Study of Energy Evaluation Control

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Abstract: One of the objectives in control theory is to ensure that a control system converges to a target state in the shortest possible time. To achieve that objective, we studied a control method that combines the Lagrangian, the Hamiltonian, and a bang-bang controller. Referred to as energy evaluation control (EEC), this method evaluates the control state using the Lagrangian, and evolves the control output using the Hamiltonian. Here, the Lagrangian and the Hamiltonian are defined for the deceleration field. The control result from the EEC is fast and robust. Moreover, EEC has the same control strategy as the sliding-mode control, and hence can be incorporated within it.

1 INTRODUCTION

The Lagrangian and Hamiltonian have been used in control theory to establish robust control methods (Bloch, et al., 2000), (Choi, et al., 1997). In addition, the control logic by using these energys are examined, and the application experiment to the inverted pendulum is conducted (Fantoni et al., 2000), (Ortega, and Spong, 2000),(Ortega, et al., 2000).

Moreover, one objective in control execution is to direct the control system towards target states in the shortest possible time. A bang-bang controller is one that is able to realize this objective. However, a bang-bang controller has one drawback in that it does not perform well when external forces are changed. Although in examining the control rule, which determines the feedback of a state quantity to the change in output of the bang-bang controller performed on the control object, the output is changed in the second-half of the cycle and has a slight complicated control structure (Vakilzadeh and Keshavarz, 1982).

In this report, we studied a control method which determines the control output of the bang-bang controller using the Lagrangian and its convergence control output using the Hamiltonian. We refer to it as the "energy evaluation controller" (EEC) because this controller evaluates the energy of the control state. The features of EEC is that a formula can be made simpler than the conventional energy method, and that a control result becomes the shortest time control because EEC based on the bang-bang controller.

We begin by explaining the control rule of the EEC for the simple control model. Next, we propose the adjustment method for the external force of the damper and friction. Finally, we explain that the EEC as a kind of sliding-mode control (SMC).

2 SWITCHING OF CONTROL OUTPUT USING THE LAGRANGIAN

A bang-bang controller is a controller that using maximum thrust enables abrupt changes in state through acceleration and deceleration of a controlled object. A simple model of the bang-bang controller (Fig. 1) and its control cycles (Fig. 2) assumes that the actuator can generate a fixed thrust $\pm F_{max}$. The notation and significance of the variables are: x: stroke, : velocity, X: target position, T: kinetic energy and $-F_{max}$(X-x): braking work. The switching of the actuator thrust is the instant when the braking energy and the kinetic energy are equal.
Next, the Lagrangian is calculated for the deceleration field which is generated by the thrust from the deceleration force of the actuator. Here, the reference position of the deceleration field is set to a target position X. The Lagrangian L is defined as the

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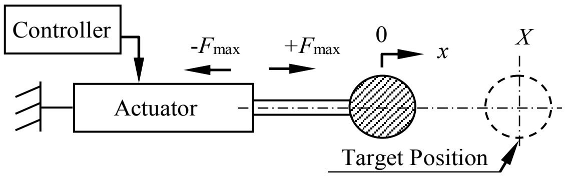

Figure 1: Simple model of a bang-bang controller.

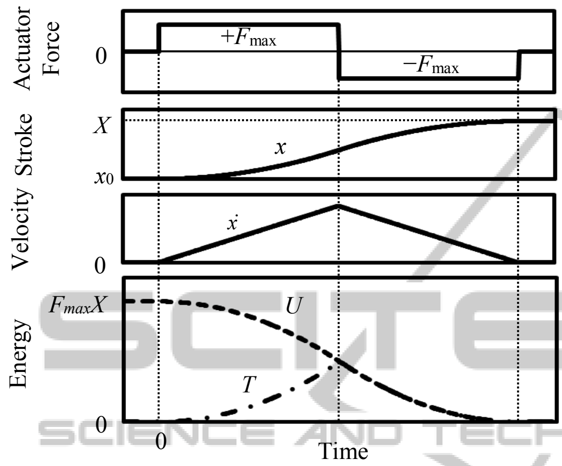

Figure 2: Operation cycle of the bang-bang Controller.

difference between kinetic energy T and potential energy U:

$$L = T - U \tag{1}$$

$$T = \frac{1}{2} m\dot{x}^2 \tag{2}$$

$$U = \int_X^x (-F_{\max})dx. \tag{3}$$

The potential energy of the decelerating field corresponds to the possible work during braking. Therefore the switching time to apply thrust from the actuator is the instance when the value of the Lagrangian takes value 0. It is necessary to consider the direction of motion of the controlled object to decide the control output. Hence the sign (plus or minus) of the velocity is added to the kinetic energy, and (1) is modified to:.

$$L' = \frac{\dot{x}}{2|\dot{x}|} m\dot{x}^2 - \int_X^x (-F_{\max})dx. \tag{4}$$

The sign of the Lagrangian L' then determines the direction of the control output u:

$$u = \frac{L'}{|L'|} F_{\max}. \tag{5}$$

Here F_{max} is the maximum thrust that can be generated by the actuator. The control is simple and seems to work fast. However, in actual control systems, it does not work properly because the estimation error for the Lagrangian is generated by the external force that cannot be predetermined. To address this issue, calculations of the Lagrangian and the control output are done repeatedly at every control cycle, in trying to move the controlled object towards the target position despite the estimation errors associated with the Lagrangian.

For the control target illustrated in Fig. 3, we obtain the EEC control result presented in Fig. 4. The simulation settings are actuator thrust: $\pm 100(\text{N})$, mass: 1(kg), initial position:-1(m), target position: 0(m). Coefficient of damping C(Ns/m) and frictional force F_f (N) is set as follows: Case A (C=10, F_f=10), Case B (C=20, F_f=0), Case C (C=0, F_f =70),. Furthermore, switching of the control output is assumed to occur abruptly. The simulation result shows that the EEC can control the object to the target and it's result seems robust. At the same time, this simulation result shows that the control out put is not shape of ideal bang-bang controller's output.

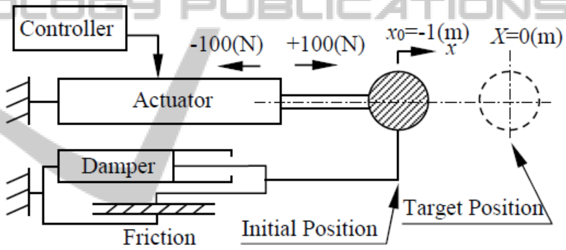

Figure 3: System of control target with dumper.

3 CORRECTION OF THE POTENTIAL ENERGY

EEC can converge the control object to the target position even if the external force act on the control ovject. However, hunting of control output occurs, as shown in Fig. 4. This hunting is caused by the estimation error associated with the potential energy of the deceleration field which generated by the damping force and frictional force. On the other hand the braking work done by damping force W_d is calculated from the equation (6) and the braking work done by frictional force W_f is given by (7). Here the Coulomb's friction model is adopted and the coefficient of friction F_f is assumed as constant. Therefor the potential energy is corrected to (8). Figure 5 shows the simulation result of using equation (8). The simulation result shows that the bang-bang control is almost realised by this correction.

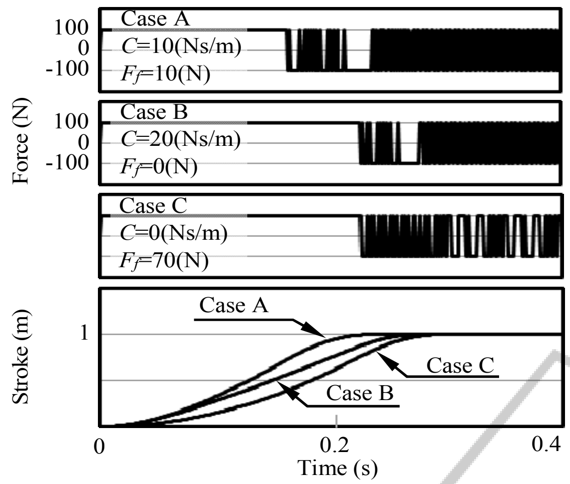

Figure 4: Control result of the EEC Simulation conditions are changed over 3 cases.

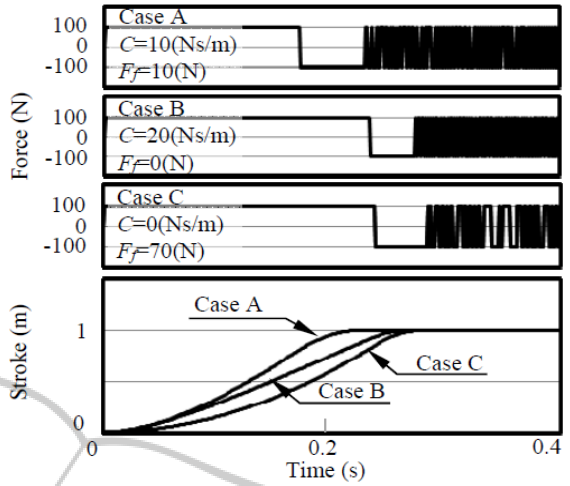

Figure 5: Control result of the EEC Effect of the potential energy correction potential energy.

$$W_c = C x_d \dot{x}_d \tag{6}$$

$$W_f = F_f x_d \tag{7}$$

$$U = \int_X^x (-F_{\max}) dx + W_c + W_f \tag{8}$$

C : coefficient of damper, F_f : frictional force.

4 CONVERGENCE OF CONTROL OUTPUT USING THE HAMILTONIAN

Figure 5 also shows the actuator output after the controlled object has reached its target state. To reduce energy consumption, it is desirable to stop the control output after control finishes (velocity is zero and positional deviation is zero). Here, the Hamiltonian H for the deceleration field (9) is introduced into the EEC procedure. As the Hamiltonian represents the total energy of the deceleration field, takes the value zero when the object velocity and positional deviation become zero. Therefore, to stop the actuator thrust at the end of the control period, the convergence rate of the actuator thrust must be determined; this is achieved by multiplying the Hamiltonian by the actuator thrust $F_{\max}$ and the gain K_h. To take into account the upper and lower limits of the control output, HK_h is restricted to values between 0 and 1. Therefore, the formula expressing the thrust of the actuator is modified to (10). Because of this, the Lagrangian is also corrected to (11). Finally, the control output is modified to give (12)

$$H = \frac{1}{2} m\dot{x}^2 + \int_X^x (-F_{\max}) dx + W_c + W_f \tag{9}$$

$$F'_{\max} = H K_h F_{\max} \quad (0 \le H \cdot K_h \le 1) \tag{10}$$

$$L'' = \frac{\dot{x}}{2|\dot{x}|} m\dot{x}^2 - \left(\int_X^x F'_{\max}\, dx + W_c + W_f \right) \tag{11}$$

$$u = \frac{L''}{|L''|} F'_{\max} \tag{12}$$

Because $F_{\max}$ is changed in the next control step, the Hamiltonian also changes:.

$$H = \frac{1}{2} m\dot{x}^2 + \int_X^x -F'_{\max}\, dx + W_c + W_f \tag{13}$$

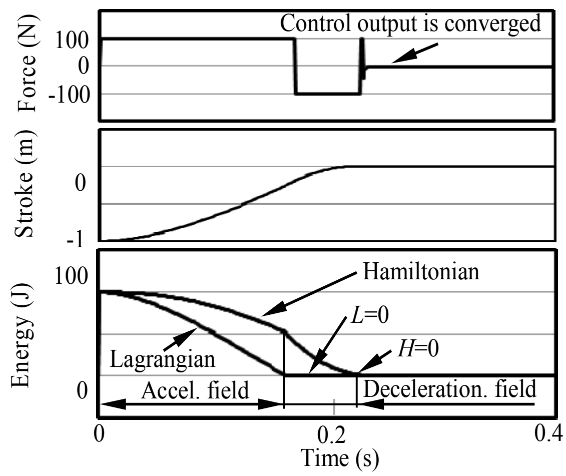

Figure 6: Using the Hamiltonian to control output convergence.

Convergence of the control output using the Hamiltonian (Fig. 6) shows that the control output has converged to zero when the control converges. On the other hand the Lagrangian is restrained to 0. It is means that the control object reached to the target position according to the deceleration field.

5 CONTROL STRATEGY OF ECC

Figure 7 shows the phase plane, obtained using (7), that determines the control output in EEC. For comparison, a switching phase plane for SMC is sho wn in Fig. 8. SMC has a switching phase plane of control output that is similar to EEC (Efe, et al., 2000), and hence these two methods have similar control structures. Usually, the displacement and velocity are associated with the axes of the phase plane of SMC. However, other axes can be used because SMC can be extended to any dimension. This implies that the EEC is included in SMC.

EEC has good characteristics in that the gradient of the output switching line is always set to −1, because the kinetic energy and braking energy have the same units (J). Moreover, the Lagrangian is restricted to zero within the SMC context. This behavior indicates that the controlled object will move naturally. Thereby EEC can simplify the problem of control, as mentioned above.

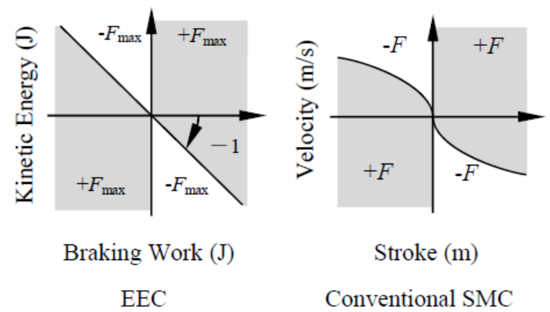

Figure 7: Phase plane for EEC and conventional SMC.

6 CONCLUSIONS

EEC was proposed that switches the control output using the Lagrangian and uses the Hamiltonian to converge the control output. And we proposed the correction method for the damping force and frictional force. Finaliy the control output of EEC becams almost same as the bang-bang controller's output. Therefor the EEC can control the controlled object in shortest time.

EEC was found to have the same phase space structure as SMC and hence is included to the SMC. However, the EEC has practical advantages in that the phase plane can be simplified by choosing energy as one of the axis variables in the phase plane for the control output. In consequence, the output switching line for EEC is the diagonal of gradient −1.

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