Determining Optimal Discount Policies in B2B Relationships

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1 STAGE OF THE RESEARCH

The research is preliminarily divided into three stages. The current work is close to the end of the first stage. At this stage two heuristic algorithms have been developed for a non-restricted one-item case. They have been tested on a simple example problem. It is planned to test models on more instances.

2 OUTLINE OF OBJECTIVES

Distinctive objectives can be outlined for each stage:

1. development of heuristic algorithms for an unrestricted one-item case (stage 1, current stage);
2. extension of the problem to a multiple item case with a capacity restriction, development of an algorithm based on metaheuristics (stage 2);
3. aplication of an alternative approach — possibly game theory — reformulation of the problem if needed (stage 3).

3 RESEARCH PROBLEM

This research deals with the question of which discounts (when and how much) a supplier needs to offer to a set of heterogeneous customers to maximize his profit.

The supplier has a possibility to regulate the demand using discounts — to increase the demand in periods when the product is produced and to lower the demand in periods with no production of the product. Savings for the supplier arise from reduction of set up and inventory cost, but the buyer gets extra inventory cost. The discount offered by the supplier should be large enough to make the buyer order anyway at the period wanted by the supplier.

The major assumptions of this study are the following:

- Monopolistic situation of the firm or very high barriers for switching suppliers imply that price levels of other firms need not be considered here.
- Perfect information about the demand and cost of each buyer.
- Buyers and the supplier have no capacity or warehouse restrictions (valid only for the first stage).
- Buyers are considered to be rational in their reaction to the discount by always choosing the lowest cost option available.
- Buyers have full information in advance about future discounts.
- Simple discount in the form of a single price reduction.

The topic is very different from standard yield management where total demand can be affected with prices. In the researched case, the total demand remains the same; there is only a question of when this demand is ordered and produced. If the price is low in one period, buyers will be induced to order in that period and there will be less demand in the adjacent periods. Thus, the problem is very distinctive in the fact that price changes in one period will affect the demand in other periods.

This approach can be applicable in situations when the buyer represents a retailer with his own customers which considers being more profitable not to introduce discounts to them. This can appear in a number of cases:

- A physical cost of changing prices can be too high — labor costs of changing shelf prices and relabelling, the costs of producing, printing, and distributing price books. These costs can take up to 40% of the reported profits (Levy et al., 1997). They are still vital especially for small retail shops lacking contemporary digital devices.
- Price decisions can be time consuming for the corporate management (Bolton et al., 2010).
- The retailer can lose its customers' loyalty,

who might consider him being too opportunistic (Mauri, 2007).

- Prices of the retailer can be fixed by the government or a franchise giver.
- Customers are heavy equipment or airplane manufacturers and have stable demand for spare parts which are only a minor component of the final product (Lal and Staelin, 1984).

In these cases despite getting items for lower price from his supplier, the buyer has no incentive to introduce discounts to its own customers. Obviously the demand of the final customers doesn't increase. In this situation the buyer's demand is insensitive to price changes, in other words price elasticity of the demand is low.

4 STATE OF THE ART

Many articles in the field of operations management have analysed ordering decisions while quantity discounts are in place (Benton and Park, 1996). The problem of when and how much discount to offer is a problem that has received less attention, although it is of equally great practical importance as how to act on a given discount.

Nevertheless, there is a number of articles addressing the problem of offering an optimal discount schedule from the supplier's side (Crowther, 1964), (Monahan, 1984), (Lal and Staelin, 1984), (Rosenblatt and Lee, 1985), (Lee and Rosenblatt, 1986), (Banerjee, 1986). Later papers in this field base their research on EOQ assumptions a well (Busher and Lindner, 2004), (Chen and Robinson, 2012). These papers share one more unifying feature — an assumption that customers' demand is independent of discounts. That is an assumption valid for the current paper as well.

The literature researched above indicates that a situation with more than two players is not considered until recently. If a number of customers is considered, they are assumed to be homogeneous, or heterogeneous only in their demand. Discount schedules offered by the supplier are constant and involve limited number of break points (very often only one). The same discount schedules are introduced for all the customers. The models presented in the articles are based on EOQ assumptions. To the best of our knowledge capacitated problems as well as cases with multiple items have not been considered yet.

More recent papers incorporate the price elasticity of demand, which makes their research closer to revenue management.

This research differs from the approaches stated above in the following way:

- some of the EOQ assumptions are not applied;
- dynamic demand and finite time horizon are supposed;
- a number of heterogeneous customers are considered, who are different not only in their demand but in their holding and order costs;
- discounts are different for every single customer;
- discounts can vary from period to period.

5 METHODOLOGY

The methodology described in this paper concerns stage 1 of the research. Currently two heuristic algorithms have been developed. Later it is planned to apply a metaheuristic (stage 2) and a game theoretical (stage 3) approaches.

5.1 Initial Situation

In the current situation the customers decide their orders based on the Wagner-Whitin algorithm (Whitin and Wagner, 1958), the supplier receives the orders and applies the Wagner-Whitin algorithm to schedule his production based on these orders.

5.2 Cost Compensation Heuristic

Exact solutions to the problem are very hard to obtain and would require an exponential amount of binary variables, representing each possible order schedule, for each customer.

Therefore, a heuristic solution approach has been developed which involves a separation between the problem when production and orders should take place, and the amount of discount that has to be offered to each costumer in each period to make them order at the periods indicated.

The following parameters are used for defining the algorithm:

d_{it} demand for every customer $i (i = 1,\dots,n$ or $i \in N)$ in every period t $(t = 1,\dots,m$ or $t \in M)$. Demand of the supplier is the summation of his customers' orders in that period;

s_i fixed order processing/set up costs for every customer i and the supplier $i = 0$;

h_i inventory holding interest rate for each customer i and the supplier to carry a monetory unit of inventory from period t to period $t+1$, assumed to be constant;

v_{it} unit price for every customer i and the supplier in every period t. Initially, unit price is set equal to the original price without discount. The unit price for the supplier is given for symmetry reasons in order to calculate inventory holding costs;

c_i costs for every customer i and the supplier before introduction of discounts.

The model operates with the following decision variables:

H_{it} inventory for every customer i and supplier $i = 0$ in every period t;

S_{it} binary variable for every customer i and supplier $i = 0$ in every period t, 1, when the order/set up is planned, 0 otherwise;

P_i total amount of compensation for every customer i;

Q_{it} order quantity for the customer i and the supplier $i = 0$ in every period t.

The solution procedure includes the following three steps:

STEP 0.

The problem for each customer and the supplier is solved using the Wagner-Whitin algorithm. Order and production patterns before introduction of discounts are obtained at this stage. They are not used in further calculations and are a benchmark for results received at Step 1.

Furthermore, a new parameter ω_{itk} for every customer i for every period t and k ($t \leq k$) is introduced and calculated. The parameter consists of the costs of ordering the demand from period t to period k in period t and the minimum cost incurred up to period t. It is calculated according to the Wagner-Whitin algorithm.

Costs c_i for customer i before introduction of discounts are received at this step:

$$c_i = \min_{t \in M} \omega_{itm} \text{ ,for } \forall i > 0 \quad (1)$$

v_{it} remains unchanged at this and the next step and is the same for every customer i and every period t.

STEP 1.

In the previous step parameter c_i which is going to be used below and parameter ω_{itk} which will be used further in Step 3 have been calculated. The order and production patterns can be determined as well which are in place before introduction of discount. They can be used as a reference while comparing the received results.

Objective (2) is minimized for the supplier:

$$\text{minimize} \sum_{t=1}^{m} h_0 v_{0t} H_{0t} + \sum_{t=1}^{m} s_0 S_{0t} + \sum_{i=1}^{n} P_i \quad (2)$$

subject to

$$\sum_{t=1}^{m} h_i v_{it} H_{it} + \sum_{t=1}^{m} s_i S_{it} + \sum_{t=1}^{m} d_{it} v_{it} - P_i \leq c_i \text{ , for } \forall i > 0 \quad (3)$$

Constraint (3) ensures that there is no cost increase for any customer i.

There are two additional constraints (4) and (5) for the customers:

$$\sum_{u=t}^{m} d_{iu} S_{it} - Q_{it} \geq 0 \text{ , for } \forall i > 0 \text{ , } \forall t \quad (4)$$

$$Q_{it} + H_{it-1} - H_{it} = d_{it} \text{ , for } \forall i > 0 \text{ , } \forall t \quad (5)$$

For the supplier constraints (6) and (7) are in place:

$$\sum_{i=1}^{n} \sum_{u=t}^{m} d_{iu} S_{0t} - Q_{0t} \geq 0 \text{ , for } \forall t \quad (6)$$

$$Q_{0t} + H_{0t-1} - H_{0t} = \sum_{i=1}^{n} Q_{it} \text{ , for } \forall t \quad (7)$$

For $t = 1$ constraints (5) and (7) are slightly modified since H_{it-1} equals to 0.

Constraints (5) and (7) ensure the continuity of the flow. They guarantee that the demand is satisfied from production or inventory.

Constraints (4) and (6) link binary variables S_{it} with continuous variables Q_{it} forcing S_{it} to take value 1, when $Q_{it} \geq 0$, and 0, when $Q_{it} = 0$.

It can be shown that the model offered above ensures both supplier and customers' cost minimisation. Supplier's costs and compensations offered by the supplier to the customers are minimized in the objective of the model. The compensations are a lump sum that would compensate the customer for ordering at the periods requested and represent the increase in customers' costs. Minimizing compensations the model minimizes not only supplier's but customers' costs. This statement can be proved, since $\sum_{i=1}^{n} P_i$ is minimized in the objective function (2), constraint (3) can be written as an equality and the objective function can be presented in the following way:

$$\text{minimize} \sum_{i=0}^{n} \sum_{t=1}^{m} h_i v_{it} H_{it} + \sum_{i=0}^{n} \sum_{t=1}^{m} s_i S_{it} + \sum_{i=1}^{n} \sum_{t=1}^{m} d_{it} v_{it} - \sum_{i=1}^{n} c_i \quad (8)$$

$\sum_{i=1}^{n} c_i$ and $\sum_{i=1}^{n}\sum_{t=1}^{m} d_{it} v_{it}$ are constants and can be omitted in the objective function. The received formula accounts for minimization of both supplier's and customers' inventory and setup costs. A comparable approach is employed by Lal and Staelin (Lal and Staelin, 1984).

STEP 2.

After application of the model described in Step 1, the new order and production patterns are determined. All the variables are fixed now. Only parameters accounting for prices — v_{it} — or discounts offered to achieve these order and production patterns are calculated at this step.

The following new parameters are introduced at this step:

ψ_{itk} amounts (demand and inventory) which depend on price in period t for an order from period t up to period k ($t \leq k$) for customer i;

x_1 period in which customers order according to the order pattern received in Step 1;

y_1 period until which the order starting at x_1 according to the order pattern received in Step 1 lasts;

x_0 period in which customers order for period x_1 given the time horizon of y_1 (according to the order pattern which is in place before discounts are introduced);

y_0 period until which the order starting at x_0 lasts (according to the order pattern which is in place before discounts are introduced).

Parameters enumarated above (except the first one) are assigned successively for every customer i. The number of assignments for every customer i is equal to the number of periods in which the order is placed according to the order pattern received in Step 1.

Parameter ψ_{itk} is calculated using formula (9) for $t = k$ and formula (10) for $t < k$ for $\forall i > 0$:

$$\psi_{itk} = d_{ik} \tag{9}$$

$$\psi_{itk} = \psi_{itk-1} + d_{ik} h_i (k - t) + d_{ik} \tag{10}$$

Parameter x_1 is successively assigned to periods when customer i places an order according to the order pattern determined at Step 1; parameter y_1 refers to the period preceding the next order in the order pattern determined at Step 1, or if there is no next order — to the last period.

Parameter y_0 initially is presumed to be equal to y_1. This predetermines that y_0 is never larger than y_1. Even if according to the order pattern which is in place if no discounts is introduced the next order is placed later than it should happen according to the order pattern determined at Step 1, the examined time horizon is contracted to y_1.

Parameter x_0 is asigned to period $t \in M$ and $t \leq y_0$ such that:

$$\omega_{ity_0} = \min_{u \in M, u \leq y_0} \omega_{iuy_0} \tag{11}$$

which refers to the minimum in column y_0 of the costs matrix calculated applying the Wagner-Whitin algorithm.

If $x_0 > x_1$, we assign $y_0 = x_0 - 1$ and recalculate x_0 using formula (11). This step is repeated until $x_0 \leq x_1$.

If $x_0 = x_1$ and $y_0 = y_1$, then the correct price is found, otherwise the new price which incorporates the discount is found using formula (12) for $x_0 < x_1$ and formula(13) otherwise, for $\forall i > 0$:

$$v_{ix_1} := v_{ix_1} - \frac{\omega_{ix_1y_1} - \min_{t \in M, t \leq y_1} \omega_{ity_1}}{\psi_{ix_1y_1}} \tag{12}$$

$$v_{ix_1} := v_{ix_1} - \frac{\omega_{ix_1y_1} - \min_{t \in M, t \leq y_1} \omega_{ity_1}}{\psi_{ix_1y_1} - \psi_{ix_0y_0}} \tag{13}$$

The formula (12) differs for the case when $x_0 < x_1$, since the items ordered in period x_0 are not ordered with the discounted price.

After calculation of the new price the costs matrix which is calculated according to the Wagner-Whitin algorithm is updated as well.

This is however a heuristic solution, because the total discount calculated at Step 2 may be larger than the increase in cost which is the compensation assumed in Step 1.

5.3 Discount Interval Heuristic

While the previous heuristic procedure is based on separation between determining the order pattern and discounts, in the current procedure the order pattern is determined based on the possible discount prices calculated in advance. Moreover, the shortest path formulation is used for the supplier in addition to the normal aggregated formulation (Eppen and Kipp, 1987).

Parameters d_{it}, s_i, h_i, v_{it}, x_0, x_1, y_0, y_1, ω_{itk}, ψ_{itk} and variables H_{it}, S_{it}, Q_{it} are defined in the same way as in the previous model.

The following new parameters are introduced:

p_{itk} reduction in price large enough for customer i to order all demand from period t to period k in period t given that no other discounts are offered;

λ original price of the item without a discount.

The following new variable is used in the model:

F_{itk} binary variable, 1 if the total demand of every customer i from period t to period k is ordered in period t, 0 otherwise. This variable can be binary due to the Wagner-Whitin property according to which if an order is placed it will cover demand for an integer number of periods.

STEP 0.

The discount for every possible combination of the starting period and the number of periods the order is placed for for every customer i is computed in advance at this step.

The algorithm described in Step 3 of the Cost Compensation Heuristic is applicable here. But the part where the prices are reset needs to be included so that at any moment there is a discount in only one period named x_1. The amount of discount required is saved:

$$p_{ix_1y_1} = \lambda - v_{ix_1} \tag{14}$$

The price itself is reset to be equal to λ and Wagner-Whitin cost matrix is updated. This is necessary because it is not known which discounts will be selected in the final solution, keeping all the calculated discounts will perturb the calculations. On the other hand, this is a deficiency of the model since the discounts applied in previous periods can influce future discounts.

STEP 1.

In Step 0 paramaters p_{itk} have been calculated and are going to be used in the model below. The parameters account for the amount of discount offered. Parameters v_{it} — prices for customer i in period t remain unchanged after this step. Objective (15) minimizes the income which is lost while introducing discounts to customers as well as supplier's inventory and set up costs:

$$\text{minimize} \sum_{i=1}^{n}\sum_{t=1}^{m}\sum_{k=t}^{m} p_{itk}F_{itk}\sum_{u=t}^{k} d_{iu} + \sum_{t=1}^{m} h_0 v_{0t} H_{0t} + \sum_{t=1}^{m} s_0 S_{0t} \tag{15}$$

Constraint (16) ensures that the amount produced by the supplier together with the inventory left in period t is enough to satisfy the demand which corresponds to the summation of customer orders that satisfy their demand from the current period t to a future period k. It is equal to constraint (7) in the previous model.

subject to

$$Q_{0t} + H_{0t-1} - H_{0t} = \sum_{i=1}^{n}\sum_{k=t}^{m} F_{itk}\sum_{u=t}^{k} d_{iu} \text{ , for } \forall t \tag{16}$$

For $t = 1$ constraint (16) is slightly modified since H_{0t-1} equals to 0.

Constraint (17) ensures that the possible maximum produced in period t is the summation of the demand for all customers t for all periods t and is equal to constraint (6):

$$\sum_{i=1}^{n}\sum_{u=t}^{m} d_{iu}S_{0t} - Q_{0t} \geq 0 \text{ , for } \forall t \tag{17}$$

The following constraints which are used while determining the shortest path and called flow balance equations (Eppen and Kipp, 1987) are added to the model.

Constraint (18) guarantees that, since it is assumed that there is no initial inventory, the demand for period 1 has to be ordered in period 1 and can be ordered up to any future period k:

$$\sum_{k=1}^{m} F_{i1k} = 1 \text{ , for } \forall i > 0 \tag{18}$$

Constraint (19) ensures that when the demand is ordered up to the previous period, then a new order must be placed in this period up to some future period k. This guarantees that the demand for all periods will be satisfied:

$$\sum_{k=1}^{t-1} F_{ikt-1} = \sum_{k=t}^{m} F_{itk} \text{ , for } \forall i > 0 \text{ ,} \forall t > a \tag{19}$$

STEP 2.

The order and production patterns are received as a result of solving the model described above. They can be displayed using the variable S_{it} which shows whether order is placed or not in period t. For the supplier $i = 0$ it can be received directly. For every customer i: $S_{it} = 1$ when $\sum_{u=t}^{m} F_{itu} = 1$ for $\forall t$.

After determining order and production patterns the same algorithm from Step 3 of the Cost Compensation Heuristic is used to calculate actual discount. This time it is applied with no changes. Actual prices v_{it} are calculated at this step.

This model is more difficult to solve than the one for the Cost Compensation Heuristic as a result of extra binary variables F_{itk} which are used in the model. At the same time the number of extra parameters calculated in Step 0 is not as big as it would be for the exact model where the discount given in period t depends on the discounts offered in the previous periods. In this heuristic it is assumed that only one discount is given at a time. That is the reason why application of the heuristic doesn't guarantee the optimal solution.

6 EXPECTED OUTCOME

Currently the heuristic algorithms developed for the simple unrestricted one-item case have been tested on one problem. It is planned to do more tests on randomly generated problems.

The data for the considered numerical example was generated randomly for the problem size of 5 customers and 20 periods. Realistically the production costs of the supplier are higher than the order cost of the customers, while the inventory holding costs of the supplier are lower than that of the customers. We assume initial inventory to be equal to 0 in this example.

Figure 1 shows order and production patterns before and after implementation of the heuristic procedures described above. It displays the period in which the order/set up is done. The first row (customer 0) represents set up schedules of the supplier.

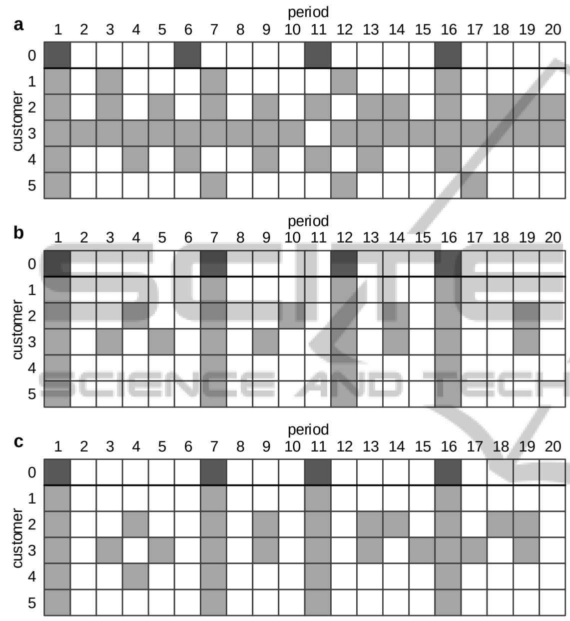

Figure 1: Difference between the initial and final order/production patterns. a — initial order pattern. b — order pattern. Cost Compensation Heuristic. c — order pattern. Discount Interval Heuristic.

It can be noticed that in the initial (before the implementation of the heuristics) pattern customers' orders rarely coincide with production periods. Customer 3 orders almost in every period. Customers 2 and 4 order very often as well. Despite rather low frequency of orders of customers 1 and 5 their order patterns are not synchronized with production patterns.

In the final patterns (after the heuristics were used) orders of customers 1, 4 and 5 totally match production periods. Customers 2 and 3 still order in-between production periods but their orders became significantly less frequent.

Result for the supplier is summarized in tables 1 and 2 .

Table 1: Result for the supplier. Cost Compensation Heuristic

Cost reduction due to coordination	45 120
Sales revenue lost because of discounts	34 805
Additional profit	10 315

Table 2: Result for the supplier. Discount Interval Heuristic

Cost reduction due to coordination	30 740
Sales revenue lost because of discounts	14 462
Additional profit	16 278

Due to the implemented discount pricing schedule the supplier's profit was improved by 10 315 (Cost Compensation Heuristic) and 16 278 (Discount Interval Heuristic). It can be seen that Cost Compensation Heuristic results in more savings for the supplier due to better coordination of the customers' orders. At the same time the supplier loses his profit offering more discounts to the customers. Discount Interval Heuristic gives less cost reduction due to coordination but in the end the supplier gets more additional profit offering less discounts. In both cases the demand doesn't increase, the revenue of the supplier decreases because of the discounts, nevertheless, he gets more profit due to the coordination of orders.

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