Resource Allocation and Scheduling based on Emergent behaviours in Multi-Agent Scenarios

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Abstract: We present our observations regarding the emergent behaviour in a population of agents following a recently presented nature inspired resource allocation / scheduling method. By having agents distribute tasks among themselves based on their local view of the problem, we successfully balance the work across agents, while remaining flexible to adapt to dynamic scenarios where tasks are added, removed or modified. We explain the approach and within it the mechanisms that give rise to the emergent behaviour; we discuss the model used for the simulations, outline the algorithm and provide results illustrating the performance of the method.

1 INTRODUCTION

Resource allocation, as discussed in (Luss, 2012), of which scheduling is a prominent type, is a wide field and applicable to a large number of commercial endeavors, e.g. (Pinedo, 2012). Many approaches to this domain exist and it is not the goal of this paper to compare between these, nor is it to champion one over the other. The pervasive nature of the topic suggests that different instances of the problem with a range of attributes and challenges will make it unlikely for any single approach to consistently outperform all others.

In this paper we would like to discuss one distributed resource allocation method (Hildmann and Martin, 2014) and, specifically, the emergence of simple properties in a population of agents using this method. Emergence is normally linked to the interaction between members of a population, not to their individual actions (Holland, 1998); we discuss the interaction between agents, and investigate the effect this has on the behaviour of the population as a whole.

The example problem we simulated to evaluate the approach is a scheduling scenario. Application of the method to civil security services like police or fire fighters is currently considered, but for the running example in this paper we chose the utility sector and the daily schedules of field service personnel.

We think the views presented in this paper are of interest to the optimization community and to research in Artificial / Swarm Intelligence; furthermore we see applications in the areas of Transportation and Scheduling (Dussutour et al., 2004).

2 PROBLEM DESCRIPTION

We simulated the dispatching of service personnel, tasked with executing a number of tasks within their shift. The time required to handle a task is predicted in advance. In day to day operations traffic conditions affect travel times between locations and, when arriving on site, a task might transpire to require less or more time than originally predicted. The problem can be stated as the sequential scheduling of service or maintenance tasks for a group of field engineers, with the cost of the schedule being calculated in terms of the time it takes to process the schedule:

Description: Let $\mathcal{A}$ be a set of agents $a_1,\ldots,a_m$, each with a finite amount of resources r_{a_i} as well as a depot d_{a_i}, let $\mathcal{T}$ be a set of tasks $t_1,\ldots,t_n$ each with a cost c_{t_j} and assume that function $f(\mathcal{L}\times\mathcal{L})\rightarrow c_{l1l2}$ maps tuples of locations (for the agents or depots) to a cost.

The problem is then to allocate the tasks to agents such that (a) all tasks are allocated to exactly one agent and (b) the minimal sum of the cost to connect the depot to all tasks allocated to this agent (i.e. the shortest path) plus the sum of the costs of the tasks themselves does not exceed the agent's capacity. ■

The presented approach is designed to handle non-static problems (Hildmann and Martin, 2014) and the following 4 aspects are considered to be dynamic:
(1) $\mathcal{A}$ (agents may join the population or drop out),
(2) $\mathcal{T}$ (tasks may be added or removed),
(3) $f()$ (tasks may change their location) and
(4) c_{t_j} (tasks may change their cost).

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3 MODEL

The model simplifies the problem in the following ways: the act of re-allocating a task from one agent to another (*stealing*) is assumed to be instantaneous and cost free. In each iteration, all agents are activated in randomized order, and finally, agents have a starting location (*depot*) and maintain individual schedules.

Parameters: The problem model has the following parameters: numbers of both, agents and tasks, the capacity of each individual agent (which is the same for all agents), the visibility range, the map size (we used 100×100 for all the simulations discussed in this paper) and finally a flag determining whether a single depot is used or whether there are multiple depots.

Furthermore, formula 1, given below, includes the tuning parameter α. Changes in α will affect the speed with which the approach converges towards a stable state, and on the other hand, the degree of change in the problem that the algorithm can handle.

Formulae: The decision to re-allocate a task from the active agent A to the passive agent B is stochastic. If agent A is *balanced* this probability (P_A^{bal}) is:

$$P_A^{bal} = 1 - \frac{(rem.cap_B)^\alpha}{(rem.cap_A)^\alpha + (rem.cap_B)^\alpha} \qquad (1)$$

with $rem.cap_X$ the remaining capacity of X. The probability for a *maximizing* agent A (P_A^{max}) is the inverse:

$$P_A^{max} = 1 - P_A^{bal} \qquad (2)$$

4 METHOD, APPROACH AND ALGORITHM

There is plenty of evidence (e.g. (Bartholdi and Eisenstein, 1996)) for the potential of nature-inspired scheduling to be extremely efficient; our work is inspired by the behavior of social insects (Camazine et al., 2001). It is known (Bonabeau et al., 2000) that colonies of social insects like bees, termites or ants seem to operate in a semi-stable pattern until some catalyzing event like e.g. the discovery of a new food source or an attack on the colony takes place. This then triggers a paradigm shift in the whole colony which persists until the event is dealt with, after which the system returns to the semi-stable state.

We present our work on 3 levels: the method, the approach and the implemented algorithm. By method we mean the generic description of how a population of agents can cooperate to collectively cope with a large number of tasks in dynamic environments. The approach describes how this method can be applied in computer science and to our example.

4.1 Method

Two opposing mechanisms govern the interaction between the agents: one to exchange tasks between themselves with the aim to balance the length of their respective schedules; and the other to re-schedule tasks in order to largely reduce some agents' loads.

Both mechanisms define interaction between exactly two agents and in both cases the decision to re-allocate a task is stochastic (using the formulae 1 and 2, respectively). The process is applied continuously; converging towards balanced or highly unbalanced workloads. As the mechanisms are stochastic they rely in a large number of interactions to achieve their goals, but on the other hand facilitate the ability to overcome deadlocks and local optima.

The first mechanism enforces a semi-stable equitable state where agents exchange tasks to maintain a level of fairness: individual tasks can be moved from one schedule to another if this *decreases* the difference in the respective agents' loads.

The second mechanism, which differs only marginally from the first, is used to achieve its exact opposite: the re-allocation of tasks between two agents is determined on the basis of whether the re-allocation *increases* the difference in loads. This mechanism is used when unallocated tasks appear / are discovered in the vicinity of an agent. The imbalance this creates reduces the load on some of the agents, thus allowing them to take on some of these unallocated tasks, even if this require a significant portion of the agent's entire capacity.

The decision which of the two mechanisms to use is based on the stance or behaviour paradigm of an agent, which corresponds to whether the agent is aware of any allocated tasks. To maintain the analogy to social insects, the presence of unallocated tasks constitutes the catalyzing event that causes a (local) behaviour shift of the population. Agents that are unaware of unallocated tasks operate under the *balanced* stance which follows the *"rich gets poorer"* paradigm. Conversely, if an agent becomes aware of unallocated tasks it switches to the *maximizing* stance and adopts the *"rich gets richer"* paradigm.

Note that as, we defined it, $P_A^{bal} = P_B^{max}$ (cf. formulae 1 and 2). This is in line with the idea that switching stances reverses the probabilities of the outcomes, i.e. if A is likely to succeed at stealing a task for *rich gets richer* then it should be equally likely to have it stolen for *rich gets poorer*.

4.2 Approach

Iteration Design: We designed the approach with a distributed implementation in mind. Agents are expected to execute their algorithms individually and repeatedly. The flow diagram in Figure 1 shows the five stages of each iteration for the agents. These are:

- `Check Capacity:` agents ensure that it has capacity left to take on tasks from another agent.
- `Determine Stance:` the agent chooses which of the two mechanisms it should use (see above):
  1. *balanced* (*"rich gets poorer"*)
  2. *maximizing* (*"rich gets richer"*)
- `Determine action:` the agent will attempt to schedule any unallocated tasks it is aware of. If no unallocated tasks are available, then the stances of the two agents determine the interaction. Since there are two stances (`bal` and `max`) four possible combinations can occur:
  1. *bal - bal*
     If both agents are balanced they will use the balanced approach between themselves. Formulae like e.g. formula 1 are used for this.
  2. *max - max*
     If both agents are set to maximization they will use the max approach between themselves using e.g. formula 2 to determine whether a task is moved from the passive to the active agent.
  3. *bal - max*
     If a balanced agent interacts with a maximizing agent, the balancing agent will always take the task from the maximising agent. The motivation for this is that a balanced agent is merely aiming to distribute the load between itself and its surrounding colleagues, while a maximizing agent is concerned with the allocation of currently unallocated tasks. By passing tasks from maximizing agents to balanced agents we effectively shift load from the problematic part of the problem space to the regions where we are merely optimizing distribution.
  4. *max - bal*
     The last case is when the active agent is maximizing and the passive agent is load balancing; in this case no task is re-allocated (with the motivation being the same as above).
- `Update Schedule:` both agent's schedules need to be updated if a task was exchanged.
- `Wait:` after a cycle an agent will wait for a certain amount of time before becoming active again.

Variations and Design Choices: We discussed the approach in the context of the example application where the resources are service engineers (agents) which are allocated to tasks (locations which the agents have to visit). As mentioned, the presented method expects interaction between exactly two agents, where one agent is taking the active part (by, amongst other things, choosing the other agent), and the other is the passive one.

This leaves two possible interpretations or variations on how to implement the approach: either the active agent is choosing the passive agent from a list of agents, or the active agent chooses a task from a list of tasks (in which case it indirectly chooses an agent, namely the one currently being assigned to this task). We implemented the latter interpretation. This is partly motivated by the desire to stick close the phenomena that inspired the approach (agents will *see* tasks in their vicinity and then potentially interact with the agent that is assigned these tasks).

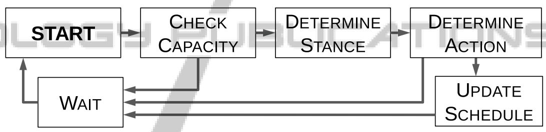


Figure 1: Basic flow diagram of the approach (for either variation, discussed above) as implemented for all agents.

Emergence: The basis of the approach is a continuous optimization of the schedules assigned to the individual members of a population of resources / agents. Pairs of spatially co-located agents attempt to pass tasks to one another with the aim to equalize their respective loads, using an easily calculated probability based on a *"rich gets poorer"* paradigm. On top of this we have designed a mechanism that enables a paradigm shift, namely the switching of the individual agent's stance to *"rich gets richer"*. This paradigm is triggered by the discovery of unallocated tasks.

By design, the mechanisms use computationally cheap formulae to increase the number of possible iterations. The result of continuously iterating is the effect that the population of agents, as a whole, behaves in a fashion that assists the individual agents.

The agents' decision whether to engage with another agent and which formula to use, is determined by their interaction, i.e. by their own stance and the stance of the agent they are interacting with. Therein lies the key to the emergence of the following behaviour: agents in a stable environment will work towards sharing the workload with their surrounding agents, while agents in areas of unallocated tasks will slowly lose tasks to their neighbours in stable environments, increasing their ability allocated new tasks.

This results in the diffusion of tasks from agents in areas of unallocated tasks to agents in areas where all tasks are covered. In other words, agents in unproblematic areas will slowly increase their average load as they take on tasks from areas where there are unallocated tasks, thereby freeing up capacity of the agents in these problematic areas. Once all (visible) tasks have been allocated, the population returns to averaging out the loads of its members, which reverse the trend of tasks gravitating towards balanced agents.

4.3 Algorithm

Algorithm 1 implements the overall approach, Algorithm 2 (where the math from §3 is used) is called by it. All agents are continuously running this algorithm at intervals, the frequency of which being determined by the wait period in line 25.

Algorithm 1 works as follows:

- First, a capacity check is performed (lines 1-4).
- Given available capacity, the agent determines its stance (line 7) on the basis of the existence of unscheduled tasks (line 6) in its vicinity (line 5).
- Independent of the agent's stance, if there are no reachable tasks (line 8, 9) no action is taken.
- If there are any reachable tasks which are not currently scheduled (lines 10, 11), one of them is chosen (line 12) and immediately scheduled (line 13).
- However, if all reachable tasks are scheduled and at least some are scheduled to a *max* agent (lines 14, 15), then one of those tasks is chosen (line 16). Depending on the stance of the agent, the task is either (in case the agent is *bal*, line 17) scheduled directly (line 18) or (in case of a *max* agent, line 19) re-allocated stochastically (line 20) using the probabilities defined in §3 and as calculated by Alg. 2.
- Finally, if all reachable tasks are scheduled but none of them is scheduled to a *max* agent (line 21), then the action is again determined by the stance of the agent: *max* agents categorically do not schedule tasks from a *bal* agent so no action is taken (line 22), but amongst themselves *bal* agents will pick a task (lines 23) and compete for it (line 24) using Alg. 2.

Regarding the choosing of a task (lines 12 or 16) this was implemented as a random choice. There are of course ways to make an informed choice to speed up convergence but in the context of evaluating the performance of the generic approach this is omitted.

Algorithm 1: Main algorithm.

```
    while True do
        // Check capacity
1       load ⟵ check_current_load()
2       if load ≥ my_capacity then
3           shed_task()
4           return
        // Determine stance
5       tasks ⟵ get_tasks_in_vincinity()
6       urgent ⟵ get_unscheduled(tasks)
7       stance ⟵ bal if urgent is empty else max
        // Determine action / Re-schedule
8       reach ⟵ get_reachable(tasks)
9       if reach is empty then return
10      unscheduled ⟵ get_unscheduled(reach)
11      if unscheduled is not empty then
12          task ⟵ pick_task(unscheduled)
13          schedule(task)
14      max_task ⟵ schedule_to_max_ag(reach)
15      if max_tasks is not empty then
16          task ⟵ pick_task(max_task)
17          if stance is bal then
18              schedule(task)
19          else
20              steal(stance,task) // cf. Alg. 2
21      else
22          if stance is max then return
23          task ⟵ pick_task(reach)
24          steal(stance,task) // cf. Alg. 2
25      wait
```

Algorithm 2: `steal`(stance, task) Stochastically determines wether a task gets re-scheduled.

```
formula_bal() uses formula 1, returns ⊤,⊥
formula_max() uses formula 2, returns ⊤,⊥
Input: stance, task
other ⟵ get_current_owner(task)
if stance is bal then
    if formula_bal(self,other) then
        schedule(task)
        return
else
    if formula_max(self,other) then
        schedule(task)
        return
```

5 RESULTS

5.1 Simulations

For the performance evaluation we ran simulations using a fixed seed for the random number generator to ensure that the initial starting positions were the same across simulations. We then investigated increasingly large number of agents. The results are presented below in the graphs shown in Figures 2 to 4.

4 separate scenarios were used for the evaluation:

1. **Scenario 1:** to evaluate the ability of the population to assimilating new tasks we added 40 tasks at the outer edge of the map at iteration 25.
2. **Scenario 2:** robustness and the ability to coped with high loads was tested by continuously adding 100 randomly located tasks every 25 iterations.
3. **Scenario 3:** for illustration purposes (i.e. screenshots in Figure 7) an additional 50 tasks, located up to 20 units outside the original map, are added at iteration 100. This guarantees that they are not in proximity of agents or original tasks.
4. **Scenario 4:** for Figure 6, an additional 50 tasks are added at iteration 100, each 10 units outside the original map and all in a straight line, making them visible to only a small number of agents.

It should be noted that determining the shortest path from the depot through all tasks in a schedule is basically the same as solving the Multi-agent Travelling Salesman Problem (MTSP); known to be NP-complete (Hassan and Al-Hamadi, 2008). We approximated the shortest path using simulated annealing.

5.2 Performance

Our results are generated using simulations: Figures 2, 3 and 5 compare single simulation runs while Figure 4 compares three separate runs of a simulation. To ensure that these results are representative we also ran parallel simulations with larger agent populations (up to 1000), higher initial task load (up to 10,000) and for up to 1000 iterations. A more detailed discussion of these is outside the scope of this paper, the interested reader is referred to (Hildmann and Martin, 2014) for a more detailed discussion of the evaluation.

Note that we intentionally omit reporting the standard deviation because the randomly generated locations for tasks and agents (which do not change) result in different lengths for the optimal schedules, making the standard deviation meaningless unless reported for much larger numbers of simulations.

Figure 2 illustrates how population size impacts the convergence properties: besides the higher load per agent for smaller populations, convergence towards stable values is consistent across populations.

Figure 3 shows the average load of the max agents for the same simulation as above. Note that a value of zero indicates the absence of max agents, (i.e. all tasks are allocated). Smaller populations take longer to assimilate the initial task load into their schedules, but the time it takes for the extra tasks seems to be identical across population sizes.

The results in Figures 2 and 3 are for agent populations operating under comparatively low loads (i.e. with a lot of excess capacity). We now briefly address the question of how the approach performs when the agents are approaching their maximum capacity.

Figure 4 shows the results of 3 separate runs. The initial condition was 250 agents and 100 tasks, each agent with a capacity of 75 and a visibility of 40. Every 25 steps another 100 tasks are added to answer the question whether the increasingly small (comparatively) additions or the decreasing remaining capacity has a significant impact and to test the adaptivity of the approach. From the graphs shown we can see that even for high task loads the population quickly reverts to the balancing stance and converge to an av-

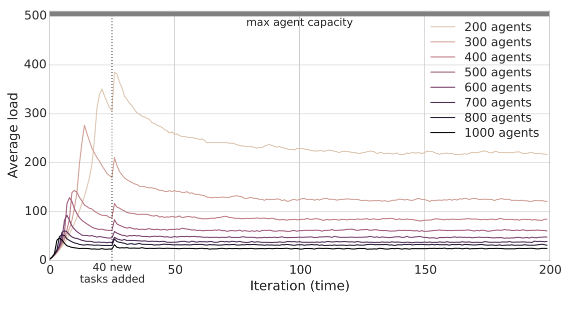


Figure 2: Scenario 1, 4000 tasks and an agent capacity of 500. The graph shows the impact of scheduling 40 new tasks at iteration 25. For populations larger than 400 the impact is minimal; all converge quickly.

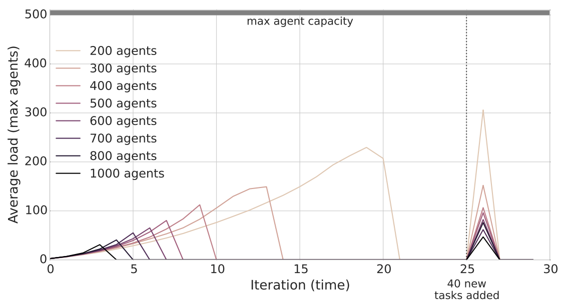


Figure 3: As in Figure 2, showing the average loads of the max agents (zero means there are no max agents). There is a correlation between population size and iterations taken for the initial task allocation; allocation of new tasks is uniform.

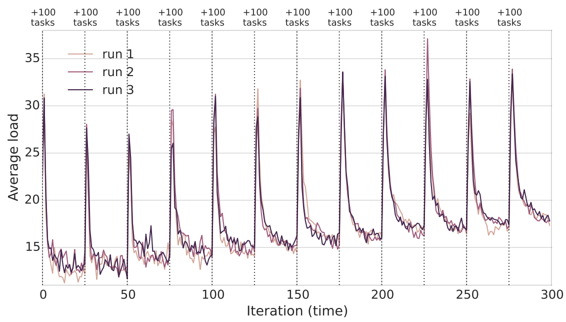


Figure 4: Scenario 2, 50 agents, capacity = 75. Every 25 iterations an additional 100 tasks are added, the graph shows the average load of the agents from 3 separate runs. The increase of the average is steadily increasing, as expected, but the approach seems to continue to perform well.

erage which is consistently increasing in a linear way.

Figure 5 shows that even when approaching their maximum capacity the performance of the population does not deteriorate. It should be noted that in the simulations the cost of a task is set to zero. If this was not the case, there would be a linear increase in the average load to match the aggregated cost of the newly added tasks. The asymptotic curve seen above results also from the fact that with increasing schedule length the cost of adding a new task will (on average) decrease (the more tasks you have the higher the probability of two tasks being very close and thus having a low distance cost associated with them).

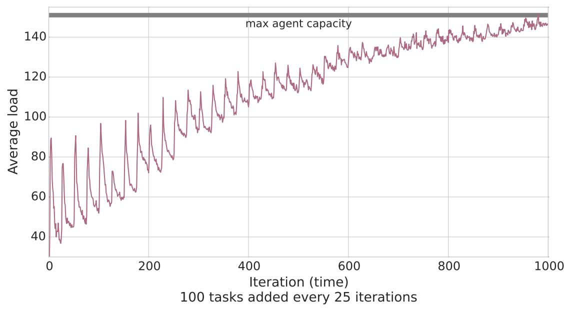


Figure 5: Scenario 2 with 50 agents, capacity = 150, 100 tasks are added every 25 iterations: the graph shows the average load of the agents. For large numbers of tasks (4000+) the average load converges towards the agent's capacity but this does not affect the time to assimilate new tasks.

The above graphs support the claim that the method does indeed scale and performs well. For a detailed evaluation cf. (Hildmann and Martin, 2014).

5.3 Emergent Behaviour

For the screenshots used to show the emergent behaviour we used scenarios 3 and 4. 50 agents were placed randomly on the map, together with 500 initial tasks (which were supplemented by 50 additional tasks at the outer edge of the map in iteration 50 (scenario 3) or 60 (scenario 4). The range of the agents was 350 and their visibility was set to 60.

We show two screenshots in Figure 6 that show the simulation at the iteration when the new tasks are introduced (a) as well as 40 iterations later when almost all the tasks have been assimilated into the agents' schedules. In comparison it is visible that the agents on the left and in the middle of the map have taken on a larger number of tasks to their respective right, i.e. freeing agents on their right.

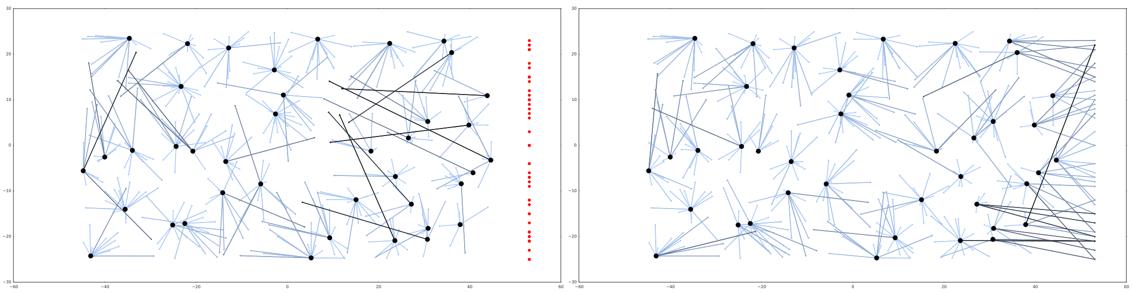
The screenshots in Figure 7 show the impact of large visibility on the behaviour of the population. Scenario 3 was used here. In addition, in this simulation the visibility was set equal to reachability, that is, any task within reach was visible. Given that the agents have quite some capacity left when the new tasks are introduced this resulted in many agents assimilating the new tasks. As a result, the average load increased substantially, because the tasks were allocated on a first come first serve basis with no regard for its distance to the agent. While this was later mitigated when the agents had returned to balancing their loads, the convergence to substantially longer.

6 DISCUSSION

We have presented an approach that, while not producing efficient solutions with regard to the aggregated workload, enable a population of agents to react to changes in the environment and to dynamically change according to changes in the problem space. The graphs in the previous section show that the approach is scalable and that small as well as large problems can be handled.

Furthermore, the preliminary investigations (cf. (Hildmann and Martin, 2014)) showed no dramatic changes for growing problem space.

It was the stated aim of this paper to present the mechanism that gives rise to the emergent behaviour in the population. Through the two separate stances the agents will, in effect, move tasks out of the realm of the agents that can address the problem of unallocated tasks. While the effect is rather hard to show on still images, it is straight forward to see why this is happening when one considers the 4 possible interactions between agents: balanced agents will balance the workload in the population of balanced agents. Max agents will split into agents with very high load and agents with very low load. The agents with a low load will eventually assimilate the unallocated tasks, while the agents with a high load are likely to lose some of their load to balanced agents.



(a) (b)

Figure 6: The two screenshots, above, from a simulation running Scenario 4 with 400 initial tasks and 40 agents, each with a maximum capacity of 220 and a visibility range of 65, illustrate the emerging behaviour of the population. In the map area, each "star" is centered around a different agent's depot, and the points connected to it radially represent the tasks the agent currently has in its schedule. The darkness of the connection corresponds to its length. The red dots on the far right of the map in (a) represent newly added and thus currently unscheduled tasks, all of which are scheduled in (b). When comparing the right half of the maps in (a) and (b) we observe a shift in the orientation of the connections between tasks and agents.

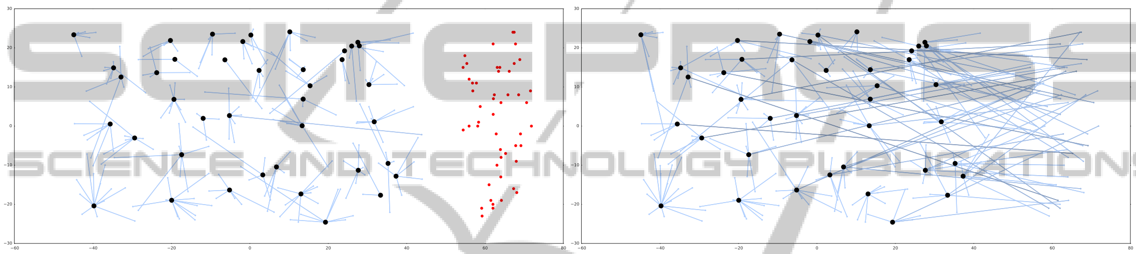


Figure 7: The two pictures above illustrate the emergent behaviour in Scenario 3, using the same representation as Figure 6. The settings are (in comparison to the above) half the initial tasks (200) for again 40 agents, which have a higher capacity (320) and a much wider visibility (200). On a small map setting a high visibility range results in a large number of agents assimilating the new tasks. This removes the collective behaviour where agents further from the tasks take on the load of agents closer to the tasks. While this will allow for an quicker assimilation of all new tasks, it will take longer to distribute the load fairly over the whole population. However, depending on the scenario, this might be a feasible parameter choice.

This should initially affect only balanced agents that are close to max agents, but in time they will share their load with other balanced agents, thereby shedding some of their load to their colleagues as well. As soon as the last unallocated task is assigned to an agent all agents will become balanced and continuously strive to balance their loads.

We presented sufficient materials to allow the reader a comparative implementation. The math provided is intentionally kept simple so as to facilitate the implementation on standard computer hardware.

Restrictions to the Approach

The proposed method relies on a number of agents working in proximity, such that the schedule assigned to a specific agent can be partly absorbed into another agent's schedule. This is expected to require a critical mass in order to outperform other approaches.

Furthermore, there is an upper limit to the degree of change over time which the method can be expected to handle well. If the changes are too rapid or dramatic, recalculating the entire solution might be the better approach. This is due to the iterative nature of the approach, which quickly adapts to changes and "follows" moving centers of gravity in the problem space. If such centers appear and disappear seemingly at random it becomes impossible to follow them, and thus the method will lose the edge over other approaches. However, we do not expect such dramatic changes in the envisioned application areas.

One final issue should be raised here: we have mentioned the Traveling Salesman Problem in the beginning, and pointed out that it is known to be NP-complete. We have then implemented our approach and used a different way to calculate the cost of a schedule. We have investigated using the shortest path from the depot through all tasks and back to the depot as a cost function for a schedule, and will report on these investigations separately; for now it suffices to say that the computational cost of calculating the least cost to address all tasks in a schedule is far outweighing the cost to calculate everything else implemented for our approach. While this indicates that our approach can be implemented efficiently it also means that for a real world application, the algorithm will need to use some insight into the problem space to decrease the computational cost associate with it. We do consider this to be a relevant restriction.

It should be noted that the chosen problem is only one example, used in this paper to illustrate the approach. By no means do we restrict the application domain to service personnel; for example: the transition cost (distance) between tasks can be interpreted as the overhead associated with the switching between tasks. Likewise, we use the term *schedule*, but do not restrict the application of the presented method nor the algorithm to schedules; e.g. the assignment of resources to elements of non-ordered sets of tasks (Beckers et al., 1994) can be addressed as well.

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