TV Minimization of Direct Algebraic Method of Optical Flow Detection Via Modulated Integral Imaging using Correlation Image Sensor

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Abstract: A novel mathematical method and a sensing system that detects velocity vector distribution on an optical image with a pixel-wise spatial resolution and a frame-wise temporal resolution is extended by total variation minimization. We applied fast total variation minimization technique for exact algebraic method of optical flow detection. Simulation result showed that directional error caused by local aperture problem decreased effectively by the virtue of global optimization. Experimental results showed edge preserving characteristics on the boundary of motion.

1 INTRODUCTION

Total variation (TV) minimization problem introduced by Rudin et. al. has the advantage of preserving edge so that applied to image analysis(Rudin et al., 1992). Chambolle developed fast algorithm with proof of convergence(Chambolle, 2004). Recently, Zach applied TV minimization to optical flow estimation (Zach et al., 2007).

Velocity field in the image can be considered to be almost uniform and smooth in the object region regardless of its texture. For example, egomotion is approximated as quadratic function of x and y. Both sides of the border has independent velocity fields so that there is clear edge on the border. TV regularization has desirable characteristic of smoothing constraint and edge preserving for optical flow estimation.

Ando et. al. applied correlation image sensor(Ando and Kimachi, 2003) and weighted integral method(Ando and Nara, 2009) to optical flow estimation(Ando et al., 2009). They started from optical flow partial differential equation(Horn and Schunk, 1981) and formulated exposure time in integral form and developed a sensing system that detects velocity vector distribution on an optical image with a pixel-wise spatial resolution and a frame-wise temporal resolution. Kurihara et. al. implemented fast optical flow estimation algorithm achieving 3ms for 640x512 pixel resolution, and 7.5ms for 1280x1024 pixel resolution using GPU(Kurihara and Ando, 2013).

In this paper, we applied total variation minimization technique for direct algebraic method of optical flow detection using correlation image sensor. The experimental results shows advantages of total variation regularization term, and the proposed method successfully reconstructed smooth and edge preserving velocity fields.

2 PRINCIPLE

2.1 Correlation Image Sensor

The three-phase correlation image sensor (3PCIS) is the two dimensional imaging device, which outputs a time averaged intensity image $g_0(x,y)$ and a correlation image $g_\omega(x,y)$. The correlation image is the pixel wise temporal correlation over one frame time between the incident light intensity and three external electronic reference signals.

The photo of the 640 × 512 three-phase correlation image sensor is shown in Figure 1, and its pixel structure is shown in Figure 2.

Let T be frame interval and $f(x,y,t)$ be instant brightness of the scene, we have intensity image $g_0(x,y)$ as

$$g_0(x,y) = \int_{-T/2}^{T/2} f(x,y,t)\,dt \tag{1}$$

Let the three reference signals be $v_k(t)$ $(k = 1,2,3)$ where $v_1(t)+v_2(t)+v_3(t) = 0$, the resulting correlation image is written like this equation.

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Figure 1: Photograph of Correlation Image Sensor(CIS).

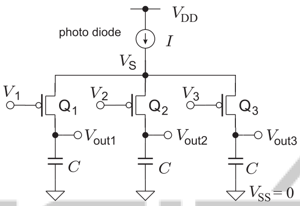


Figure 2: Pixel structure of the correlation image sensor.

$$c_k(x,y) = \int_{-T/2}^{T/2} f(x,y,t)\, v_k(t)\, dt \qquad (2)$$

Here we have three reference signals with one constraint, so that there remains 2 DOF for the basis of the reference signal. We usually choose orthogonal sinusoidal wave pair $(\cos\omega t, \sin\omega t)$ as the basis, which means $v_1(t) = \cos\omega t, v_2(t) = \cos(\omega t + \frac{2}{3}\pi), v_3(t) = \cos(\omega t + \frac{4}{3}\pi)$.

Let the time-varying intensity in each pixel be

$$I(x,y,t) = A(x,y)\cos(\omega t + \phi(x,y)) + B(x,y,t). \quad (3)$$

Here $A(x,y)$ and $\phi(x,y)$ is the amplitude and phase of the frequency component ω, and $B(x,y,t)$ is the other frequency component of the intensity including DC component. Due to the orthogonality, $B(x,y,t)$ doesn't contribute in the outputs c_1, c_2, c_3. Therefore the amplitude and the phase of the frequency ω component can be calculated as follows(Ando and Kimachi, 2003)

$$A(x,y) = \frac{2\sqrt{3}}{3}\sqrt{(c_1-c_2)^2+(c_2-c_3)^2+(c_3-c_1)^2} \qquad (4)$$

$$\phi(x,y) = \tan^{-1}\frac{\sqrt{3}(c_2-c_3)}{2c_1-c_2-c_3} \qquad (5)$$

From the two basis of the reference signal $(\cos n\omega_0 t, \sin n\omega_0 t)$, we can rewrite amplitude and phase using complex equation.

$$g_\omega(x,y) = \int_{-T/2}^{T/2} f(x,y,t)e^{-j\omega t}dt \qquad (6)$$

Here $\omega = 2\pi n/T$. $g_\omega(x,y)$ is the complex form of the correlation image, and it is a temporal Fourier coefficient of the periodic input light intensity.

2.2 Total Variation Minimization

We review TV minimization method proposed by Chambolle(Chambolle, 2004). Let f be observed brightness with noise. Solve minimizing problem

$$\min_u \left[\int_\Omega |\nabla u| d\Omega + \frac{1}{2\lambda}\int_\Omega (u-f)^2 d\Omega\right] \qquad (7)$$

where λ is Lagrangian multiplier. Desired denoised brightness is the solution u.

The auxiliary variable $\mathbf{p}$ was introduced to represent

$$|\nabla u| = \max\{\mathbf{p}\cdot\nabla u \mid |\mathbf{p}| \le 1\} \qquad (8)$$

Then transform the minimization problem by using $\mathbf{p}$, we obtain

$$\min_u \max_{|\mathbf{p}|\le 1} \left[\int_\Omega \mathbf{p}\cdot\nabla u d\Omega + \frac{1}{2\lambda}\int_\Omega (u-f)^2 d\Omega\right]. \qquad (9)$$

Exchanging min and max and from Euler-Lagrangian equation we obtain

$$u = f + \lambda\nabla\cdot\mathbf{p}. \qquad (10)$$

The variable $\mathbf{p}$ for the optimized solution is calculated by the iteration

$$\mathbf{p}_{n+1} = \frac{\mathbf{p}_n + \tau\nabla(f+\lambda\nabla\cdot\mathbf{p}_n)}{1+\tau|\nabla(f+\lambda\nabla\cdot\mathbf{p}_n)|}. \qquad (11)$$

Chambolle(Chambolle, 2004) showed convergence condition as $\tau < 1/8$.

2.3 Direct Algebraic Solution for Optical Flow Detection

We consider the brightness f on the object observed in the moving coordinate system is constant. Then we have well-known optical flow constraint

$$(u\partial_x + v\partial_y)f(x,y,t) + \partial_t f(x,y,t) = 0 \qquad (12)$$

where u, v is optical flow velocity, and $\partial_x = \partial/\partial_x$, $\partial_y = \partial/\partial_y$, $\partial_t = \partial/\partial_t$. Traditional optical flow estimation based on the OFC obtains the unknown velocity u, v from $\partial_x f, \partial_y f, \partial_t f$ as the observed quantities.

These days, Ando et.al proposed weighted integral method for parameter estimation method for partial differential equation and applied for optical flow estimation(Ando et al., 2009). It is based on identity relation

$$(u\partial_x + v\partial_y + \partial_t)f(x,y,t) = 0 \ \forall t \in [-\frac{T}{2}, \frac{T}{2}]$$
$$\leftrightarrow \int_{-T/2}^{T/2} \{(u\partial_x + v\partial_y + \partial_t)f(x,y,t)\} w(t)dt \ \forall w(t). \qquad (13)$$

The temporal integration means exposure time of an image sensor. As the weight function $w(t)$, we can consider an arbitrary set of complete function. Here, we restrict our attention to the complex exponential function set $\{e^{-j\omega t}\}$, $\omega = 2\pi n/T, n = 0,1,2,\cdots$ for implementation by a correlation image sensor.

Then, evaluation of integral form of optical flow equation using integral by parts leads to

$$\int_{-T/2}^{T/2} \{(u\partial_x + v\partial_y + \partial_t) f(x,y,t) e^{-j\omega t} dt = (u\partial_x + v\partial_y) g_\omega(x,y) + j\omega g_\omega(x,y) + \left[f(x,y,t)e^{-j\omega t}\right]_{-T/2}^{T/2} = 0 \tag{14}$$

where

$$g_\omega(x,y) = \int_{-T/2}^{T/2} f(x,y,t) e^{-j\omega t} dt \tag{15}$$

is the correlation image. The unknown variables are u, v and difference term $\left[f(x,y,t)e^{-j\omega t}\right]_{-T/2}^{T/2}$ between the instantaneous images at the beginning and end of the frame.

Letting $\omega = 0$, we obtain another relation on the intensity image as

$$(u\partial_x + v\partial_y) g_0(x,y) + \left[f(x,y,t)e^{-j\omega t}\right]_{-T/2}^{T/2} = 0 \tag{16}$$

By using Eq.(14) and Eq.(16) for eliminating difference term $\left[f(x,y,t)e^{-j\omega t}\right]_{-T/2}^{T/2}$, then we obtain a complex equation

$$(u\partial_x + v\partial_y)\{g_\omega(x,y) - g_0(x,y)\} = -j\omega g_\omega(x,y). \tag{17}$$

for $n = 1$.

Decomposing this equation into real part and imaginary part, we obtain matrix-vector form equation

$$\begin{bmatrix} \partial_x H & \partial_y H \\ \partial_x K & \partial_y K \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} h \\ k \end{bmatrix} \tag{18}$$

where, $H = \Re[g_\omega - g_0]$, $K = \Im[g_\omega - g_0]$, $h = \Re[j\omega g_\omega]$, $k = \Im[j\omega g_\omega]$, and $\Re$ and $\Im$ denote the real and the imaginary part, respectively. This matrix equation hold in each pixel, u and v can be solved in each pixel and every frames.

When we assume u and v are uniform in the small region Ω, then we obtain

$$J = \frac{1}{2}\int_\Omega (uH_x + vH_y + h)^2 d\Omega + \frac{1}{2}\int_\Omega (uK_x + vK_y + k)^2 d\Omega. \tag{19}$$

Differentiating by u and v respectively,

$$\frac{\partial J}{\partial u} = S_{xx}u + S_{xy}v + S_x = 0 \tag{20}$$

$$\frac{\partial J}{\partial v} = S_{xy}u + S_{yy}v + S_y = 0 \tag{21}$$

where $S_{xx} = \int_\Omega (H_x^2 + K_x^2) d\Omega$, $S_{yy} = \int_\Omega (H_y^2 + K_y^2) d\Omega$, $S_{xy} = \int_\Omega (H_x H_y + K_x K_y) d\Omega$, $S_x = \int_\Omega (H_x h + K_x k) d\Omega$, $S_y = \int_\Omega (H_y h + K_y k) d\Omega$.

We also obtain

$$\begin{bmatrix} S_{xx} & S_{xy} \\ S_{xy} & S_{yy} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = -\begin{bmatrix} S_x \\ S_y \end{bmatrix} \tag{22}$$

2.4 Total Variation Minimization of Optical Flow Estimation

We introduce regularization term in Eq.(19). That is to find a solution of minimization problem

$$J = \int \lambda(|\nabla u| + |\nabla v|) d\Omega + \frac{1}{2}\int_\Omega \{(uH_x + vH_y + h)^2 + (uK_x + vK_y + k)^2\} d\Omega. \tag{23}$$

To solve the above problem, we introduce auxiliary variables u' and v' with parameter θ,

$$J = \int \lambda(|\nabla u| + |\nabla v|) d\Omega + \frac{1}{2\theta}\int \left((u-u')^2 + (v-v')^2\right) d\Omega + \frac{1}{2}\int \left\{(u'H_x + v'H_y + h)^2 + (u'K_x + v'K_y + k)^2\right\} d\Omega. \tag{24}$$

The second term means distance between u and u' and between v and v' . When we set θ sufficiently small, the differences between u and u' and between v and v' are expected to be sufficiently small.

We solve this minimization problem by iteration in terms of u, v and u', v' one after another.

1. For fixed u, v, solve

$$\min\left\{\frac{1}{2\theta}\int \left((u-u')^2 + (v-v')^2\right) d\Omega + \frac{1}{2}\int (u'H_x + v'H_y + h)^2 d\Omega + \frac{1}{2}\int (u'K_x + v'K_y + k)^2 d\Omega.\right\} \tag{25}$$

From

$$\frac{\partial J}{\partial u'} = \frac{1}{\theta}(u-u')(-1) + (u'H_x + v'H_y + h)H_x + (u'K_x + v'K_y + k)K_x = 0 \tag{26}$$

we obtain matrix equation

$$\begin{bmatrix} 1+\theta(H_x^2+K_x^2) & \theta(H_xH_y+K_xK_y) \\ \theta(H_xH_y+K_xK_y) & 1+\theta(H_y^2+K_y^2) \end{bmatrix} \begin{bmatrix} u' \\ v' \end{bmatrix} = \begin{bmatrix} u-\theta(H_xh+K_xk) \\ v-\theta(H_yh+K_yk) \end{bmatrix} \tag{27}$$

u', v' can be solved in each pixel.

2. For fixed u', v', solve

$$\min\left\{\int \lambda(|\nabla u|+|\nabla v|)d\Omega + \frac{1}{2\theta}\int\left((u-u')^2+(v-v')^2\right)d\Omega\right\}. \tag{28}$$

The variables u and v are independent. So we can apply Chambolle approach in section 2.2.

From

$$\frac{\partial J}{\partial u} = \frac{1}{\theta}(u-u'),\ \frac{\partial J}{\partial u_x} = \lambda\frac{\partial_x u}{|\nabla u|},\ \frac{\partial J}{\partial u_y} = \lambda\frac{\partial_y u}{|\nabla u|} \tag{29}$$

$$\frac{\partial J}{\partial v} = \frac{1}{\theta}(v-v'),\ \frac{\partial J}{\partial v_x} = \lambda\frac{\partial_x v}{|\nabla v|},\ \frac{\partial J}{\partial v_y} = \lambda\frac{\partial_y v}{|\nabla v|}, \tag{30}$$

we obtain

$$u = u' + \lambda\theta\nabla\cdot\mathbf{p} \tag{31}$$

$$v = v' + \lambda\theta\nabla\cdot\mathbf{q} \tag{32}$$

where $\mathbf{p} = \frac{\nabla u}{|\nabla u|}$, $\mathbf{q} = \frac{\nabla v}{|\nabla v|}$. The parameter $\mathbf{p}$ and $\mathbf{q}$ are solved by iteration of

$$\mathbf{p}_{n+1} = \frac{\mathbf{p}_n + \tau\nabla(u'+\lambda\theta\nabla\cdot\mathbf{p}_n)}{1+\tau|\nabla(u'+\lambda\theta\nabla\cdot\mathbf{p}_n)|} \tag{33}$$

$$\mathbf{q}_{n+1} = \frac{\mathbf{q}_n + \tau\nabla(v'+\lambda\theta\nabla\cdot\mathbf{q}_n)}{1+\tau|\nabla(v'+\lambda\theta\nabla\cdot\mathbf{q}_n)|} \tag{34}$$

3 EXPERIMENTS

3.1 Simulation

To confirm proposed principle, we evaluate global optimization result. Figure 3 shows results.

By moving random dot pattern (Fig. 3(a)) in the direction of $(v_x, v_y) = (10.6, 5.7)$, we compared conventional method and TV regularization method. Each of the result are shown in Fig. 3(e) and (f) with the color chart of Fig. 3(b). Conventional method shows clearly some directional error caused by aperture problem. On the other hand, TV regularization outputs global optimization results therefore the output flow of each pixel is quite uniform.

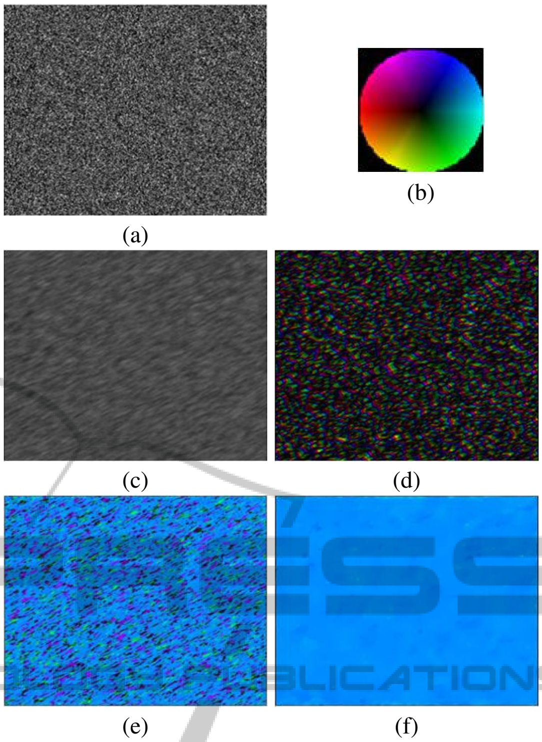


Figure 3: Simulation results of optical flow estimation. The instantaneous pattern(a) is moved toward $v_x = 10.6$pixel/frame, $v_y = 5.7$pixel/frame. Conventional method outputs some directional error caused by aperture problem. TV regularization outputs global optimization therefore the output flow of each pixel is quite uniform. (a)instantaneous brightness, (b)color chart of velocity representation, (c)intensity image, (d)correlation image(amplitude and phase is shown by brightness and hue, respectively.) (e)conventional method, (f)result of total variation optimization.

3.2 Real Images on the Moving Vehicle

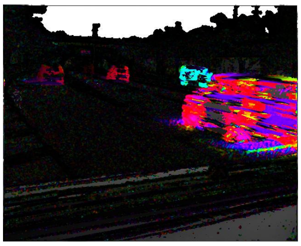
To confirm effect of proposed method, we compared proposed method to conventional method by using real images captured from the moving vehicle.

The results are shown in Fig. 4- 7. In the Fig. 4, the output of the correlation image sensor from the parking car is shown. In the result of conventional method, there are lots of black region on the building wall and on the vehicle moving left to right. This is caused by textureless area. Horn & Shunk method with modification for correlation images shows smoothing effect on the motion field resulting blurred edge on the motion boundaries, but fills black region depending on number of iterations. Opposing to that, in the proposed method, there are smooth velocity field on the frontal wall, and velocity boundary on the edge of the car is successfully reconstructed.

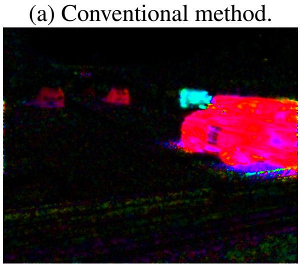
(a) Intensity image.

(b) Correlation image.

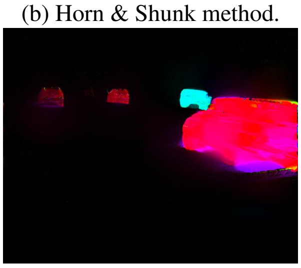
Figure 4: Example of output images of the correlation image sensor. (b)correlation image only captures the area of brightness changes, which is moving object in this example.



(a) Conventional method.



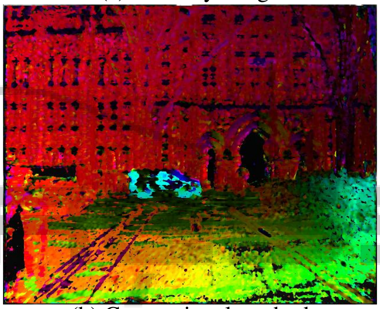
(b) Horn & Shunk method.



(c)Proposed method.

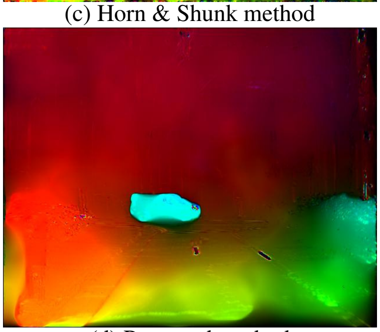
Figure 5: Optical flow of Fig. 4. In the result of conventional method, there are lots of black region on the object. Opposing to that, in the proposed method, there are smooth velocity field on the object.

(a) Intensity image



(b) Conventional method

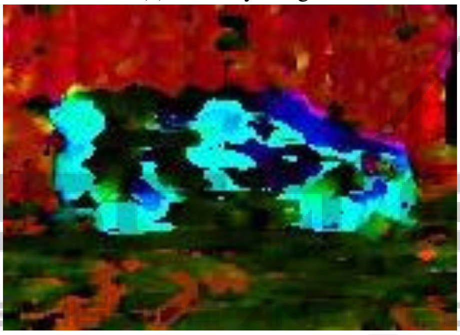
(c) Horn & Shunk method



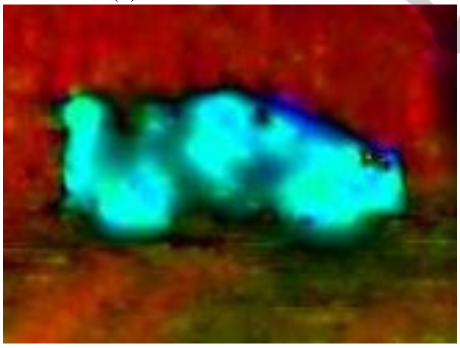
(d) Proposed method

Figure 6: Result of optical flow estimation for on-vehicle moving images. Each color represents optical flow speed and direction by the chart in Fig. 3(b). In the result of conventional method, there are lots of black region on the building and vehicle moving left to right caused by textureless area. Opposing to that, in the proposed method, there are smooth velocity field on the frontal wall, and velocity boundary on the edge of the car is successfully reconstructed.

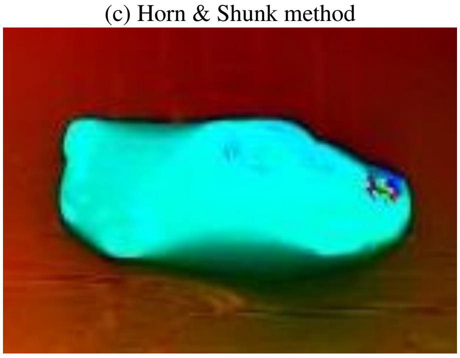
(a) Intensity image



(b) Conventional method



(c) Horn & Shunk method



(d) Proposed method

Figure 7: Center of the result in Fig. 6. In the conventional method(b), there are lots of uncalculated region in the uniform intensity area. Horn & Shunk method modified for correlation image(c) also shows uncalculated region and blurred edge on the motion boundaries. But in proposed method(d), the result shows embedded smooth flow region in the internal area of the vehicle with preserving flow boundary on the vehicle edge.

4 CONCLUSIONS

A novel sensing scheme and algorithm for optical flow detection with maximal spatio and temporal resolution was proposed with the conjunction of Total variation optimization scheme. It can outputs edge preserving flow under global optimization, which is suitable for optical flow analysis, especially for textureless region or flow boundary of the object edge. An experimental system was constructed with a 640 512 pixel 3PCIS and showed good results for moving vehicle images.

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