# SymPaD: Symbolic Patch Descriptor 

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#### Abstract

We propose a new local image descriptor named SymPaD for image understanding. SymPaD is a probability vector associated with a given image pixel and represents the attachment of the pixel to a previously designed shape repertoire. As such the approach is model-driven. The SymPad descriptor is illumination and rotation invariant, and extremely flexible on extending the repertoire with any parametrically generated geometrical shapes and any desired additional transformation types.


## 1 INTRODUCTION

The origins of research on qualitative image structures date back to Marr (Marr, 1982) in 1980's. In his three-step representation framework, Marr described primal sketch of images qualitatively in terms of the feature categories of edges, lines and blobs based on the quantitative responses of linear filters (Marr, 1982). Marr's scheme was further improved by (Koenderink, 1984) so that the localization of detected edges is estimated on the points of highest gradient rather than simply near them. (Griffin, 2007) extended the idea in 2000's by using $1^{\text {st }}$ order, $2^{\text {nd }}$ order and $1^{\text {st }}$ and $2^{\text {nd }}$ order filters with which a wider range of image symmetries can be probed compared to Marr's "edge", "line", "blob" feature sets. This set of approaches leads to the paradigm of model-guided shape dictionaries to describe images via pixel neighbourhoods.

The alternative paradigm in classifying and categorizing images or recognizing objects in images via local features uses data-driven dictionaries. As a case in point, SIFT (Lowe, 1999) or HOG (Dalal and Triggs, 2005) features represent local image structures based on the magnitude and orientation of gradients at pixel locations. A visual dictionary is then constructed typically by the k-means clustering algorithm (Csurka et. al, 2004).

Both approaches have their own advantages and disadvantages. In the data-driven scheme, which is presently by far more popular in the literature, one
learns the descriptor prototypes statistically from local image descriptors computed on a training set of image patches. This entails some dependency on the training dataset, hence might limit generalizability and there may be some loss of accuracy due to the clustering algorithm chosen (Jurie and Triggs, 2005). In contrast, model-driven approaches do not need an elaborate training step to learn a visual dictionary. The dictionary is created based on variations of shape models such as ramps, ridges, valleys, corners, lines, spots, and their various combinations. In this study, we pursue the model-driven framework to generate new descriptors that would best capture the image characteristics in a database-independent manner.

Basic Image Features (BIFs) proposed by (Crosier and Griffin, 2010) is the most current model-driven dictionary construction study in the literature. It is based on determining image symmetries by using reparameterized derivatives of gradient filters (DtG). The method is invariant to some geometric transformations such as rotation, reflection, and some grayscale transformations such as intensity multiplications and addition of a constant intensity (Griffin and Lillholm, 2007). They re-parameterize the jet space of DtG filters, called $2^{\text {nd }}$ order local image structure solid, which they partition into Voronoi-like regions in order to obtain seven feature categories corresponding to the symmetries of "flat", "ramp", "dark/light line", "dark/light circle", "saddle" patches on natural images.

BIF features tested on texture classification
(Crosier and Griffin, 2010), object recognition (Lillholm and Griffin, 2008), handwriting recognition (Newell and Griffin, 2014) with a bag of words implementation yield modest performance. (Crosier and Griffin, 2010) then introduce a multi-scale version of with BIFs and achieve state-of-the-art performance competing with the data-driven schemes.

Our proposed Symbolic Patch Descriptor (SymPaD) uses a shape repertoire, as detailed in the sequel, based on patch shapes described by sigmoidals of polynomials and some transcendental functions. Pixels are then characterized by their posterior probability vector to belong to the members of the shape repertoire. The probability vectors from all pixels of an image are then accumulated into a frequency vector, in much the same way as the Bag-of-Words approach. The advantages of SymPaD can be summarized as follows:

- The model driven approach makes SymPaD a dataset independent tool bypassing the computational step required by clusteringbased dictionary construction methods.
- SymPaD is illumination invariant as we use the BRIEF features (Calonder, et. al, 2010), that essentially encode the signs of local image derivatives. Furthermore robustness against rotation and scaling is obtained by accomodating the prototypical shapes in an adequate number of orientations and scales.
- The repertoire of patch shapes can be enriched by incorporating different parametric functions and/or by considering their linear and nonlinear combinations.
At the first section of the paper, we define the main components of the framework of proposed descriptor SymPaD. We present the performance of the SymPaD for an object recognition application and compare it with state of the art at the second section. Finally we address the conclusion and the future work.


## 2 SymPaD FRAMEWORK

The SymPaD framework consists of three computational components: (i) Generation of the Primitive Shape Library (PSL), in other words, the model-driven dictionary, (ii) Posterior computation module in which we compute the SymPaD vectors on dense image points, and (iii) Pooling module in which we construct the final descriptor or code vector for the input image. The block diagram of the system is given


Figure 1: SymPaD framework.
in Figure 1 and the pooling module is illustrated in Figure 2.

### 2.1 Primitive Shape Library (PSL)

The primitive shape library contains a variety of shape appearances generated by input functions in Table 1, which are fed into the standard logistic function shown in Eq. 1.

$$
\begin{equation*}
F(x, y)=\frac{1}{1+e^{-\alpha f(x, y)}} \tag{1}
\end{equation*}
$$

In a preliminary proof of concept study (Aslan, et. al, 2014), we experimented with a limited set of the dictionary (functions $f_{0}$ to $f_{7}$ ).

If the primitive shape structure of a natural image is probed in patches of characteristic size, it would mostly appear as step edges in various orientations and scales (Griffin et. al, 2004) or as flat regions However, if an image is probed in constant size patches, the primitive local structure can have appearances in different forms, such as combinations of oriented, translated or scaled step edges, circular or elliptical pits or hills, ridges corners, saddles, three step edges etc. all at different orientations and scales.

We use sigmoidal outputs of the chosen $\{f(x, y)\}$ shape functions, as in Table 1, in order to add one more control parameter, $\alpha$, that adjust the steepness of the shapes. An alternative monotonically increasing function generating sigmoidal curves is hyperbolic tangent function, whose outcome is symmetric around the origin. While this symmetry behaviour has some advantages for the neural networks, (LeCun et. al, 2012), in our case this is not relevant.

To achieve invariance against rotation and scaling effects, each $m \times m$ sized prototypical shape is generated in an adequate number of orientations and scales. The orientations are created by the $\theta$

Table 1: Parametric patch generators $\left(x_{\theta}, y_{\theta}\right.$ denote the rotated versions of $x$ and $y$ with angle $\theta, a$ is the minor and $b$ is the major axes of the elliptic shape).

| Generator function | Appearance |
| :--- | :--- |
| $f_{0}(x, y)=c$ |  |
| $f_{1}(x, y)=x_{\theta}+y_{\theta}$ |  |
| $f_{2}(x, y)=\left(x_{\theta}+y_{\theta}\right)^{2}$ |  |
| $f_{3}(x, y)=-\left(x_{\theta}+y_{\theta}\right)^{2}$ |  |
| $f_{4}(x, y)=\left(x^{2}+y^{2}\right)$ |  |
| $f_{5}(x, y)=-\left(x^{2}+y^{2}\right)$ |  |
| $f_{6}(x, y)=x_{\theta}{ }^{2} / a+y_{\theta}{ }^{2} / b$ |  |
| $f_{7}(x, y)=-\left(x_{\theta}{ }^{2} / a+y_{\theta}{ }^{2} / b\right)$ |  |
| $f_{8}(x, y)=x_{\theta}{ }^{2}-y_{\theta}{ }^{2}$ |  |
| $f_{9}(x, y)=x_{\theta}+y_{\theta}{ }^{2}$ |  |
| $f_{10}(x, y)=-\left(x_{\theta}+y_{\theta}{ }^{2}\right)$ |  |
| $f_{11}(x, y)=x_{\theta} \times y_{\theta}{ }^{2}$ |  |
| $f_{12}(x, y)=\left(x_{\theta}+y_{\theta}\right)^{3}$ |  |
| $f_{13}(x, y)=x_{\theta}{ }^{3}+y_{\theta}{ }^{3}$ |  |
| $f_{14}(x, y)=x_{\theta}{ }^{2} \times y_{\theta}{ }^{2}$ |  |
| $f_{15}(x, y)=-\left(x_{\theta}{ }^{2} \times y_{\theta}{ }^{2}\right)-4 N$ |  |
| $f_{16}(x, y)=e^{x_{\theta} \times y_{\theta}}$ |  |
| $f_{17}(x, y)=e^{x_{\theta}+e^{y_{\theta}}}$ |  |
| $f_{18}(x, y)=-\left(e^{x_{\theta}}+e^{y_{\theta}}\right)$ |  |
| $f_{19}(x, y)=x_{\theta} \times \cos y_{\theta} / 2$ |  |
| $f_{20}(x, y)=x_{\theta} \times \cos y_{\theta}$ |  |

parameter as in Table 1; the scales are controlled by the $\alpha$ parameter in Eq. 1. In addition, elliptical trenches and ridges have the eccentricity parameter. To generate patch varieties, we randomly sample the $[\theta, \alpha]$ plane with $K$ instances for every PSL class.

### 2.2 Posterior Computation Module

We characterize each $m \times m$ patch (test or prototype) by its BRIEF feature vector with length $n_{d}=256$ using a sampling geometry of (Calonder et. al, 2010) that corresponds to random point locations drawn from the uniform distribution.

BRIEF features $b_{p}$ for a patch centered on a pixel $p$ are computed densely on the image, that is, on every pixel of the image. Similarly, BRIEF features $\left\{b_{l_{1}}, b_{l_{2}}, \ldots, b_{l_{M}}\right\}$ of the PSL patches $\left\{l_{1}, l_{2}, \ldots, l_{M}\right\}$ are precomputed where $M=K \times D, K$ is the number of scale and orientation varieties for a shape as in Table 1 that construct the class of that particular shape and $D$ is the number of words in the dictionary or number of functions in Table 1 taking place in PSL generation.

When the BRIEF feature $b_{p}$ of a test image patch
$p$ is given, we estimate its class posterior probability among $\left\{b_{l_{1}}, b_{l_{2}}, \ldots, b_{l_{M}}\right\}$ by counting votes among the $K$-nearest neighbours ( $K \gg 1$ ). We execute the linear $K-N N$ search with FLANN library (Muja and Lowe, 2012) using Hamming distance. For each test patch $p$, let the $n n_{k_{p}}, k=1, \cdots, K$ be the nearest neighbour class occurrences. Then the patch posterior probability is computed by Eq. 2 where $c \in$ $\{1,2 \ldots, D\}, \delta(u, v)=1$ if $u=v$, and 0 otherwise.

$$
\begin{equation*}
\tilde{P}\left(c \mid n n_{k_{p}}\right),=\frac{\sum_{i=1}^{K} \delta\left(c, n n_{k}\right)}{K} \tag{2}
\end{equation*}
$$

### 2.3 Pooling Module

In Section 2.2, the image has been converted into a D band image where each pixel is represented by the jet of the estimated posterior probabilities. We examined two types of pooling methods, one with max pooling rule, where we assign the label of maximum posteriori of $\tilde{P}\left(c \mid n n_{k_{p}}\right)$ to the pixel p as in Eq. 3:

$$
\begin{equation*}
L_{p}=c \text { if } L=\underset{c=1,2, \ldots, D}{\operatorname{argmax}} \tilde{P}\left(c \mid n n_{k_{p}}\right) \tag{3}
\end{equation*}
$$

An image is then represented by the histogram of pixels $L_{p}$ consisting of $D$ number of bins. The other scheme does not discard the probability estimates in the second, third ranking decisions. Instead, we build a separate histograms for each of $R$ ranks, where the first histogram is as in the max pooling case; the $R^{t h}$ histogram is considers label assignments at the $R^{t h}$ rank among the $K$ labels resulting in the K-NN scheme. Here $R \ll D$. The pooling process of posterior jets is illustrated in Figure 2.

## 3 EXPERIMENTS

We examined the proposed descriptor on the COIL20 "processed" corpus, which contains 20 object categories with a pose interval of 5 degrees between 72 images of $128 \times 128$ pixels in each category. For the test setup, we tried two scenarios: (i) in coil20_rand, we randomly select 6,12 and 24 images from each object category for training, and use the remaining ones for testing and repeat the whole process ten times, (ii) in coil20_seq, for the training set, we chose images with the pose interval of 15 degrees for coil20_seq_tr24, 30 degrees for coil20_seq_tr12, and 60 degrees for coil20_seq_tr6 sequentially and throw the remaining ones into the test set for each object category, this scheme was also used in (Shekar, et. al, 2013). We also compare the
accuracy of SymPaD with the conventional clustering-based dictionary construction method using dense SIFT features with the default stride parameter in (Law et. al, 2014) and with equal number of visual words in the dictionary used in SymPaD. Since we are interested in the performance of the descriptor, we did not use an advanced classifier but a simple K-nearest neighbour classifier using chi-square distance with 5-fold cross validation to accomplish object recognition.

First we used the visual dictionary in (Aslan, et. al, 2014) that consists of eight shape classes that are generated by input functions $f_{0}$ to $f_{7}$ in Table 1 into the sigmoid function in Eq. 1. Then the whole process is repeated for the extended dictionary created by using the whole set of functions $f_{0}$ to $f_{20}$ Since the Flat label is assigned mostly to the (background region in images, we can exclude it during the coding to exploit the effect of structural regions arising from the foreground object. Hence, we also considered the without flat scheme by omitting the effect of the PSL class generated by $f_{0}$ function.

We generated $K=50$ number of $15 \times 15$ sized gray-level patches of varying orientation and coarseness level for each of the $D$ PSL classes. Since the higher values of $K$ represents the characteristics of uniform distribution better, more transformational variations could be simulated by generating a higher number of patches in a PSL class. $K=20, K=50$, $K=100$ are examined and we observed that the decision of $K$ should be given by considering the pooling method used, that is, in hard assignment based pooling higher $K$ performs better than the lower ones, however in soft-assignment based pooling
performance of both gets similar. This is not a big surprise that, since we exploit the uncertainty of pixel labels on the image pixels about their affiliation to the PSL classes by soft assignment, the performance improves.

Orientation: Since the shapes generated by the functions $f_{0}, f_{4}$ and $f_{5}$ exhibit rotational symmetry, we do not assign orientation to them, ramp-like structures generated by the functions $f_{1}, f_{11}, f_{12}, f_{13}$, and $f_{16}$ has the orientation range of $[0,2 \pi]$, and the remaining ones has the orientation range of $[0, \pi]$. So, for each shape class we randomly sample $K$ number of orientation values drawn from uniform distribution in each function's orientation range in order to be able to represent every possible orientation of a form in its own class.

Transition rate: Since we want an approximately uniformly distributed appearances of coarseness levels for a particular shape, we sampled the values of $\alpha$ from exponential distribution with pdf in Eq. 4, with mean of $\mu$, and we shift the sampled values by a constant $\tau$ to prevent collapse on the coarsest and finest levels.

$$
f_{\alpha}(x)=\left\{\begin{array}{cl}
\frac{1}{\mu} \exp \left(-\frac{x}{\mu}\right) & \text { if } x \in[0, \infty)  \tag{4}\\
0 & \text { otherwise }
\end{array}\right.
$$

The values of $(\mu, \tau)$ that we used for the functions with degree of (i) 1 are $(\mu=0.3, \tau=0.3$ ), (ii) 2 are ( $\mu$ in [0.004, 0.1], $\tau=0.04$ ), (iii) 3 are ( $\mu$ in [0.01,0.02], $\tau$ $=0.01$ ), (iv) 4 are ( $\mu=0.0035, \tau=0.0008$ ), and (v) for the exponential functions ( $\mu$ in $[0.15,0.3], \tau=0.15$ ) and (vi) transcendental functions ( $\mu=0.3, \tau=0.3$ ).


Figure 2: Pooling scheme of SymPaD framework for a $r \times c$ sized input image.

The performance results obtained with the dictionary of eight words (of functions $f_{0}$ to $f_{7}$ ) are given in Table 2 and 21 words (of functions $f_{0}$ to $f_{20}$ ) are given in Table 3. We also present the results of (Shekar, et. al, 2013) which used the same test setup on COIL20 in Table 4. Some outcomes of the tests are:

- SymPaD performs best in every test setup, when soft assignment based encoding with rank 4 was used. We could also outperform the results given in (Shekar, et. al, 2013).
- For $D=8$ hard assignment based encoding, withFlat is slightly better than withoutFlat scheme. However, when $D=21$ is used, the situation becomes reversed, probably because when $D=21$ is used, classes except Flat becomes more descriptive compared to the same case when $D=8$ is used
- Soft-assignment based encoding improves performance whether Flat was included or not.
The amount of improvement is higher for $D=$ 8 than $D=21$ which can be interpreted as, for the low dimensional dictionaries soft
assignment based encoding has a more vital role.
- Both methods, SymPaD and conventional Dense SIFT + BoW, exhibited performance achievements when the training set was designed with sequentially selected images of each category, however, when the smaller sized dictionary was used, amount of improvement acquired by conventional method was higher than the amount of improvement acquired by SymPaD, that shows robustness of SymPaD for the test setup designed by random elements.
- The larger dictionary provided improvement when hard assignment based encoding was used, but it did not have a significant effect when soft assignment based encoding was used.


## 4 CONCLUSIONS

In this study, we propose a new descriptor, generated by a model-driven framework. Since model-driven

Table 2: Recognition performance. ( $D=8$, HA: Hard Assignment, SA: Soft Assignment).

| Test | SymPaD, withFlat |  | SymPaD, withoutFlat |  | Dense SIFT + BoW |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | HA | SA_Rank $(1: 4)$ | HA | SA_Rank $1: 4)$ |  |
| coil20_rand_tr6 | $78.84 \pm 1.72$ | $85.86 \pm 1.01$ | $77.15 \pm 2.71$ | $\mathbf{8 7 . 5 5} \pm \mathbf{2 . 5 0}$ | $77.58 \pm 1.27$ |
| coil20_rand_tr12 | $86.81 \pm 1.46$ | $92.66 \pm 1.36$ | $84.28 \pm 3.55$ | $\mathbf{9 4 . 3 6} \pm \mathbf{1 . 0 5}$ | $85.04 \pm 1.03$ |
| coil20_rand_tr24 | $92.35 \pm 0.73$ | $96.72 \pm 0.80$ | $91.86 \pm 0.62$ | $\mathbf{9 8 . 4 2} \pm \mathbf{0 . 6 2}$ | $90.91 \pm 1.24$ |
| coil20_seq_tr6 | 79.39 | 85.61 | 80.38 | $\mathbf{9 0 . 9 1}$ | 79.92 |
| coil20_seq_tr12 | 89.83 | 95.42 | 89.17 | $\mathbf{9 7 . 0 0}$ | 89.92 |
| coil20_seq_tr24 | 93.65 | 97.71 | 94.58 | $\mathbf{9 9 . 3 8}$ | 95.00 |

Table 3: Recognition performance. ( $\mathrm{D}=21, \mathrm{HA}$ : Hard Assignment, SA: Soft Assignment).

| Test | SymPaD, withFlat |  | SymPaD, withoutFlat |  | Dense SIFT + BoW |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | HA | SA_Rank(1:4) | HA | SA_Rank(1:4) |  |
| coil20_rand_tr6 | $82.69 \pm 1.96$ | $85.27 \pm 1.93$ | $83.65 \pm 1.92$ | $\mathbf{8 4 . 3 7} \pm \mathbf{2 . 1 3}$ | $83.06 \pm 1.87$ |
| coil20_rand_tr12 | $90.81 \pm 1.39$ | $92.54 \pm 1.00$ | $92.19 \pm 1.16$ | $\mathbf{9 2 . 9 7} \pm \mathbf{1 . 1 1}$ | $90.25 \pm 1.17$ |
| coil20_rand_tr24 | $95.77 \pm 0.72$ | $96.94 \pm 0.48$ | $97.22 \pm 0.56$ | $\mathbf{9 7 . 6 6} \pm \mathbf{0 . 4 5}$ | $95.39 \pm 0.99$ |
| coil20_seq_tr6 | 85.83 | 88.56 | 87.65 | $\mathbf{8 9 . 3 9}$ | 86.66 |
| coil20_seq_tr12 | 94.5 | 95.83 | 94.92 | $\mathbf{9 6 . 2 5}$ | 95.00 |
| coil20_seq_tr24 | 97.29 | 97.71 | 98.65 | $\mathbf{9 8 . 9 6}$ | 98.54 |

Table 4: Overall comparison.

| Test | SymPaD, Without Flat, SA_Rank(1:4) |  | Dense SIFT + BoW | Results in (Shekar, et. al, 2013) |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | D $=8$ | D $=21$ |  | KID | SIFT | SURF | ORB |
| coil20_seq_tr6 | 90.91 | 89.39 | 86.66 | 86.97 | 84.47 | 48.94 | 81.81 |
| coil20_seq_tr12 | 97.00 | 96.25 | 95.00 | 96.67 | 93.42 | 72.00 | 92.25 |
| coil20_seq_tr24 | 99.38 | 98.96 | 98.54 | 98.75 | 96.56 | 83.33 | 95.73 |

approaches do not need to be tuned for databases of different image understanding applications, we believe that a carefully designed system would be a solution for generalizability. We worked in single scale in this study and it is a fact that some of the PSL shape structures such as circulars or ellipticals, or the star shape generated by $f_{14}$ and $f_{15}$ could only be met explicitly at some particular scales. Hence, as a future work, we intend to examine the method in multiscale. Moreover, we plan to extend the dictionary by (i) judiciously quantizing the shape parameter space to generate the shape varieties, (ii) deriving new shape classes by various combinations of the current shape classes that would be filtered by a feature selection method. Finally, we need to consider the localization of the descriptors on the image in a more accurate coding scheme. SymPaD will also be examined in various databases as a future work.

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