Image Halftoning with Turing Patterns

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Abstract: This paper presents an image halftoning algorithm with a reaction-diffusion system in which periodic patterns called Turing patterns autonomously emerge. Image halftoning refers to conversion of a gray level image to a binary image so that the human visual system can perceive the original gray level image from the converted binary one. The reaction-diffusion system has activator and inhibitor distributions, and creates the Turing type periodic patterns in the distributions from an initial noisy distributions under the condition of long-range inhibition. Characteristics of the Turing patterns depend on a parameter of the reaction-diffusion system. Thus, by modulating the parameter distribution according to an image brightness distribution, the proposed algorithm creates Turing patterns of which characteristics distribute spatially; the human visual system can perceive distribution of the Turing patterns as the original image. Application of the proposed algorithm to a test image demonstrates its qualitative performance and convergence.

1 INTRODUCTION

Application of Turing patterns to computer graphics and image processing is an interesting topic from a biological point of view. Some animals have periodic patterns on their skin surface (Shoji et al., 2002). Turing originally proposed a scenario explaining how biological systems organize stable periodic patterns (Turing, 1952). He considered a reaction-diffusion system consisting of two diffusion processes coupled with some reaction functions on activation and inhibition, and showed that strong diffusive coupling or long-range inhibition causes the periodic patterns. The periodic patterns autonomously emerge from initial noisy distributions in the reaction-diffusion system under the long-range inhibition. Since it is possible to numerically compute the reaction-diffusion system, Witkin and Kass proposed to apply Turing type periodic patterns to computer graphics for drawing patterns observed on animal skin surfaces (Witkin and Kass, 1991).

Image halftoning refers to conversion of a gray level image to a binary image; that is a kind of image quantization. A simple algorithm for image halftoning is binarization with a fixed threshold level for image brightness. However, halftoning algorithms are required to create binary images that can be perceived as their original gray level images by the human visual system. The simple binarization algorithm does not satisfy this requirement. Thus, there have been proposed many halftoning algorithms (Lau and Arce, 2008), eg pattern dithering algorithm, error diffusion algorithm and a direct binary search algorithm. Recently, Schmaltz et al. proposed a physics-based halftoning algorithm, which utilizes an electrostatic phenomenon (Schmaltz et al., 2010).

If focusing on nature inspired algorithms in image processing, we can find interesting work on image segmentation. For example, Kuhnert demonstrated that a two-dimensional light-sensitive chemical reaction-diffusion system works as image segmentation (Kuhnert et al., 1989). However, the chemical reaction system did not satisfy the condition of the long-range inhibition; it did not create stable results of image segmentation. The segmentation result transiently emerged as time proceeds. In addition, since the image segmentation was the simple binarization, we did not perceive the result as the original image.

The purpose of this research work is to present a novel halftoning algorithm with Turing patterns created by a reaction-diffusion system under the long-range inhibition. Characteristics of periodicity and average level on the Turing patterns depend on a parameter of the reaction-diffusion system. In two-dimensional space, the periodic patterns appear as circular patterns. Thus, by modulating the characteristics of periodicity and average level according to image brightness distribution, the algorithm converts

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image brightness levels to different Turing type periodic patterns. When a local area has pixels of higher brightness level, it is filled with high density of larger white circular patterns; when a local area has pixels of low brightness level, it is filled with low density of small white circular patterns. Spatial distributions of these different periodic (circular) patterns bring brightness perception of the original image for the human visual system. Application of the proposed algorithm to a test image demonstrates qualitative performance and convergence of the algorithm, in comparison with previous representative algorithms.

2 THE ALGORITHM

2.1 Reaction-diffusion System and Turing Patterns

A reaction-diffusion system generally consists of time-evolving partial differential equations of diffusion equations coupled with reaction functions. Schnakenberg proposed a reaction-diffusion system consisting of the following set of equations with two distributions: activator $u(x,y,t)$ and inhibitor $v(x,y,t)$ defined in a two-dimensional space (x,y) and time t (Schnakenberg, 1979), as follows:

$$\frac{\partial u}{\partial t} = \nabla^2 u + \gamma(a - u + u^2 v) \quad (1)$$

$$\frac{\partial v}{\partial t} = D\nabla^2 v + \gamma(b - u^2 v) \quad (2)$$

in which ∇^2 is a two-dimensional Laplacian operator and D is a diffusion coefficient on $v(x,y,t)$; a, b and γ are constants; a diffusion coefficient on $u(x,y,t)$ is fixed at 1.0. The reaction-diffusion system described with Eqs. (1) and (2) creates periodic patterns on the two distributions u and v, when the inhibitory diffusion coefficient D is much larger than the activatory one $(D \gg 1)$ and their initial conditions are as follows:

$$u(x,y,t=0) = a + b + n_1 \quad (3)$$

$$v(x,y,t=0) = \frac{b}{(a+b)^2} + n_2 \quad (4)$$

in which n_1 and n_2 are random noise. Turing patterns refer to periodic patterns emerging in the reaction-diffusion system under the condition of $D \gg 1$, that is, under the condition of the long-range inhibition due to the strong inhibitory diffusion. The noise components n_1 and n_2 added to $u = a + b$ and $v = b/(a+b)^2$ initiate emergence of the Turing patterns. Equations (1) and (2) can be solved numerically with a finite difference method with a spatial difference δh and a temporal difference δt.

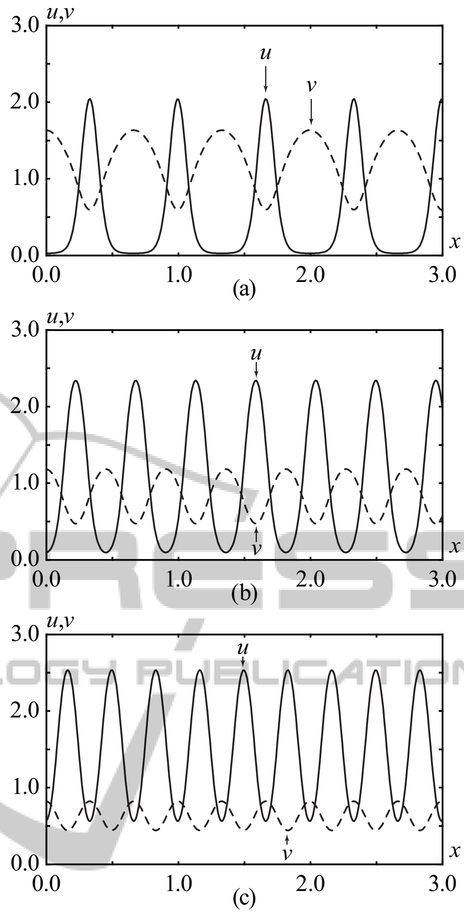

Figure 1: Three examples of one-dimensional Turing patterns created with Eqs. (1) and (2) with (a) $b = 0.50$, (b) $b = 1.00$ and (c) $b = 1.50$. Solid lines indicate spatial distributions of $u(x, t = 5.0)$ and broken lines indicate those of $v(x, t = 5.0)$. Equations (3) and (4) provided initial conditions of u and v. Other parameter settings were fixed at $a = 0.025, D = 20, \gamma = 1.0 \times 10^3, \delta h = 1.0 \times 10^{-2}$ and $\delta t = 1.0 \times 10^{-5}$.

Figure 1 shows three examples of one-dimensional Turing type periodic patterns created with Eqs. (1) and (2). Initial distributions of Eqs.(3) and (4) became unstable and spatial periodic patterns autonomously emerged. Interestingly, spatial wave length of the periodic patterns depends on the parameter b; a lager value of b brought shorter wave length. In addition, a smaller value of b brought a larger value of v in comparison with a smaller value of u. That is, it is possible to modulate the wave length and the difference between the average levels of u and v with modulation of the parameter b.

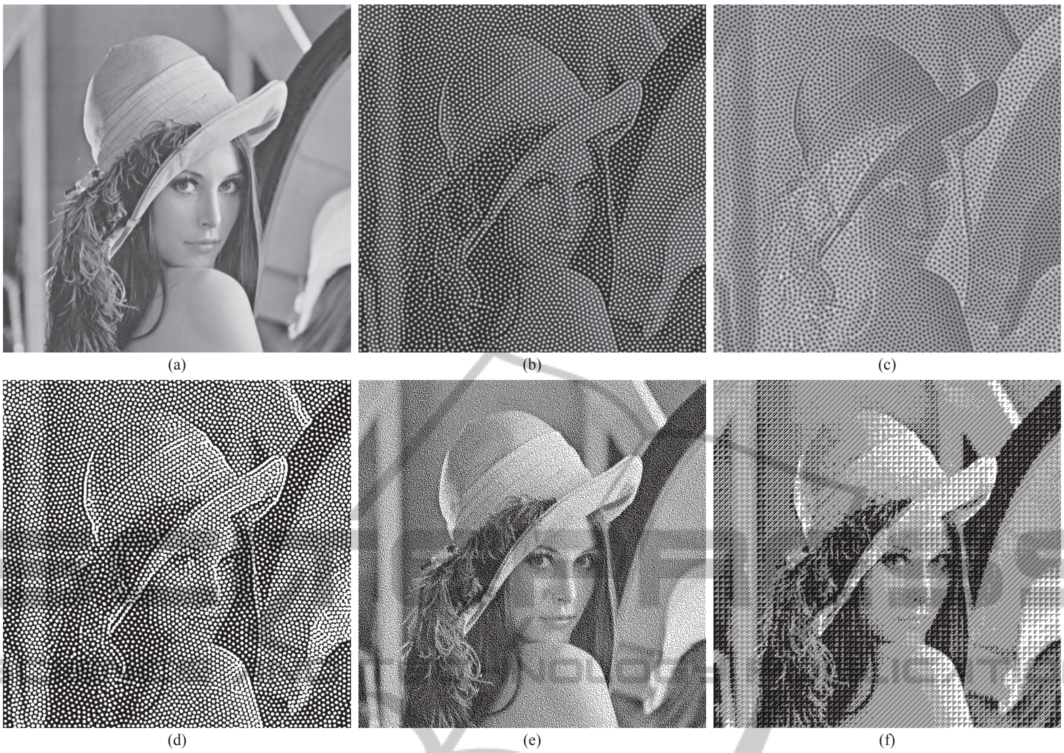

Figure 2: Results of halftoning for the gray level image LENA (Wakin, 2003). (a) Original image with the size of 512×512 (pixels) and 256 brightness levels; (b) a distribution of $u(x,y,t=10)$ created by Eq. (1); (c) a distribution of $v(x,y,t=10)$ created by Eq. (2); (d) a halftone image $H(x,y,t=10)$ created by the proposed algorithm; (e) a halftone image created by an error diffusion algorithm; (f) a halftone image created by a pattern dithering algorithm. Parameter settings of the proposed algorithm were $a=0.025, b_{\min}=0.50, b_{\max}=1.50, D=20, \gamma=1.0\times 10^3, \delta h=0.05$ and $\delta t=1.0\times 10^{-5}$. The results (e) and (f) were created by an image processing software (Adobe Photoshop CS4) for comparison.

2.2 Image Halftoning

Upon the one-dimensional experimental results of Fig. 1, this paper proposes an image halftoning algorithm by employing the Schnakenberg type reaction-diffusion system (Schnakenberg, 1979) under the long-range inhibition. Since the characteristics of the Turing patterns depend on the parameter b, the algorithm linearly modulates the parameter value of b according to a normalized image brightness distribution $I(x,y)$, as follows:

$$b(x,y) = b_{\min} + (b_{\max} - b_{\min}) I(x,y) \tag{5}$$

in which $b_{\min}$ and $b_{\max}$ are the minimum and maximum values of b.

The image halftoning algorithm consists of the following three steps. In the first step, we prepare the initial conditions of u and v described with Eqs. (3) and (4). In the second step, we numerically compute temporal developments of $u(x,y,t)$ and $v(x,y,t)$ with the reaction-diffusion system of Eqs. (1) and (2), in which the parameter b spatially distributes according to Eq. (5). In the final step, after sufficient duration of time, we create a halftoning image $H(x,y,t)$ with

$$H(x,y,t) = \begin{cases} 1 & \text{if } \ u(x,y,t) \geq v(x,y,t) \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

in which $H=1$ indicates a white pixel and $H=0$ does a black pixel. Recall that a smaller value of b brings a longer wave length of periodic pattern and a larger average level of v, and thus the area having smaller values of b filled with high density of black pixels (see also Fig. 1).

3 EXPERIMENTAL RESULT

This section presents an example of image halftoning with the proposed algorithm. Figure 2 shows a halftone image created by the proposed algorithm for the gray level image LENA (Wakin, 2003), in comparison with two other representative algorithms of halftoning. Figure 3 shows temporal changes of u and v in the proposed algorithm. Computation time of the proposed algorithm was four and half hours on a standard computer system with an Intel CPU.

We can roughly perceive the original image from the halftone image created by the proposed algorithm [Fig. 2(d)]. Thus, the proposed algorithm would be better than a simple binarization algorithm. When comparing the halftone image with that of the error diffusion algorithm [Fig. 2(e)], we recognize that quality of the image created by the proposed algorithm is insufficient. We can state that the proposed algorithm roughly achieved its convergence as shown in Fig. 3, and the algorithm needs quite longer computation time than the previous ones. The convergence of the algorithm implies that the proposed algorithm also has the function of image pooling.

Future research work for the proposed algorithm is as follows. In order to evaluate quantitative performance of the proposed algorithm, we need any evaluation method for image quality. Kawasaki et al. proposed a quantitative evaluation method for halftone image, by modeling the human brightness perception (Kawasaki et al., 2002). Their method is one of candidates for quantitative evaluation of image halftoning algorithms. Previous halftoning algorithms were also applied to multi-level and color halftoning. Extension of the proposed algorithm is also an interesting topic as an image processing. A cellular neural network (CNN) approach can implement a reaction-diffusion system with a circuit system (Crounse et al., 1993). We can expect that the proposed algorithm implemented with CNN performs in real time. Stable image pooling is also one of application areas of the proposed algorithm.

4 CONCLUSIONS

This paper presented a novel halftoning algorithm with Turing patterns emerging in a reaction-diffusion system. Characteristics of the Turing patterns depend on a parameter value of the system. Thus, in order to convert a gray level image to a binary image, the algorithm modulated the parameter of the system according to image brightness distribution. Although the human visual system can perceive the resulting halftone image as its original image, the quality of the halftone image was poor, in comparison with other previous representative algorithms. As future research work, in addition to improvement of image quality and reduction of computation time, it is also interesting to consider how image is represented with a bio-inspired reaction-diffusion system.

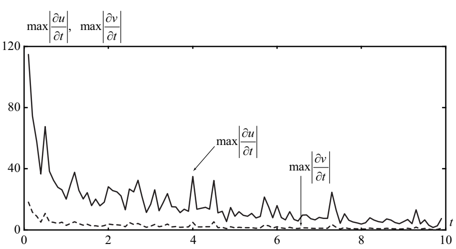

Figure 3: Temporal changes of $\max_{(x,y)} |\partial u/\partial t|$ (the solid line) and $\max_{(x,y)} |\partial v/\partial t|$ (the broken line) computed for the two distributions of u and v in the process of image halftoning performed for LENA (Wakin, 2003) (see also Fig. 2).

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