High Dimensional Similarity Search with Bundled Query Processing on Hilbert R-Tree*

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Abstract: Hilbert R-tree is an R-tree, which is a B-tree-like multiway balanced tree, such that data objects with high dimensions are sorted along the Hilbert curve. In this paper, we first point out that the compact Hilbert R-tree, which is a Hilbert R-tree without preserving Hilbert values, realizes the same performance as the standard Hilbert R-tree, by using the Hilbert sort and the Hilbert merge. Then, to improve search time for high dimensional objects in the compact Hilbert R-tree, we propose a *bundled query processing*. Furthermore, we introduce two methods, the *pre-processing by the Hilbert merge* and the *control for the order of visiting nodes*. From experimental results, we observe that, in the similarity search of sound and image data, the bundled query processing is about 30% faster than the combinations of individual query processing.

1 INTRODUCTION

In information retrieval for multimedia data, it is a general method to extract feature data from objects and to construct hierarchical spatial index structure. An *R-tree* (Guttman, 1984) is an extension of a B-tree to treat high dimensional objects, and many variations of the R-tree have been developed (*cf.*, (Samet, 2006)). Almost all of the variations have been designed for a dynamic environment.

A *Hilbert R-tree* (Kamel and Faloutos, 1993; Kamel and Faloutos, 1994) is an adequate R-tree for high dimensional objects to impose a linear ordering on the data along the Hilbert curve, which is one of the space filling curves (Bader, 2013). The ordering on nodes in an R-tree by using the Hilbert curve has to be good, in the sense that it should group similar data rectangles together, to minimize the area and perimeter of the resulting minimum bounding rectangles (MBRs).

We can apply the Hilbert R-tree to both static and dynamic environment. In particular, to realize dynamic operations such as insertion or deletion of objects, every entry of the Hilbert R-tree has a Hilbert value of an object. On the other hand, the explicit construction of the Hilbert curve to obtain the Hilbert Values falls into the inefficient construction of the Hilbert R-tree.

To solve this problem, in this paper, we adopt the *Hilbert sort* introduced by Tanaka (Tanaka, 2001). The Hilbert sort is a method which sort objects along the Hilbert curve without constructing Hilbert curves explicitly nor using Hilbert values. Furthermore, Tashima (Tashima, 2011) has introduced a variation of Hilbert R-tree without Hilbert values, which is constructed by using the Hilbert sort, named *compact Hilbert R-tree*. He has also designed the *Hilbert merge* to realize the insertion of objects in the compact Hilbert R-tree with the similar performance as the standard Hilbert R-tree.

In this paper, to improve search time for high dimensional objects in the compact Hilbert R-tree, we propose a *bundled query processing*, which is a method to reduce the IO costs to read nodes in the compact Hilbert R-tree from files with spatial indices. In particular, we introduce two effective methods into the bundled query processing, called the *pre-processing by the Hilbert merge* and the *control for the order of visiting nodes*. In the former, we com-

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pute data objects adjacent to queries along the Hilbert curve, and then obtain the initial radius for each query in the similarity search. In the latter, by computing the minimum distances between the bundle of queries and the nodes, we determine the order of visiting nodes.

Finally, as experiments, we apply the bundled query processing to similarity search of image and sound data. We observe that our bundled query processing is faster than the combinations of individual query processing about 30% for the search time.

This paper is organized as follows. In Section 2, we introduce some notions for later discussion. In Section 3, we explain the Hilbert curve, the Hilbert sort, the compact Hilbert R-tree and the Hilbert merge. In Section 4, we investigate the bundled query processing for the compact Hilbert R-tree. In Section 5, we give experimental results for the bundled query processing. Section 6 concludes this paper.

2 PRELIMINARIES

Let $\mathcal{U} = \mathbf{R}^N$ be a space of objects, where $\mathbf{R}$ is the set of real numbers and N is a dimension of objects. Also let $d : \mathcal{U} \times \mathcal{U} \to \mathbf{R}^+$ be a distance function on $\mathcal{U}$ and $\mathcal{D} = (\mathcal{U}, d)$ a distance space. In this paper, we assume that d is a metric, that is, d satisfies the following conditions for every $x, y, z \in \mathcal{U}$.

1. $d(x,y) \geq 0$.
2. $d(x,y) = 0 \iff x = y$.
3. $d(x,y) = d(y,x)$.
4. $d(x,z) \leq d(x,y) + d(y,z)$ (triangle inequality).

Let $x, y \in \mathcal{U}$, where $x = (x_1, \ldots, x_N)$ and $y = (y_1, \ldots, y_N)$. In this paper, as a distance function on $\mathcal{U}$, we adopt the L_1-distance $d_1(x,y) = \sum_{i=1}^{N} |x_i - y_i|$,which is more natural than L_2-distance for many multimedia data. We use *Simple-Map* (Shinohara and Ishizaka, 2002) as a dimension reduction technique which is applicable to any metric.

In the similarity search in this paper, we use *nearest neighbor queries* (NN-queries, for short). The NN-query searches for an object in $\mathcal{U}$ nearest to a given query (point) with respect to d_1. In the implementation of NN-queries, we first set the radius r of a query q to ∞ and then update r to the distance between q and an object within the current radius r of q.

In NN-queries, it is not necessary to compute the exact distances between a query q and objects outside the radius of q. In other words, for a query $q = (q_1, \ldots, q_N)$ with the radius r and an object $x = (x_1, \ldots, x_N)$, if there exists a K $(1 \leq K \leq N)$ such that $\sum_{i=1}^{K} |q_i - x_i| > r$, then it is not necessary to compute the distance between q and x. Then, with incrementing K from 1 to N, we can break off computing the distances between q and x when the above inequality holds. By using this method, we can reduce the number of computing distances. In experimental results represented in Section 5, we will use such breaking off computing distances implicitly.

3 COMPACT HILBERT R-Tree

In this paper we adopt the compact Hilbert R-tee supported by two algorithms Hibert-sort (Tanaka, 2001) and Hilbert-merge (Tashima, 2011), which are also used in our algorithms for bundled query processing. However, there are no English papers for them. Therefore, here, we briefly introduce them.

3.1 Hilbert Curve

A *space filling curve* (Bader, 2013) visits all the points in a high dimensional grid exactly once and never crosses itself. The *Hilbert curve* (Bader, 2013; Butz, 1971; Lawder and King, 2001a; Lawder and King, 2001b) is one of the space filling curves. The Hilbert curve of order 1 on a 2×2 grid is shown in Figure 1 in 2-dimensional case. To derive a curve of order i, each vertex of the basic curve is replaced by the curve of order $i-1$, which may be appropriately rotated and/or reflected. Figure 1 also shows the Hilbert curves of order 2 and 3. The Hilbert curve can be generalized for higher dimensions.

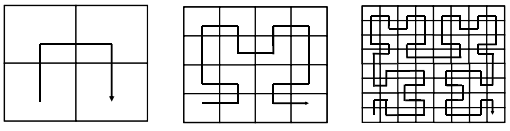


Figure 1: The Hilbert curves in 2 dimension.

3.2 Hilbert Sort

For a space $\mathcal{U}$ with N dimension, we can construct the Hilbert curve by repeating the procedure that divides a subspace of $\mathcal{U}$ into 2^N subspaces and then sorts objects recursively, until all of the objects are sorted. We call the value representing the order of an object on the Hilbert curve the *Hilbert value* of it. Then, for n objects in $\mathcal{U}$ and the m division of $\mathcal{U}$, we compute the Hilbert value in $O(n2^N m)$ time (*cf.*, (Bader, 2013)), which is very expensive in general.

In order to solve this problem, Tanaka (Tanaka, 2001) has introduced the *Hilbert sort*, which is a method to sort data objects with high dimensions along the Hilbert curve, without constructing the Hilbert curve explicitly. The Hilbert sort repeats the procedure that divides a subspace of $\mathcal{U}$ into two for some coordinate, not 2^N subspaces, with dividing objects in subspaces, recursively, until a subspace contains at most one object.

Figure 2 illustrates the running image of the Hilbert sort applied to five objects from *a* to *e* in *xy*-plane. In Step 1, the Hilbert sort divides a whole space into two for the *y*-axis with dividing objects into $\{a,d,e\},\{b,c\}$. In Step 2, it divides two subspaces containing $\{a,d,e\}$ and $\{b,c\}$ into two for the *x*-axis with dividing objects into $\{d,e\},\{a\}$ and $\{b,c\},\emptyset$, respectively. In Step 3, it divides two subspaces containing $\{d,e\}$ and $\{b,c\}$ into two for the *y*-axis with dividing objects into $\{e\},\{d\}$ and $\{b\},\{c\}$. Here, the Hilbert sort halts the division of subspaces for the subspaces containing $\{a\}$ and no object, illustrated by dotted boxes in Step 3 and 4.

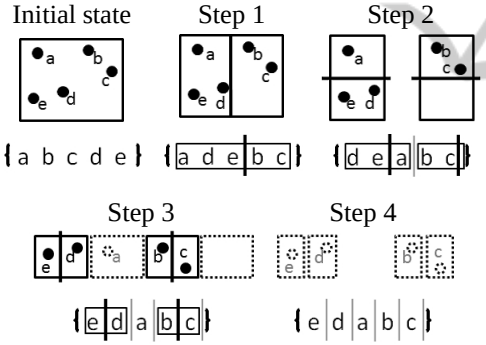


Figure 2: The running image of the Hilbert sort.

In contrast to the above $O(n2^N m)$ time, by using the Hilbert sort, we can sort n objects in N dimensional space in $O(nNm')$ time, where m' is the number of divisions such that $m' < m$ in general. Figure 3 illustrates the running time (sec) of the Hilbert sort (black line) and the ordering by Hilbert values (grey line) when varying the number of data under fixed 8 dimensions (upper) and varying the number of dimensions under fixed $7,000,000$ data (lower) pointed by the *x*-axis. Hence, the Hilbert sort is more efficient than the ordering by Hilbert values.

3.3 Compact Hilbert R-tree

The *R-tree*, introduced by Guttman (Guttman, 1984), is an extension of the B-tree for high dimensional objects. A geometric object is represented by its minimum bounding rectangle (MBR). Then, internal

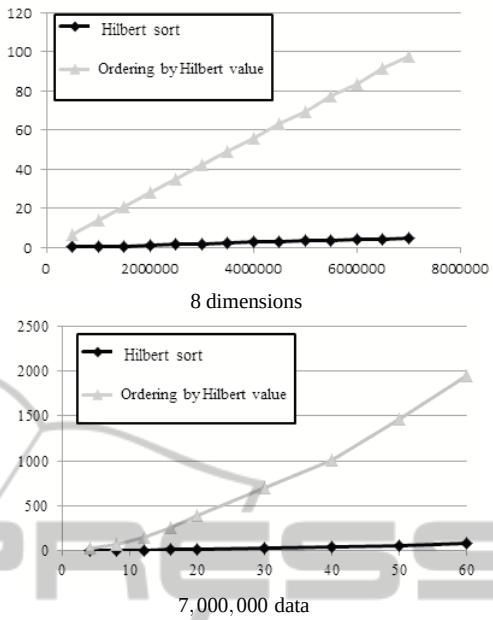


Figure 3: The running time of the Hilbert sort and the ordering by Hilbert values.

nodes in an R-tree contain entries of the form (R, ptr), where *ptr* is a pointer to a child nodes in the R-tree and R is the MBR that covers all rectangles in the child nodes; leaf nodes contain entries of the form (obj, R), where *obj* is a pointer to the object description and R is the MBR of the object. The R-tree is known to be adequate to both static and dynamic environment (Guttman, 1984).

The *Hilbert R-tree* (Kamel and Faloutos, 1993; Kamel and Faloutos, 1994) is an R-tree such that data objects with high dimensions are sorted along the Hilbert curve. Then, internal nodes in a Hilbert R-tree contain entries of the form (R, ptr, LHV), where *LHV* is the *largest Hilbert value* among the data rectangles enclosed by R; leaf nodes contain entries of the form (obj, HV), where *HV* is the Hilbert value of objects. We can apply the Hilbert R-tree to both static and dynamic environment.

Note that, since the explicit construction of the Hilbert curve to obtain the Hilbert values falls into the inefficient construction of the Hilbert R-tree. On the other hand, by using the Hilbert sort, we can construct the Hilbert R-tree without constructing the Hilbert curve explicitly.

The *compact Hilbert R-tree*, introduced by Tashima (Tashima, 2011), is a Hilbert R-tree constructed from the Hilbert sort without using Hilbert values. Internal nodes in a compact Hilbert R-tree contain entries of the form (des_obj, R, ptr), where *des_obj* is the leftmost object *obj* in the leftmost child node; leaf nodes contain entries of the form (obj).

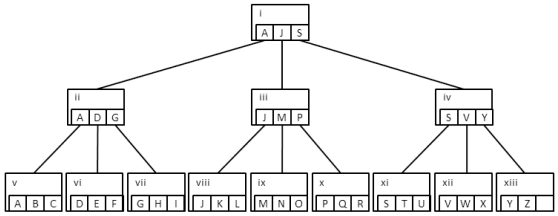


Figure 4: The compact Hilbert R-tree.

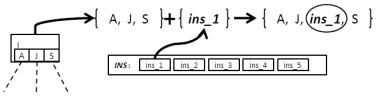
Figure 4 illustrates the outline of the compact Hilbert R-tree omitting R and ptr. Here, every alphabet from A to Z is an object such that the alphabetical order coincides with the order along the Hilbert curve.

3.4 Hilbert Merge

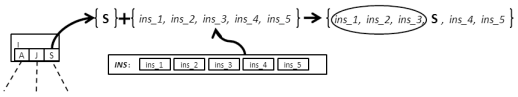
The remained problem for the compact Hilbert R-tree is to design the method for insertion of objects in the Hilbert R-tree with the similar performance as the standard Hilbert R-tree. Also we cannot apply the method of mass update to the Hilbert R-tree, because the objects are stored by ordering along the Hilbert curve. In order to solve these problems, Tashima (Tashima, 2011) introduced the *Hilbert merge* to insert a group of objects ordered by the Hilbert sort in a mass like as merging leaf nodes in the compact Hilbert R-tree.

We explain the Hilbert merge by using an example to insert a group $INS = \{ins\_1, ins\_2, ins\_3, ins\_4, ins\_5\}$ of objects to the compact Hilbert R-tree illustrated in Figure 4. Here, suppose that *INS* is ordered by the Hilbert sort from left to right. We call the order along the Hilbert curve a *Hilbert order* simply.

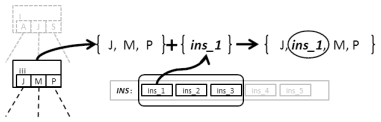
In Step 1, the Hilbert merge adds *ins_1* to the leftmost descendant group $\{\mathtt{A}, \mathtt{J}, \mathtt{S}\}$ of objects in the node (i) and determines the position of *ins_1* in the group as the Hilbert order. As a result, suppose that the order is given as $\{\mathtt{A}, \mathtt{J}, ins\_1, \mathtt{S}\}$. Then, *ins_1* is inserted to the descendant nodes (viii), (ix) and (x) of J.



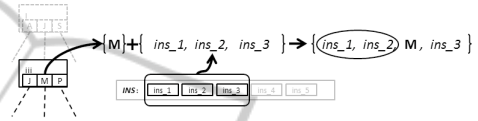
In Step 2, the Hilbert merge adds S as the right of J to *INS* and determines the position of S in *INS* as the Hilbert order. As a result, suppose that the order is given as $\{ins\_1, ins\_2, ins\_3, \mathtt{S}, ins\_4, ins\_5\}$. Then, not only *ins_1* but also *ins_2* and *ins_3* are inserted to the descendant nodes (viii), (ix) and (x) between J and S.



In Step 3, the group $\{ins\_1, ins\_2, ins\_3\}$ is sent to the child node (iii) of J as the group of insertion objects. Suppose that the Hilbert merge determines the position of *ins_1* in the leftmost descendant group $\{\mathtt{J}, \mathtt{M}, \mathtt{P}\}$ of objects in the node (iii) as $\{\mathtt{J}, ins\_1, \mathtt{M}, \mathtt{P}\}$.



In Step 4, the Hilbert merge determines the position of M in $\{ins\_1, ins\_2, ins\_3\}$ as the Hilbert order similar as Step 2.



As a result, suppose that the order is given as $\{ins\_1, ins\_2, \mathtt{M}, ins\_3\}$. In this case, both *ins_1* and *ins_2* are inserted to the leaf node (viii).

The Hilbert merge has the following properties.

1. The Hilbert merge visits every node in the Hilbert R-tree at most once.
2. The Hilbert merge arranges the MBR of a node in the Hilbert R-tree after finishing the insertion to all of its children nodes.
3. When it is necessary to divide a node in the Hilbert R-tree, the Hilbert merge inserts the object that the node is a child node to an upper level in the Hilbert R-tree as a new object.

Hence, by using the Hilbert sort and the Hilbert merge, the compact Hilbert R-tree realizes the same performance as the standard Hilbert R-tree.

4 BUNDLED QUERY PROCESSING

In the remainder of this paper, we call an NN-query a query simply, and assume that a query is set to some radius. In this section, we discuss the processing for the bundle of queries, called a *bundled query processing*, to search for high dimensional objects on the compact Hilbert R-tree. In contrast, we call the combinations of query processing for every query in bundles an *individual processing*.

In the remainder of this section, we denote the bundle consisting of queries $q_1, \ldots, q_n$ by $\langle q_1, \ldots, q_n \rangle$. For a bundle Q of queries and a query q_i, we denote that Q contains q_i by $q_i \in Q$. Also we denote the radius of q_i by r_i.

4.1 Naive Bundled Query Processing

In the naive bundled query processing, the bundle of queries visits nodes in the compact Hilbert R-tree by using the depth-first search from the root node. Note that naive processing does not consider the order of visiting nodes.

When Q visits an internal node whose entry is (des_obj, R, ptr), we determine whether $q_i \in R$ for every $q_i \in Q$, and, if such a q_i exists, then Q visits the children of the node. When Q visits a leaf node whose entry is (obj), we compute the distance $d(o, q_i)$ between each $o \in obj$ and each $q_i \in Q$, and, if the radius r_i of q_i is greater than $d_i = \min\{d(o, q_i) \mid o \in obj\}$, then we set r_i to d_i.

The number of visiting nodes in the naive bundled query processing is much smaller than one in the individual processing. Hence, the naive bundled query processing reduces the IO costs for reading files.

4.2 Pre-processing by Hilbert Merge

On the other hand, the number of distance computaions in the naive bundled query processing is possible to be larger than one in the individual processing. To avoid this situation, we introduce the *pre-processing by the Hilbert merge*, which follows from the following two properties.

1. Two objects such that one is near to the other in the Hilbert order are that one is near to the other in a space of objects with high probability. Hence, an object near to a query within the Hilbert order is contained in the radius of the query with high probability.
2. By using the Hilbert merge, we can insert the group of objects to the nodes in the Hilbert order simultaneously.

In the pre-processing by Hilbert merge, first we sort the bundle Q of queries in the Hilbert order by using the Hilbert sort. Next, we simultaneously search for nodes where each query $q_i \in Q$ is inserted by using the Hilbert merge. Then, after computing distances $d(o, q_i)$ between each $q_i \in Q$ and each $o \in obj$ in the searched leaf node whose entry is (obj), we set the initial radius r_i of q_i to $\min\{d(o, q_i) \mid o \in obj\}$.

Hence, the number of distance computaions in the bundled query processing with the pre-processing by the Hilbert merge is smaller than one in the naive bundled query processing.

4.3 Control for the Order of Visiting Nodes

In order to reduce the number of distance computaions, in the bundled query processing, we control the order of visiting nodes in the compact Hilbert R-tree as similar as the R-tree, by using the ABL (Active Branch List) L.

When Q visits an internal node, we determine whether or not Q must visit its child nodes, and, if so, then we insert the set C of such child nodes to L. Here, let Q' be a sub-bundle Q' of Q which will visit C and R_c an MBR of $c \in C$. Then, C in L is sorted by ascending order of $\min\{d(q_i, R_c) \mid q_i \in Q'\}$. Hence, by visiting nodes in the first element of L, Q can visit a node $c \in L$ such that $d(q_i, R_c)$ is minimum for every $q_i \in Q$.

5 EXPERIMENTAL RESULTS

In this section, we give experimental results for the bundled query processing in similarity search on the compact Hilbert tree.

For image data, data objects consist of about $7,000,000$ pictures extracted from about $2,800$ video data whose features have 64 dimensions, and queries consist of $90,000$ pictures extracted from 100 video data. On the other hand, for sound data, data objects consist of about $7,000,000$ sound fragments extracted from about $1,500$ musical data whose features are 96 dimensions and queries consist of $90,000$ fragments extracted from 30 musical data. Here, $90,000$ queries consist of near queries to far queries. Queries is not intended to completely match in the database. The number of queries in bundles varies 10, 100 and $1,000$. The computer environment is Intel Core i7-3770 3.40GHz CPU with 16GB RAM.

Table 1 and 2 describe the results of the bundled query processing for image data and sound data, respectively. Here, #queries, #nodes and #distance denote the number of queries in bundles, the number of visiting nodes and the number of distance computations, respectively. The individual processing adopts the pre-processing of the Hilbert merge and the control for the order of visiting nodes.

As shown in row (3) in Table 1 and 2, in the bundled query processing with the control for the order of visiting nodes, increasing the number of queries decreases the number of visiting nodes, as well as in rows (1) and (2). Also, the search time is shortest when the number of queries in bundles is 100, not $1,000$, for both image data and sound data.

Table 1: The results for image data.

method	#queries	#nodes	#distance	search time (min)
(0) individual processing	@	$\mathbf{1.11 \times 10^9}$	$\mathbf{5.99 \times 10^{10}}$	**138.17**
(1) naive bundled query processing	10	3.50×10^8	1.33×10^{11}	196.07
	100	5.45×10^7	1.33×10^{11}	187.60
	1,000	$\mathbf{6.39 \times 10^6}$	$\mathbf{1.33 \times 10^{11}}$	**186.32**
(2) (1) with pre-processing by Hilbert merge	10	2.93×10^8	9.34×10^{10}	135.87
	100	5.11×10^7	9.34×10^{10}	128.29
	1,000	$\mathbf{6.32 \times 10^6}$	$\mathbf{9.34 \times 10^{10}}$	**127.02**
(3) (2) with control for the order of visiting nodes	10	2.14×10^8	6.05×10^{10}	101.03
	100	$\mathbf{4.12 \times 10^7}$	$\mathbf{6.23 \times 10^{10}}$	**95.51**
	1,000	6.17×10^6	6.73×10^{10}	104.42

Table 2: The results for sound data.

method	#queries	#nodes	#distance	search time (min)
(0) individual processing	@	$\mathbf{7.98 \times 10^8}$	$\mathbf{4.59 \times 10^{10}}$	**138.76**
(1) naive bundled query processing	10	2.57×10^8	7.79×10^{10}	166.52
	100	4.23×10^7	7.79×10^{10}	158.12
	1,000	$\mathbf{5.62 \times 10^6}$	$\mathbf{7.79 \times 10^{10}}$	**156.97**
(2) (1) with pre-processing by Hilbert merge	10	2.08×10^8	5.99×10^{10}	123.99
	100	3.81×10^7	5.99×10^{10}	117.24
	1,000	$\mathbf{5.42 \times 10^6}$	$\mathbf{5.99 \times 10^{10}}$	**115.94**
(3) (2) with control for the order of visiting nodes	10	1.78×10^8	4.66×10^{10}	106.68
	100	$\mathbf{3.35 \times 10^7}$	$\mathbf{4.79 \times 10^{10}}$	**100.61**
	1,000	5.04×10^6	4.95×10^{10}	102.69

As shown in rows (2) and (3) when the search time is shortest denoted by bold faces, the number of distance computations decreases about 30% and 20% to the bundled query processing without the control (in the row (2)) in the bundled query processing with the control (in the row (3)) for image data and sound data, respectively. Also the search time decreases about 20% and 10%, respectively.

As shown in the rows (0) and (3) when the search time is shortest denoted by bold faces, the number of visiting nodes much decreases to the individual processing (in the row (0)) in the bundled query processing with the control for image data and sound data, respectively. However, the number of distance computations increases 10% for both data. Nevertheless, the search time decreases about 30% for both data.

6 CONCLUSION

In this paper, we have proposed the bundled query processing on compact Hilbert R-trees for high dimensional data and, in particular, introduced two methods that the pre-processing by the Hilbert merge and the control for the order of visiting nodes. Then, we have given the experimental results for the bundled query processing in similarity search of image and sound data. Hence, we have succeeded that our bundled query processing is more efficient than the individual processing about 30% for the number of visiting nodes and the search time.

However, the number of distance computations in our bundled query processing is larger than one in the individual processing. It is a future work to design an appropriate method for bundled query processing to reduce the number of distance computations.

In addition, it is necessary to verify this technique for other tree structures such as *M-tree* (Ciaccia and M. Patella, 1997).

REFERENCES

Bader, M. (2013). *Space-filling curves*. Springer.

Butz, A. R. (1971). Alternative algorithm for Hilbert's space-filling curves. *IEEE Trans. Computers*, C-20:424–426.

Ciaccia, P. and M. Patella, P. Z. (1997). M-tree: An efficient access method for similarity search in metric spaces. *Proc. 23rd Int. Conf. on Very Large Data Bases*, pages 426–435.

Guttman, A. (1984). R-trees: A dynamic index structure for spatial searching. In *Proc. SIGMOD'84*, pages 47–57.

Kamel, I. and Faloutos, C. (1993). On packing R-trees. In *Proc. CIKM'93*, pages 490–499.

Kamel, I. and Faloutos, C. (1994). Hilbert R-tree: An improved R-tree using fractals. In *Proc. VLDB'94*, pages 500–509.

Lawder, J. K. and King, P. J. H. (2001a). Querying multidimensional data indexed using the Hilbert space-filling curve. *ACM SIGMOD Record*, 30:19–24.

Lawder, J. K. and King, P. J. H. (2001b). Using state diagrams for Hilbert curve mappings. *Internat. J. Computer Math.*, 78:327–342.

Samet, H. (2006). *Foundations of multidimentional and metric data structures*. Morgan Kaufmann.

Shinohara, T. and Ishizaka, H. (2002). On dimension reduction mappings for approximate retrieval of multidimensional data. *Progress in Discovery Sciencer (LNCS 2281)*, pages 224–231.

Tanaka, A. (2001). *Study on a fast ordering of high dimensional data to spatial index*. Master Thesis, Kyushu Institute of Technology.

Tashima, K. (2011). *Study on efficient method of insertion for spatial index structure by using Hilbert sort*. Master Thesis, Kyushu Institute of Technology.