delta_\lambda}(\Delta_\lambda(x))$ for the ratio $\Delta_\lambda = (\mu_{\lambda+1}(Z^P_{\lambda+1}(x)))/(\mu_\lambda(Z^P_\lambda(x)))$ of satisfactory degrees and the minimal satisfactory level $\hat{\delta}_{\Delta_\lambda}$. Solve the following maxmin problem.

$$\left.\begin{array}{l} \underset{x \in X}{\text{maximize}} \min_{l=\lambda+1,\ldots,K} \left\{\mu_l(Z^P_l(x))\right\} \\ \text{subject to } \mu_1(Z^P_1(x)) \geq \hat{\delta}_1 \\ \qquad \mu_{\Delta_2}(\Delta_2(x)) \geq \hat{\delta}_{\Delta_2} \\ \qquad \vdots \\ \qquad \mu_{\Delta_\lambda}(\Delta_\lambda(x)) \geq \hat{\delta}_{\Delta_\lambda} \end{array}\right\} \quad (10)$$

Repeat this step until $\lambda = K - 1$.

Step 7: If the current solution satisfies the termination conditions, DM$K - \lambda'$ accepts it, and $K - \lambda' = 1$, then the procedure stops and the current solution is determined to be a satisfactory solution. Otherwise, ask DM$K - \lambda'$ to update the minimal satisfactory level $\hat{\delta}_{\Delta_{K-\lambda'}}$. If $K - \lambda' = 1$, ask DM1 to update the minimal satisfactory level $\hat{\delta}_1$. Go to step 8.

Step 8: Solve the following problem.

$$\left.\begin{array}{l} \text{maximize } v \\ \text{subject to } x \in X \\ \qquad 0 \leq v \leq 1 \\ \qquad \mu_1(Z^P_1(x)) \geq \hat{\delta}_1 \\ \qquad \mu_{\Delta_2}(\Delta_2(x)) \geq \hat{\delta}_{\Delta_2} \\ \qquad \vdots \\ \qquad \mu_{\Delta_{K-1}}(\Delta_{K-1}(x)) \geq \hat{\delta}_{\Delta_{K-1}} \\ \qquad \Pi^{K-1}_{l=K-\lambda'+1}\hat{\Delta}_l \mu_{K-\lambda'+1}(Z^P_{K-\lambda'+1}(x)) \geq v \\ \qquad \cdots \\ \qquad \hat{\Delta}_{K-1}\hat{\Delta}_{K-2}\mu_{K-2}(Z^P_{K-2}(x)) \geq v \\ \qquad \hat{\Delta}_{K-1}\mu_{K-1}(Z^P_{K-1}(x)) \geq v \\ \qquad \mu_K(Z^P_K(x)) \geq v \end{array}\right\} \quad (11)$$

It should be noted that all problems (6), (7), (8), (9), (10) and (11) in the interactive fuzzy programming algorithm can be solved by tabu search based on strategic oscillation (Hanafi and Freville, 1998).

4 NUMERICAL EXAMPLE

As an example for a stochastic multi-level 0-1 programming problem, consider the following three-level problem:

$$\left.\begin{array}{ll} \underset{\text{DM1 (Level 1)}}{\text{minimize}} & c_{11}(\omega)x_1 + c_{12}(\omega)x_2 + c_{13}(\omega)x_3 \\ \underset{\text{DM2 (Level 2)}}{\text{minimize}} & c_{21}(\omega)x_1 + c_{22}(\omega)x_2 + c_{23}(\omega)x_3 \\ \underset{\text{DM3 (Level 3)}}{\text{minimize}} & c_{31}(\omega)x_1 + c_{32}(\omega)x_2 + c_{33}(\omega)x_3 \\ \text{subject to} & A_1x_1 + A_2x_2 + A_3x_3 \leq b(\omega) \\ & x_1 \in \{0,1\}^{n_1}, \ldots, x_3 \in \{0,1\}^{n_3} \end{array}\right\} \quad (12)$$

where $x_1 = (x_1, \ldots, x_{15})^T, x_2 = (x_{16}, \ldots, x_{30})^T, x_3 = (x_{31}, \ldots, x_{45})^T$; each entry of 15-dimensional row constant vectors $c_{ij}, i, j = 1, 2, 3$, and each entry of 3×15 coefficient matrices A_1, A_2, and A_3 are random

In step 1 of the interactive fuzzy programming, DM1 specifies satisficing levels $\beta_i, i = 1, 2, \ldots, 9$ as:

$$(\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8, \beta_9)^T = (0.95, 0.80, 0.85, 0.90, 0.90, 0.85, 0.85, 0.95, 0.80)^T.$$

For the specified satisficing levels $\beta_i, i = 1, 2, \ldots, 9$, in step 2, minimal values $z^E_{l,\min}$ and maximal values $z^E_{l,\max}$ of objective functions $E\{\bar{z}_l(x_1, x_2, x_3)\}$ under the chance constraints are calculated. By considering these values, the DMs subjectively specify permissible levels.

In step 3, maximal values $p_{l,\max}$ of $p_l(x_1, x_2, x_3)$ are calculated. Assume that the DMs identify the linear membership function whose parameter values are determined by the Zimmermann method (Zimmermann, 1978).

In step 4, the maxmin problem is solved. The obtain result is shown at the column labeled "1st" in Table 1.

In step 5, for the obtained optimal solution, then, the ratio of satisfactory degrees Δ_1 is equal to 0.9837. Since DM1 is not satisfied with this solution, DM1 sets the minimal satisfactory level $\hat{\delta}_1$ to 0.75. (9) for $\hat{\delta}_1 = 0.75$ is solved. For the obtained optimal solution to (9), $\mu_1(Z_1^P(x)) = 0.7772$, $\mu_2(Z_2^P(x)) = 0.6122$, and $\mu_3(Z_3^P(x)) = 0.6618$, shown at the column labeled "2nd" in Table 1.

In step 6, DM2 sets the membership function $\mu_{\Delta_2}(\Delta_2(x))$ for the ratio Δ_2 of satisfactory degrees and the minimal satisfactory level as $\hat{\delta}_{\Delta_2} = 0.80$. (10) for $\hat{\delta}_{\Delta_2} = 0.80$ is solved. The obtained result is shown at the column labeled "3rd" in Table 1. For the obtained optimal solution to (10), $\mu_1(Z_1^P(x)) = 0.7696$, $\mu_2(Z_2^P(x)) = 0.6923$, $\mu_3(Z_3^P(x)) = 0.6118$ and $\mu_{\Delta_2}(\Delta_2(x)) = 0.9767$.

In step 7, since the ratio of satisfactory degrees Δ_2 is greater than $\hat{\delta}_{\Delta_2} = 0.80$, the condition of termination of the interactive process is fulfilled. Then, DM1 is asked whether he is satisfied with the obtained solution. Since DM1 is not satisfied, and he updates the minimal satisfactory level $\hat{\delta}_1$ from 0.75 to 0.80 in order to improve $\mu_1(Z_1^P(x))$ and sets $\hat{\delta}_{\Delta_1} = 0.80$.

In step 8, (11) for $\hat{\delta}_1 = 0.80$ and $\hat{\delta}_{\Delta_2} = 0.8837$ is solved. The obtained result is shown at the column labeled "4th" in Table 1. For the obtained optimal solution to (11), $\mu_1(Z_1^P(x)) = 0.8095$, $\mu_2(Z_2^P(x)) = 0.6585$, $\mu_3(Z_3^P(x)) = 0.6001$ and $\mu_{\Delta_1}(\Delta_1(x)) = 0.9325$.

In step 6, since the current solution satisfies all termination conditions of the interactive process and DM1 is satisfied with the current solution, the satisfactory solution is obtained and the interaction procedure is terminated.

Table 1: Interaction process.

Interaction	1st	2nd	3rd	4th
$\hat{\delta}_1$		0.7500	0.7500	0.8000
$\hat{\delta}_{\Delta_1}$				0.8000
$\hat{\delta}_{\Delta_2}$			0.8000	
$\mu_1(Z_1^P(x))$	0.7160	0.7772	0.7696	0.8095
$\mu_2(Z_2^P(x))$	0.7043	0.6122	0.6923	0.6585
$\mu_3(Z_3^P(x))$	0.6856	0.6618	0.6118	0.6001
$\Delta_1(x)$	0.9837	0.7877	0.8996	0.8135
$\Delta_2(x)$	0.9734	1.0801	0.8837	0.9123
$\mu_{\Delta_1}(\Delta_1(x))$				0.9325
$\mu_{\Delta_2}(\Delta_2(x))$			0.9767	0.8775

5 CONCLUSIONS

In this paper, we focused on stochastic multi-level 0-1 programming problems with random variable coefficients in both objective functions and constraints. Through the use of the probability maximization model in chance constrained programming, the stochastic multi-level 0-1 programming problems are transformed into deterministic 0-1 programming ones under some appropriate assumptions for distribution functions. Taking into account vagueness of judgments of the DMs, interactive fuzzy programming has been proposed. In the proposed interactive method, after determining the fuzzy goals of the DMs at all levels, a satisfactory solution is derived efficiently by updating the satisfactory degree of the DM at the 1st level with considerations of overall satisfactory balance among all levels. It is significant to note here that the transformed deterministic problems to derive an overall satisfactory solution can be solved through tabu search based on strategic oscillation. An illustrative numerical example for a three-level 0-1 programming problem was provided to demonstrate the feasibility of the proposed method. Extensions to other stochastic programming models will be considered elsewhere. Also extensions to multi-level 0-1 programming problems involving fuzzy random variable coefficients will be required in the near future.

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